TABLE IV. Calculated values of $R^3q'(R)$ in atomic units.

| $R$ | $R^3q'(R)$ |
|---|---|
| 1.2 | 0.603,69 |
| 1.3 | 0.547,18 |
| 1.4 | 0.488,86 |
| 1.5 | 0.430,96 |

TABLE V. Values of the Dunham[a] coefficients as calculated from spectroscopic data.[b] $a_0$ is in $\text{cm}^{-1}$.

| | HD | $D_2$ |
|---|---|---|
| $a_0$ | 79,795 | 79,873 |
| $a_1$ | $-1.608{,}2$ | $-1.589{,}2$ |
| $a_2$ | $+1.846{,}4$ | $+1.750{,}7$ |
| $a_3$ | $-1.840$ | $-1.708$ |
| $a_4$ | $+1.664$ | $+2.07$ |

[a] See reference 6.
[b] See reference 7.

found in the value for HD and $D_2$. The values of $q'$ given above are consistent with this observation. Taking the average of the two values of $q'$ for HD and $D_2$, $Q_D$ is found to be $2.766\times10^{-27}$ cm². On the basis of the errors discussed in Section 3, a probable error of 0.6 percent is assigned to $q'$. Combining this with the error of 0.6 percent in H''' gives $\sqrt{2}(0.6)=0.9$ percent as the probable error in $Q_D$:

$$Q_D=(2.766\pm0.025)\times10^{-27}\ \text{cm}^2. \tag{10}$$

This differs from the value $2.73\times10^{-27}$ cm² ±2 percent obtained by Nordsieck[2] chiefly because the latter performed the average over the zero-point vibration using a Morse function fitted to give the correct binding energy. Though the new value does not differ much from the value $2.79\times10^{-27}$ cm² obtained by Ishiguro, the values of $q'(1.4)$ differ by as much as five percent. On the basis of the James-Coolidge density values of Table III, the errors in Ishiguro's density are very badly distributed in the vicinity of the nucleus. The errors are negative for points $e$ and $b$, presumably also for points in between, whereas the errors at $a$ and $c$ are both definitely positive. The percentage error in $q(R)$ can be considerably larger than the percentage error in $\rho$ due to the cancellation in the integral of the spherically symmetric part of the distribution. Thus one can account for the five percent difference in the values of $q'(1.4)$. That the two values of $Q_D$ are so close is partly due to the 0.7 percent correction of the new value made in Section 4 and partly because Ishiguro used a Morse function for his average over the zero-point vibration, probably the same one used by Nordsieck.

The author wishes to thank Professor A. Nordsieck for suggesting the problem and for helpful advice.

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# A Hodoscope Study of Penetrating Cosmic-Ray Showers. I. Local Showers*

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Showers produced locally in a lead absorber were studied with a hodoscope arrangement. It is shown that, in order to record "local penetrating showers," a rigid selection demanding high penetrating power of the secondaries as well as of the primary must be used. If this is done, some of the discrepancies of recent investigations are removed. For showers capable of penetrating at least 200 g/cm² Pb one finds a collision mean free path of (162±10) g/cm², and for showers of at least 100 g/cm² penetration a mean free path of (196±13) g/cm². Besides, "local soft showers" due to electronic or photonic secondaries of ordinary $\mu$-mesons were studied. Their frequency at 3260-m elevation is about 1 in $10^5$ traversals of the lead absorber.

## I. THE EXPERIMENTAL TECHNIQUE

IN its present stage, work on penetrating showers is particularly strongly affected by the limitations of current experimental methods. Counter arrangements, though suitable for detection and for selection of showers, cannot usually provide a unique answer to questions concerning the nature of the particles involved, or concerning details of their interactions. Cloud chambers, on the other hand, could adequately answer this purpose, but it is troublesome and difficult to stack in them a sufficiently large amount of absorber, and they tend to become slow in operation because of the inevitably long interval between successive expansions of a large chamber, and its equally inevitable unfavorable solid angle of detection. It appeared, therefore, that in this field the use of what one might consider a compromise technique, namely a hodoscope arrangement, is well justified. It offers speedier work than a cloud chamber, as better solid angles and recovery times can be obtained, and it offers more extended evidence than a simple counter arrangement would reveal, though, of course, it is inferior in this respect to the cloud chamber.

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* The expense of constructing the equipment and of running the experiment were partly provided by an AEC contract.

In deciding upon the experimental arrangement one might consider still another point. For the purpose of analyzing the shower phenomena in a hodoscope set it would appear desirable to use as large a number of narrow counters as possible, thus approaching the picture recorded in a cloud chamber with a single non-stereoscopic camera. This, however, has obviously two serious drawbacks. The selection of the events under investigation is usually based on a suitable "master pulse" formed by a certain group of counters. Using a large number of narrow hodoscope counters to define the master pulse one might then frequently find it difficult to eliminate undesired secondary effects without rejecting too many showers of the type one wishes to study. Furthermore, it necessitates an unduly complicated electronic circuit, thus again reducing the efficiency of the arrangement because of the inevitable time losses due to faults which these circuits develop during a long period of operation.

For the purpose of the present experiment it was decided, therefore, to use independent "master" and "hodoscope" counters. Groups of larger counters were used to select the showers, and additional narrow counters to indicate the number and the paths of the particles present in the shower. With a large number of counters, this apparent complication actually results in a simplification of the arrangement.

A schematic view of the arrangement is given in Fig. 1. The counter trays *A*, *B*, and *C* contain the "master pulse" counters of dimensions 16×1 in. in *A* and *B*, and 24×2 in. in *C*. Trays *a*, *b*, and *c* operate on the neon lamp system; their diameters are $\frac{1}{2}$ in. in *a* and *b*, and 1 in. in *c*. The thickness of the top absorber, $\Sigma_1$, was varied in the different runs of the experiment between 0 and 350 g/cm$^2$. The absorber $\Sigma_2$ remained at 100 g/cm$^2$ throughout the experiment; $\Sigma_3$ was 200 g/cm$^2$ during the first, and 100 g/cm$^2$ during the second series of recordings. An additional thin absorber sheet, either $\frac{1}{2}$ in. of lead or $\frac{3}{4}$ in. of iron, was placed between "master pulse" counters and "hodoscope" counters at all three levels. Heavy shielding by 6 to 7 in. of lead protected the counter trays against side showers.

## II. SELECTION OF LOCAL SHOWERS

In several recent papers[1-3] measurements of the collision mean free path of charged particles producing local penetrating showers have been reported. The results, however, were conflicting: thus, while Tinlot and Gregory,[1] working at 4300-m altitude, find an exponential decrease of the shower rate with absorber thickness and a mean free path of 350 g/cm$^2$ for lead and 200 g/cm$^2$ for iron, Cocconi's[2] experiments indicate a variation of the collision mean free path with absorber thickness. At 3260-m altitude, for instance, he finds in lead between 0 and 500 g/cm$^2$ absorber thickness collision mean free paths between 160 and 380 g/cm$^2$, and in iron in the same region a variation of the mean free path between 135 and 310 g/cm$^2$.

It might be noted here that the evaluation of Cocconi's measurements which led him to these striking results is not quite correct and tends to an overestimate of the variation with absorber thickness of the collision mean free path, if such a variation exists. If the absorption curve is not purely exponential, that is, if the collision probability varies with the amount of absorber traversed, this will affect not only the collisions in the top absorber, but also those in the "production layer," and the simple procedure of calculating the collision mean free path as the reciprocal of derivative of the logarithmical intensity curve is no longer adequate, since it supposes that a constant fraction of the incident single particles will produce showers in the production layer. Even so, a re-evaluation of Cocconi's results would only bring the maximum values for his mean free paths down to about 300 in lead, and 250 g/cm$^2$ in iron, thus leaving unexplained the disagreement both in shape and in slope of his absorption curves and those of Tinlot and Gregory.[1]

Both these experiments, however, are open to another more serious criticism. The "penetrating showers" recorded were events in which a single penetrating particle strikes the top counter tray, and two or more particle counters of the lower tray, separated from each other by a few inches of lead and from the top tray by a heavier shield, 4 or 6 in. of lead. The selection thus guarantees the penetrating nature of the primary par-

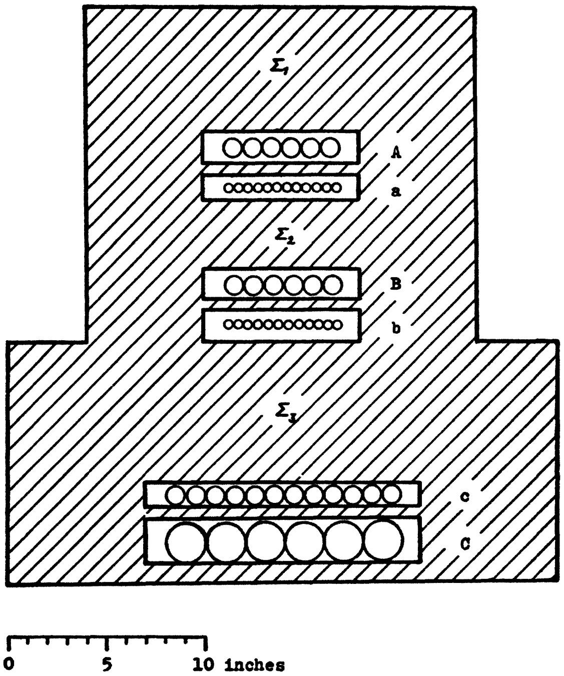

FIG. 1. Schematic diagram of the experimental arrangement. The "master pulse" counters *A*, *B*, and *C* select the events, the "hodoscope" counters *a*, *b*, and *c* operate the neon lamp system.

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[1] J. Tinlot and B. Gregory, Phys. Rev. **75**, 519 (1949).
[2] G. Cocconi, Phys. Rev. **75**, 1074 (1949).
[3] W. B. Fretter, Phys. Rev. **76**, 511 (1949).

TABLE I. Rate of local penetrating showers as function of the top absorber thickness $\Sigma_1$, taken for two different values of the bottom absorber thickness $\Sigma_3$.

| Event / Absorber thickness $\Sigma_1$ | | LPS | | | LPS+*E* | | |
|---|---|---|---|---|---|---|---|
| | | Counts | Time (hr.) | Rate/hr. | Counts | Time (hr.) | Rate/hr. |
| 0 | $\Sigma_3=200$ | 411 | 107.75 | 3.82±0.18 | Not recorded | | |
| | $\Sigma_3=100$ | 479 | 103.25 | 4.64±0.21 | | | |
| 170 | $\Sigma_3=200$ | 200 | 149.5 | 1.34±0.08 | 2 | 140.5 | 0.014 |
| | $\Sigma_3=100$ | 272 | 142.5 | 1.91±0.12 | 3 | 140.0 | 0.021 |
| 270 | $\Sigma_3=200$ | 120 | 164.75 | 0.73±0.06 | 1 | 161.0 | 0.006 |
| | $\Sigma_3=100$ | 195 | 172.25 | 1.13±0.05 | 2 | 163.0 | 0.012 |
| 350 | $\Sigma_3=200$ | 82 | 189.25 | 0.43±0.05 | 0 | 187.5 | 0.000 |
| | $\Sigma_3=100$ | 149 | 188.75 | 0.79±0.06 | 1 | 185.5 | 0.005 |

ticle, but not of the shower particles. It appeared quite possible that at least some of the showers recorded with such an arrangement would not be "penetrating showers" at all, but consist of a penetrating particle with soft secondaries. Therefore, the main purpose of the present experiment was to repeat absorption measurements of local showers under conditions of more rigid control and separation between these events.

Cocconi, continuing his experiments at sea level and underground, has more recently[4] come to a similar conclusion. He concludes that his arrangement recorded an appreciable component of showers, probably of electronic nature, produced by ordinary $\mu$-mesons. At the time when the experiment reported here was prepared these conclusions were not yet known; nevertheless, the separation into "real" penetrating showers and soft showers produced by penetrating primaries, as attempted in the present hodoscope study, can be expected to yield a pertinent check for the corrections deduced in his paper. In the following, the notation "local penetrating shower" (LPS) for the first type, and "local soft shower" (LSS) for the second type of events will be used.

For the selection of LPS it is essential to use all three counter trays *A*, *B*, and *C* in Fig. 1. An event was called an LPS if only one counter in *A*, at least two counters in *B*, and at least three counters in *C*, were struck. For this selection, tray *B* was subdivided into two groups of three not-neighboring counters, and it was demanded that at least one counter in each group be triggered. No such subdivision of tray *C* was deemed necessary; any event in which at least three of its counters were struck was registered. Using these precautions one makes practically certain that all the showers recorded were produced in the middle absorber $\Sigma_2$, as of the few which might originate in $\Sigma_3$ and emit backward particles capable of penetrating $\Sigma_3$ and the additional $\frac{1}{2}$ in. of lead between the trays *B* and *b*, still only one-half will strike the right group of the *B* counters. In fact, the hodoscope photographs showed that only about one in 20 of the LPS came from $\Sigma_3$, so that it is a good picture to consider $\Sigma_2$ as the producing layer, while $\Sigma_3$ determines the penetrating power one wishes to demand for the shower particles. As a possible variation of the collision mean free path with absorber thickness would be much more clearly exhibited if a thin producing layer was used, and would disappear for very thick $\Sigma_2$, a layer of 100 g/cm$^2$ was chosen. In two series of measurements with identical counter geometry and production layer the penetration conditions were then varied by using for $\Sigma_3$ absorbers of 200 and 100 g/cm$^2$, respectively.

If an LPS was accompanied by an electron shower striking at least three counters of an extension tray *E*, an additional neon lamp was lit. Tray *E* consisted of six counters of 2 in. diameter and 24 in. length, and was placed near the "*P*-set" shown in Fig. 1. These mixed showers will be called "events LPS+*E*."

In the events selected as LSS only one penetrating particle was required, if it was accompanied by at least two secondaries at either tray *B* or tray *c*. The tray of "hodoscope" counters *c* had to be used in order to arrange identical conditions in both levels. Like tray *B*, *c* was subdivided into two groups of not-neighboring counters, and a shower was recorded if all three counters of a subgroup of *B*, or at least three counters of a subgroup of *c*, were struck together with one counter in each of the two other levels. As tray *c* was placed on top of the 2-in. counter tray *C* directly under the absorber, the LSS were required to extend over at least 3 in. in lead.

All the electronic circuits used were of the conventional type, with crystal diodes applied both in the mixing circuits and in the coincidence sets. The hodoscope was operated by applying the master pulse to the

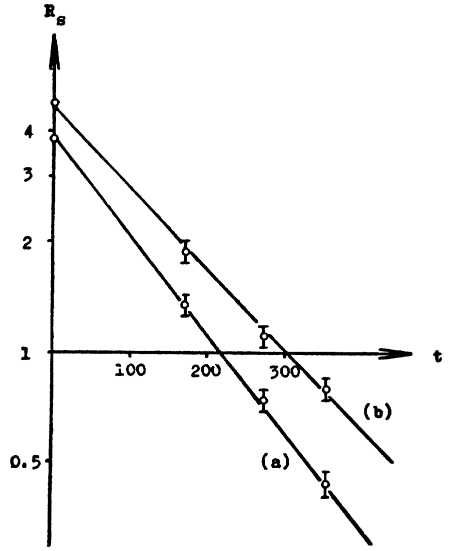

FIG. 2. Rate of local penetrating showers per hour, as function of the absorber thickness *t* (g/cm$^2$ Pb). Curve (a): Minimum penetration 200 g/cm$^2$ Pb. Curve (b): Minimum penetration 100 g/cm$^2$ Pb.

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[4] G. Cocconi, Phys. Rev. **76**, 984 (1949).

suppressor grid, and the pulses from the indicator counters to the control grid of 6SJ7 tubes. The lengthened pulses were then led to the grid of another tube, with a neon lamp connected in parallel across the plate resistance. Tests with artificial pulses proved an efficiency and reliability of the neon lamp system of more than 99 percent. The experiment was carried out at Echo Lake, Colorado, at 3260-m altitude.

## III. RESULTS ON LOCAL PENETRATING SHOWERS

It has been shown above that the arrangement recorded mostly showers produced in $\Sigma_2$. For the purpose of this experiment, moreover, the knowledge of the exact point of origin is irrelevant; no attempt was thus made to correct for the few events generated in $\Sigma_3$. Corrections for threefold knock-on showers with one secondary at $B$ and two at $C$ are small because of the 2-in. separation at $C$, but not negligible. They have been computed from observations made with the same arrangement on single penetrating particles in air showers.

Another correction which might play an important part in some cases, but proves to be negligibly small in the present experiment, is the difference between the knock-on events in lead and in air. The incident knock-on accompaniment is widely spread out if no lead shield covers the top tray, and only very rarely a penetrating particle will be rejected because one of its knock-on secondaries strikes a top counter. In lead, however, the lateral spread is much reduced and if the counters are not placed directly under the absorber one might lose an appreciable fraction of the penetrating events. With the tight arrangement of Fig. 1, a check on knock-on events observed in the hodoscope pictures reveals that this effect amounted to less than one percent of the number of penetrating particles.

Measurements were performed with four thicknesses of $\Sigma_1$: 0, 170, 270, and 350 g/cm². During the first series of the experiment the bottom absorber was 200 g/cm², during the second 100 g/cm². The observed rates, corrected for knock-on showers, are summarized in Table I. To each value of the top absorber thickness in column 1, the upper line refers to the first, the lower line to the second series of measurements.

The LPS-rates are also plotted in the graphs of Fig. 2. Here curve (a) refers to the first series, with a minimum penetration exceeding 200 g/cm², and curve (b) to the second, with $\Sigma_3 = 100$ g/cm². In a logarithmic plot, both can be represented well by a straight line, but with a slightly different slope. From (a) one obtains a collision mean free path

$$\lambda = (162 \pm 10)\ \text{g/cm}^2,$$

while (b) leads to

$$\lambda' = (196 \pm 13)\ \text{g/cm}^2.$$

It should be noted that even in this second series, though the separation between $B$ and $C$ was in total only 4½ in. of lead, or not much more than the lateral counter separation in Cocconi's experiment, one has good reason to expect no appreciable admixture of LSS of the kind which distorted these earlier experiments. The reason is that here three secondaries were required, against two in the Cocconi experiment.

TABLE II. Rate of local soft showers as function of the top absorber thickness $\Sigma_1$, taken for two different values of the bottom absorber thickness $\Sigma_3$.

| Event / Absorber thickness $\Sigma_1$ | | Time (hr.) | Counts | Tray $B$ Rate/hr. | Counts | Tray $c$ Rate/hr. |
|---|---|---|---|---|---|---|
| 0 | $\Sigma_3=200$ | 107.75 | 9 | 0.084±0.028 | 7 | 0.068±0.026 |
| | $\Sigma_3=100$ | 103.0 | 6 | 0.058±0.024 | 8 | 0.078±0.027 |
| 170 | $\Sigma_3=200$ | 140.5 | 9 | 0.064±0.021 | 10 | 0.071±0.022 |
| | $\Sigma_3=100$ | 140.0 | 10 | 0.071±0.022 | 10 | 0.071±0.022 |
| 270 | $\Sigma_3=200$ | 161.0 | 8 | 0.050±0.018 | 11 | 0.068±0.020 |
| | $\Sigma_3=100$ | 163.0 | 14 | 0.086±0.023 | 9 | 0.056±0.019 |
| 350 | $\Sigma_3=200$ | 187.5 | 14 | 0.075±0.020 | 12 | 0.065±0.019 |
| | $\Sigma_3=100$ | 185.5 | 11 | 0.059±0.018 | 11 | 0.059±0.018 |

The hodoscope pictures were furthermore used to compare the numbers of recorded particles in the two series. Again one finds a slight but unmistakable difference: The average number of $c$ counters struck in the first series was

$$\langle n_{200} \rangle = 4.18 \pm 0.18,$$

and with the thinner absorber

$$\langle n_{100} \rangle = 4.68 \pm 0.17.$$

This difference, moreover, is apparently not, or not entirely, due to a new group of showers of more particles and smaller penetrating power, but rather to a gradual shift in the distribution curve. One finds in the second series fewer showers with the minimum number of three penetrating particles: 105 in 1042 analyzed showers, against 201 in 751 analyzed showers of more than 200 g/cm² penetration. Thus it must be concluded that in penetrating showers in lead the fraction of particles with ranges between about 150 and 250 g/cm² is by no means negligible.

As to the penetrating showers that are accompanied by air showers, Table I shows that the correlation is only about one percent. This result is in satisfactory agreement with earlier observations of Tinlot and Gregory;[5] it will be discussed in the following analysis of the air shower experiment.

Finally, one can compute the directional intensity of the charged radiation that produces hard showers in lead. Unfortunately, the uncertainty of this estimate is much larger than the statistical error, due to the difficulties encountered in correctly accounting for geometry and zenith angle distribution. The values derived from the data given above are $(1.6 \pm 0.3) \times 10^{-5}$ cm$^{-2}$ sec.$^{-1}$ sterad.$^{-1}$ for the more penetrating, and $(2.2 \pm 0.3) \times 10^{-5}$ cm$^{-2}$ sec.$^{-1}$ sterad.$^{-1}$ for the less penetrating

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[5] J. Tinlot and B. Gregory, Phys. Rev. 75, 520 (1949).

showers.† These figures are only slightly higher than those obtained by Hazen, Randall, and Tiffany[6] with a very different experimental method, and agree very well, as far as can be checked, also with the observations of Broadbent and Jánossy,[7] of Cocconi[2,4] and of others.

## IV. RESULTS ON LOCAL SOFT SHOWERS

In the study of LSS two striking features became soon apparent: First, the hodoscope records of the *b* tray seemed frequently in disagreement with the selection which demanded at least three counters of the *B* tray struck. Very many photographs showed that only one or two of the *b* counters discharged. (No such effect could be observed at the lower level, where the hodoscope counters *c* were also used to transmit the master pulse of the LSS records.) Second, there was still much less correlation between these LSS and air showers; in fact, during almost 1000 hr. of observations only one such event (LSS+$E$) was recorded.

There could be no doubt that the apparent unreliability of the *b* tray was "selective:" it continued to record reliably everything but the LSS events. It became evident, therefore, that the secondaries produced in the LSS are either very weakly penetrating indeed so that they are partly removed by the $\frac{1}{2}$-in. lead plate between *B* and *b*, or that they are very inefficiently recorded by G-M counters, or, of course, both.

In order to investigate this point, tray *a* was placed during the second series directly under *B*, so that the $\frac{1}{2}$-in. absorber now separated the two hodoscope trays *a* and *b*. For about half the time of each run this absorber was then replaced by a $\frac{3}{4}$-in. iron plate.

The total numbers of LSS recorded and the counting rates per hour are given in Table II. Again the upper of the two lines to each value of $\Sigma_1$ refers to the series with $\Sigma_3=200$ g/cm$^2$, and the lower to $\Sigma_3=100$ g/cm$^2$.

The most striking feature in these results is the insensitivity of the shower rate to absorber thickness, and that both for the top absorber $\Sigma_1$ and for the bottom absorber $\Sigma_3$. This, and the absence of a correlation between LSS and air showers, prove that the primaries which generate these local showers must be $\mu$-mesons of very high energies. If one compares the rate of LSS with the total flux of single penetrating particles through the arrangement (about 200/min.) one computes that about one in $10^5$ $\mu$-mesons originate either near the *B* tray or near the *c* tray a LSS of the type recorded. Radiation processes and high energy knock-on secondaries may well account for such a rate of soft showers.

During the first run, with $\Sigma_3=200$ g/cm$^2$, the pictures of 40 LSS discharging the *B* tray revealed altogether 59 activated counters of the *b* tray, under $\frac{1}{2}$ in. of lead. During the second run, with $\Sigma_3=100$ g/cm$^2$, 19 showers originating at *B* were observed with 28 *b* counters discharged when the same $\frac{1}{2}$ in. lead shield was used, and 46 particles in 22 showers when the lead was replaced by $\frac{3}{4}$ in. of iron. Thus, the number of particles recorded per shower was $1.47\pm0.16$ under $\frac{1}{2}$ in. of lead, and $2.09\pm0.30$ under the iron shield of equivalent mass. Tray *a*, placed above these shields and immediately under *B* during this second run, showed 44 counters discharged in the 19 showers with the lead shield, and 54 in the 22 showers with the iron shield: rates of $2.3\pm0.35$ and $2.45\pm0.35$, respectively. This latter fact, that even with no absorber between the master pulse tray *B* and the hodoscope tray *a* less than the three particles which discharge *B* are registered in the adjacent tray, indicates that the counters are not fully efficient in recording these showers. Together with the strong absorption in lead, and much less strong absorption in iron, this evidence proves that the soft secondaries in these showers are electrons and photons rather than of nucleonic nature.

## V. SUMMARY

The results of the present investigation stress the necessity of a more rigid selection of "penetrating" events. If at least three particles capable of penetrating more than 200 g/cm$^2$ are demanded, the cross section for shower production is of the order of the geometrical nuclear cross section. These showers contain in average about 4.2 such highly penetrating particles, but also others of a penetration of between 100 and 200 g cm$^{-2}$. This corresponds to a total kinetic energy of all charged shower particles of at least 2 to 3 Bev, with the lower figure valid if most of the particles are mesons, and the higher if they are protons. If only 100 g/cm$^2$ penetration is demanded, or in other words a minimum kinetic energy of 1.5 to 2.5 Bev, the cross section is somewhat smaller, and the number of particles recorded about 4.7. It would be interesting to restrict the measurements to that lower energy region and thus to determine the cross section for low energy nuclear interactions: this was, however, not possible from the data of the present experiment.

If the penetration requirements are still more reduced, extra-nuclear interactions of ordinary $\mu$-mesons resulting in the production of a local soft shower of electrons and photons become increasingly important.

In conclusion the author wishes to express his gratitude to the Inter-University High Altitude Laboratories, and in particular to Drs. Cohn and Iona of the University of Denver, for the permission to use the facilities at Echo Lake Laboratory and for their help.

## APPENDIX. ON THE ABSORPTION OF THE PARTICLES THAT PRODUCE PENETRATING SHOWERS

In the following an attempt is presented to derive, from a summary of empirical data and from a few plausible general assumptions, a number of characteristics of fast nuclear collisions

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† The zenith angle distribution was assumed to follow a $(\cos\theta)^6$ law which was well compatible with observations, and theoretically the most likely correct distribution at a depth of 700 g/cm$^2$, or about six absorption mean free paths, under the top of the atmosphere.

[6] Hazen, Randall, and Tiffany, Phys. Rev. **75**, 694 (1949).

[7] D. Broadbent and L. Jánossy, Proc. Roy. Soc. (A)**190**, 497 (1947).

resulting in the production of "penetrating showers." Let it be emphasized that this attempt makes no claim to be a theory of penetrating showers; its aim is only to outline certain general features such a theory would need to embrace, and to deduce certain directives for such a theory.

The mathematical methods which will be used, at this stage, are rather crude. Fluctuations will be neglected, and simplifications, like that of assuming a power law with constant exponent for the energy spectrum of the cosmic-ray primaries, will be made. No further apologies are offered later on whenever such procedures of dubious purity are applied, and no better results can be hoped for than those obtained, for instance, in kinetic gas theory by assuming a uniform velocity of all particles. In a word, it is believed that a more accurate treatment will change many details, but leaves unchanged the basic conclusions.

The essential empirical facts on which any theory must be based are:

(i) The *interaction* cross section for fast nuclear collisions initiating penetrating showers is very nearly equal to the geometrical cross section of the absorber nucleus (e.g., Rossi[8]).[‡]

(ii) The *absorption* cross section is considerably smaller than the interaction cross section. In air, for instance, an absorption thickness of about 120 g/cm$^2$ has been found (e.g., Tinlot[9]), while the interaction mean free path, according to (i), is only about 70 g/cm$^2$.

(iii) The ratio of the intensities of neutral and charged shower primaries is nearly unity both at sea level and at mountain altitudes (e.g., Jánossy and Rochester.[10] Walker.[11]

(iv) The total kinetic energy carried by the charged particles of a penetrating shower is in most experiments at least 2 to 3 Bev.

(v) The total intensity of the charged radiation which originates in lead penetrating showers of the energies quoted in (iv) is at sea level about $0.95\times 10^{-6}$ cm$^{-2}$ sec.$^{-1}$ sterad.$^{-1}$ (Hazen, Randall, and Tiffany;[6] Section III of this paper).

(vi) At sea level, no noticeable geomagnetic latitude effect is found between equator and moderate latitudes (Appapillai and Mailgavanam[12]).

Let us denote the *collision* mean free path by $\lambda$ and the *absorption* mean free path by $\Lambda$, and assume that, in the region of large energies which we are to consider, both are constant. Assume further a power law for the integral energy spectrum of the incident primary radiation:

$$S(E)=(E/E_c)^{-\gamma}$$

with $E_c$=cut-off energy, and $\gamma\sim 1.5$. It will be shown later that we have to deal with an energy region in which this value is plausible. However, a change in the value of $\gamma$ would only affect some of the numerical results found, without invalidating the conclusions drawn from them.

Assume next that on the average each particle in initiating a penetrating shower retains a fraction $\alpha E$ of its initial energy $E$, and transfers a fraction $\beta E$ to each of $m$ secondaries that are capable of further shower production ("$N$ component"). The remainder, $(1-\alpha-m\beta)E$, is dissipated in ejection of slow secondaries ("evaporation"), or of particles which cannot reproduce further penetrating showers. In collisions in air, for instance, the energy transferred to $\pi$-mesons belongs to the "dissipated" fraction, as for all energies below $\sim 10^{11}$ ev their mean free path for decay is much shorter than that for collision. In dense materials $\pi$-mesons belong to the "reproducing" $N$ component.

Assume finally that the collision mean free path of the "reproducing" secondaries is the same as that of the primary incident particle; i.e., no distinction can be made between "real" primaries and their descendants which are still capable of originating hard showers.

Let $E_s$ be the minimum energy a particle must have in order to produce showers of penetrating secondaries. If $E_s \gtrsim E_c$ [which, according to (iv), is plausible at least at moderate latitudes], the number of *primary* cosmic-ray particles participating in shower production at depth $x$ from the top of the atmosphere (if measured in units of the collision mean free path this depth will be denoted by $y=x/\lambda$) is

$$N_s^{(0)}=N_0\{(E_s/E_c)^{-\gamma}\cdot e^{-y}+(E_s/\alpha E_c)^{-\gamma}\cdot e^{-y}\cdot y/1! \\ +(E_s/\alpha^2E_c)^{-\gamma}\cdot e^{-y}\cdot y^2/2!+\cdots\} \quad (1)$$

($N_0$=primary intensity). The sum gives

$$N_s^{(0)}=N_0\cdot(E_s/E_c)^{-\gamma}\cdot\exp[-(1-\alpha^\gamma)y]. \quad (2)$$

Secondaries emitted in preceding collisions contribute, after $n$ "secondary" generations

$$N_s^{(n)}=m^n\cdot N_0\cdot(E_s/E_c)^{-\gamma}\cdot[(\beta^\gamma y)^n/n!]\cdot\exp[-(1-\alpha^\gamma)y], \quad (3)$$

one-half of which can be assumed charged, and one-half neutral. The total number of shower-producing secondaries is then

$$\sum_{n=1}^{\infty} N_s^{(n)}=N_0(E_s/E_c)^{-\gamma}\cdot\exp[-(1-\alpha^\gamma)y]\cdot\{\exp(m\beta^\gamma y)-1\}, \quad (3a)$$

again divided equally into charged and neutral particles. This gives a total intensity of shower-producing radiation.

$$N_s=N_0+\sum_{n=1}^{\infty} N_s^{(n)}=\sum_{n=0}^{\infty} N_s^{(n)}.$$

$$N_s=N_0(E_s/E_c)^{-\gamma}\cdot\exp[-(1-\alpha^\gamma-m\beta^\gamma)y], \quad (4)$$

and if we divide into charged and neutral components, we obtain for the charged radiation

$$(N_s)^p=\tfrac{1}{2}N_0(E_s/E_c)^{-\gamma} \\ \cdot\exp[-(1-\alpha^\gamma-m\beta^\gamma)y]\cdot\{1+\exp(-m\beta^\gamma y)\} \quad (4a)$$

and for the neutral radiation

$$(N_s)^n=\tfrac{1}{2}N_0(E_s/E_c)^{-\gamma} \\ \cdot\exp[-(1-\alpha^\gamma-m\beta^\gamma)y]\cdot\{1-\exp(-m\beta^\gamma y)\}. \quad (4b)$$

These results permit several important conclusions and estimates:

(a) The absorption mean free paths for both charged and neutral particles are, for not too small depths, very nearly the same:

$$\Lambda\approx\lambda/(1-\alpha^\gamma-m\beta^\gamma). \quad (5)$$

In air, with $\lambda\approx 70$ g/cm$^2$ and $\Lambda\approx 120$ g/cm$^2$, we have thus

$$(1-\alpha^\gamma-m\beta^\gamma)\approx 0.6. \quad (5a)$$

(b) The ratio of the intensities of neutral and charged shower primaries is

$$(N_s)^n/(N_s)^p=[1-\exp(-m\beta^\gamma y)]/[1+\exp(-m\beta^\gamma y)], \quad (6)$$

and hence approaches unity at sea level ($y\approx 15$) or mountain altitudes ($y\approx 8-10$) only if an appreciable fraction of the incident energy goes into shower-reproducing secondaries. If we permit a ratio $(N_s)^n/(N_s)^p=0.75$ at mountain altitudes, we have to demand $(m\beta^\gamma)\sim 0.20$ to 0.25. Together with (5a) this gives $\alpha^\gamma\sim 0.15$ to 0.20, or, with $\gamma=1.5$, $\alpha\sim 0.28$ to 0.34. Thus we expect the primary to retain on the average about one-third of its incident energy, and to transfer an about equal amount to shower-producing secondaries, while between one-third and one-fourth of the energy is "dissipated."

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[8] B. Rossi, M.I.T. Technical Report No. 26 (April 4, 1949).

[‡] Professor K. Greisen has kindly informed me that the measurements of the Cornell group give for the collision mean free path in carbon a value which is somewhat larger than that corresponding to the geometrical nuclear cross section of C. This means that light nuclei are apparently not quite opaque. In the following computations no corrections for the partial transparency of air nuclei have been made, but it should be noted that this effect would have very little influence even on the numerical results given later.

[9] J. Tinlot, Phys. Rev. **73**, 1476 (1948); **74**, 1197 (1949).

[10] L. Jánossy and G. D. Rochester, Proc. Roy. Soc. **(A)182**, 180 (1943).

[11] W. D. Walker, Echo Lake Symposium on Cosmic Rays (1949).

[12] V. Appapillai and A. W. Mailgavanam, Nature **162**, 374 (1949).

(c) With these estimates, it is easy to interpret the absence of a geomagnetic effect. Write the number $N_s$ of shower-producing particles from (4) as a sum indicating the number $n$ of collisions the individual particles have suffered:

$$N_s = N_0(E_s/E_c)^{-\gamma}\cdot\exp[-(1-\alpha^{\gamma})y]\cdot\sum_{n=0}^{\infty}(m\beta^{\gamma}y)^n/n!. \tag{4c}$$

As $m\beta^{\gamma}\sim 0.2$, the maximum contribution to $N_s$ comes from particles that have already suffered about two to three previous collisions, and hence must have started with an energy $\gtrsim(E_s/\alpha^{2.5})$, or about 15 $E_s$, when incident on the top of the atmosphere. Thus, being descendants of very energetic primaries, the particles that originate penetrating showers will show no appreciable geomagnetic latitude effect at sea level.

(d) The ratio $(E_s/E_c)$ can be estimated if one compares the observed flux of shower-producing particles with the incident primary flux at the top of the atmosphere. Even if the entire contribution of the heavier particles is neglected, and all local penetrating showers are related to the primary proton flux of about 0.1 $\text{cm}^{-2}$ $\text{sec.}^{-1}$ $\text{sterad.}^{-1}$ and their nucleonic secondaries, one gets from (4a) applied to sea-level data $(x/\Lambda\approx 8.75)$:

$$(E_s/E_c)^{-\gamma}\approx(2N_s/N_0)\cdot e^{8.75}=(1.9\times10^{-6}\times6.3\times10^{3})/0.1\approx 0.12,$$

or again with $\gamma=1.5$,

$$E_s\approx 4E_c,$$

that is, at medium latitudes, about 15 to 20 Bev, and incidentally even more if one believes in a lower value for $\gamma$. However, as at lower altitudes at least the bulk of the showers evidently are due to very energetic primaries for which the energy spectrum certainly is not flat, it appears to be justified to approximate it by this particular power law.

This last surprising result merits some further consideration. It may be stated that the following remarks do not depend on any of the special assumptions made above; they are essentially based only on the experimental fact that the flux of shower-producing particles at sea level is less than one-tenth of the flux which one would expect if all primary particles incident at the top of the atmosphere were to contribute to the production of local penetrating showers, and if the absorption at all altitudes did not exceed that observed at sea level or moderate heights.

The following reasons might explain this remarkably small flux: (1) Particles in the lower energy region may be absorbed more readily than those of higher energies. (2) Collisions of low energy particles ($E\lesssim 15$ Bev) will in general not result in the emission of a penetrating shower.

One can easily see that the first is not a very likely explanation. The interaction cross section cannot exceed the geometrical cross section, but attains or approaches this value for high energies. Thus an entirely different mechanism would have to be found to account for an absorption process favoring the low energy region. The simplest would be the assumption that on the average not a constant *fraction* of the incident energy is transferred in a collision, but a constant *amount* of energy. Such a transfer law would lead to a quicker elimination of the slower particles. However, it would also lead to an absorption law which, far from being exponential, resembles the range curves of absorption due to ionization losses. Only with very artificial assumptions could a "selective" absorption of low energy particles be combined with an exponential or near-exponential absorption curve.

Hence the second assumption appears to be the more plausible one. It means that, although for all primaries, and indeed all nucleons and possibly $\pi$-mesons of energies $\gtrsim E_c$, the interaction cross section may be equal to the geometrical nuclear cross section, the probability that the energy transfer to several secondaries is large enough so that a penetrating shower results from the collision, is small for small incident energies and increases with the energy. The simplest representation of such a transfer law is by a probability function which depends on the *fractional* transfer $U/E$ only ($U$ is the energy transferred in the collision):

$$p(U,E)dU=p(U/E)d(U/E) \tag{7}$$

and which decreases with increasing fractional transfer. It is well known that one encounters a transfer probability of this kind for instance in radiative collisions. From analogy reasons, a probability function of the type (7) has therefore, been chosen arbitrarily by Heitler and Jánossy[13] in their theory of fast nuclear collisions. From the arguments presented here it appears that there is experimental evidence as well as reasons of analogy to support this rather than other transfer laws.

It should be recalled that, if this interpretation is accepted, $\alpha$ stands for the *average* fractional energy retained by the incident particle. In a similar way, $E_s$ is the *average* energy above which the production of penetrating secondaries in a nuclear collision becomes very likely, while for $E<E_s$ the probability for these processes becomes increasingly small. In this region of lower energies collisions still occur with about equal frequency, but they result in a more star-like "shower" with no or few penetrating secondaries, most likely "knock-on protons," but no mesons.

There is, finally, a simple check on the validity of these estimates and conclusions. If it is so that, on the average, mesons are produced only in high energy collisions, then it must be possible to derive the entire $\mu$-meson component from these collisions. In other words. One can first compute the average number of mesons produced in a single collision by comparing the intensity of the $\mu$-meson component at sea level, corrected for decay while traversing the atmosphere, with the total number of collisions the primary cosmic-ray particles of energy $E\gtrsim E_s$ and their energetic descendants have suffered. One can then re-check by computing the energy each meson would carry off, and comparing it with the observed meson energies. Details of these computations will not be given here; it may suffice to state that not only a very reasonable value for the average multiplicity of the meson production is obtained ($n\approx 4$ to 5), but also a rather satisfactory representation of the altitude dependence of the meson flux.

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[13] W. Heitler and L. Jánossy, Proc. Phys. Soc. **A62**, 374 and 669 (1949).