The Low States of Li^7

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An attempt is made to obtain consistency between the positive quadrupole moment of Li^7 as well as other known ground state data (spin; magnetic moment; $Be^7 \xrightarrow{K} Li^7$, Li^{7*} lifetime and branching ratio) and the assumption that the wave function belongs to an s^4p^3 configuration of nucleons. This is found to be possible with only one particular form for the spin and angular dependence of the function. This form is very different from that predicted by previous theory; in particular the D and P state probabilities are found to be 80 percent and 20 percent, respectively. It should be emphasized that the validity of the analysis depends on the accuracy of the assumptions that the quadrupole moment is positive and due only to the charge distribution of the nucleons, with no contribution arising from their interaction.

An investigation is also made of the spin of the Li^7 480-kev excited state. Previous arguments for a high spin are reviewed. Using the ground state wave function indicated, and assuming the s^4p^3 configuration also for the excited state, one can by further analysis show that an excited state of spin 5/2 or 3/2 would give fair agreement with experimental data (Be^7 K-capture data; $Li^{7*} \rightarrow Li^7 + \gamma$ lifetime). An excited state of spin 1/2, however, would give poor agreement. On the assumptions made no agreement is possible for spin greater than 5/2.

1. INTRODUCTION

THIS work was prompted by the failure of current nuclear theory to give agreement with the recently reported[1] positive value of the Li^7 electric quadrupole moment. Although it is possible that an error[2] in the interpretation of the quadrupole coupling for the molecules used in the measurement has led to an incorrect value for the quadrupole moment, this investigation proceeds on the assumption that the sign and magnitude of the quadrupole moment of Li^7 are as reported. The most widely used theory[3] of the structure of light nuclei is based on the independent particle model and simple assumptions on nuclear forces. The ground state wave function predicted by this theory for Li^7 gives a negative value for the quadrupole moment.

The first problem treated here concerns the extent to which the measured properties of the Li^7 nucleus determine the form of its wave function. No explicit assumptions on nuclear forces are made; therefore, no attempt is made to solve the dynamical problem of finding a wave function from a potential energy function. It is necessary, of course, to make some simplifying assumptions to aid in the calculations. The same approximation is used here as in reference 3: that of assuming an s^4p^3 configuration, with the four s particles (two neutrons and two protons) forming a closed s shell. This assumption is consistent with current ideas about nuclear shell structure. The aim of the investigation, then, is to establish whether or not the s^4p^3 configuration is capable of describing the Li^7 ground state and, if so, to what extent the wave function is determined by the values of the measured properties.

Because of the symmetry of the closed s shell, the nuclear properties considered here should be determined only by the p-nucleons. Consequently the calculations need involve only these particles. The choice of configuration and the value of the total angular momentum do not fix the spin and angular dependence of the wave function, since the angular momenta of the three p-nucleons can be combined to give eight linearly independent eigenfunctions of $J=3/2$. In the theory of reference 3 this arbitrariness in the spin and angular dependence of the ground state is completely eliminated by the requirement that the orbital part of the wave function be symmetric under permutations of the three p-nucleons (this property to be referred to as space-symmetry). This requirement is a consequence of the assumption that the nuclear forces are spin and charge independent and predominantly of the Majorana (space-exchange) type. Since no assumptions on nuclear forces are to be made here, the requirement of space-symmetry is ignored and thus there remains considerable freedom in the wave function.

The data to be considered are given in Table I. Odd parity is obtained by the choice of an s^4p^3 configuration. The transition probability for K-capture in Be^7 leading to the ground state of Li^7 can be treated as a property of the ground state if it is assumed that the ground state of Be^7 is identical with that of Li^7 except for interchange of neutrons and protons. This assumption has recently received rather convincing confirmation. A single excited state has been observed[4] in Be^7 at about 430 kev, in excellent correspondence with the 480-kev excited state of Li^7.

It is found possible to obtain reasonable agreement with all the data simultaneously, and the solution has the advantage of definitely singling out one spin and angular dependence for the wave function as better than any other. The wave function designated as best does not seem to have a simple interpretation, except

* This work was supported by the AEC.
[1] P. Kusch, Phys. Rev. **76**, 138 (1949).
[2] Townes, Foley, and Low, Phys. Rev. **76**, 1415 (1949).
[3] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[4] Brown, Chao, Fowler, and Lauritsen, Phys. Rev. **78**, 88 (1950). Johnson, Laubenstein, and Richards, Phys. Rev. **78**, 413 (1950).

that it is very different from the space-symmetric wave function given by the theory of reference 3.

The Li^7 nucleus appears to have only one low lying excited state. There has recently been considerable speculation as to the nature of this 480-kev bound level. Its spin (J^*) was long thought to be 1/2, on the assumption that the ground state and the excited state are the two members of a $^2P_{3/2,1/2}$ doublet. Support for this assignment was given by the fact that it is consistent with the theory of reference 3. The lowest lying state according to that theory is a space-symmetric ^2P, which can have total angular momentum 1/2 or 3/2. Since $J=3/2$ for the ground state, it was entirely reasonable to assume that a space-symmetric $^2P_{1/2}$ level is nearby. Since results obtained here indicate that the ground state should not be treated as a $^2P_{3/2}$ state belonging to an s^4p^3 configuration, there remains little reason to assume that the excited state is to be described as the corresponding $^2P_{1/2}$ state.

Assuming spin 3/2 for the Be^7 ground state, it is possible to assign an upper limit of 5/2 to J^* on the basis of the K-capture lifetime. Thus, only 1/2, 3/2, 5/2 are considered as possible values for J^*. The measured value of 3 for the spin of B^{10} together with the branching ratio of the reaction

$$B^{10}+\text{thermal neutron}\rightarrow\alpha+Li^7,\ Li^{7*}$$

makes the assignment of spin 1/2 to the excited state doubtful. This argument has been re-investigated and the conclusion is reached that the evidence favors, but does not insure, $J^*>3/2$.

There are two experimental data, in addition to that obtained from the $B^{10}(n,\alpha)$ reaction, which can be used to give information on the excited state: the lifetime for $Li^{7*}\rightarrow Li^7+\gamma$, and the lifetime for $Be^7\xrightarrow{K}Li^{7*}$. In each case the data specify a value for a matrix element between the excited state and the appropriate ground state. Using for the ground state wave functions of both Li^7 and Be^7 the form already found, these two data have been used to give further evidence on the value of J^*. If one assumes that the excited state belongs to the same configuration as assumed for the ground state (s^4p^3), the possibility of fitting the γ-ray and K-capture data has been investigated for each of the cases: $J^*=1/2$, 3/2, 5/2. Full consistency with the data is found to be impossible with any of these spins. The margin of disagreement for $J^*=5/2$ or 3/2, however, is not very large. $J^*=1/2$ seems to be definitely unsatisfactory.

2. THE COMPLETE SET OF FUNCTIONS FOR THE GROUND STATE

Since the closed s shell is neglected completely, a set of wave functions corresponding to a p^3 configuration is required. This set of functions is to be complete in spin and angular dependence under the restriction that two of the three particles are identical and $J=3/2$. The angular momenta of two p-nucleons can combine to

TABLE I. Data for the Li^7 ground state.

Total angular momentum (J)	3/2[a]
Magnetic moment (μ)	+3.25 n.m.[a]
Parity	Odd[b]
Quadrupole moment (Q)	+1/50 barn[c]
Half-life for $Be^7\xrightarrow{K}Li^7$ (ground states)	5.1×10^6 sec.[d]

[a] J. Mattauch and S. Fluegge, *Nuclear Physics Tables* (Interscience Publishers, Inc., New York, 1946).
[b] D. R. Inglis, Phys. Rev. **74**, 21 (1948).
[c] See reference 1.
[d] E. Segrè and C. Wiegand, Phys. Rev. **75**, 43 (1949); R. M. Williamson and H. T. Richards, Phys. Rev. **76**, 614 (1949).

form ^1S, ^3P, ^1D states. These, in turn, can combine with the p-proton to give the following eight states:

$$\begin{array}{ll} (^1S,\,^2p)^2P & (^1D,\,^2p)^2D \\ (^1D,\,^2p)^2P & (^3P,\,^2p)^2D \\ (^3P,\,^2p)^2P & (^3P,\,^2p)^4D \\ (^3P,\,^2p)^4P & (^3P,\,^2p)^4S. \end{array} \tag{1}$$

The notation is "(neutron state, proton state) combined state." $(^3P,\,^2p)^2S$ and $(^1D,\,^2p)^2F$ are not acceptable because they cannot give $J=3/2$. The eight functions give the required complete set.

Since no information concerning the radial behavior of the nuclear wave function is available, the following simplifying assumptions are made. The radial dependence is taken to be the same for each nucleon, and to be independent of the spin and angular dependence of the wave function. The wave function of Li^7 is thus to be approximated by a product of a factor of the form $f(r_1)f(r_2)f(r_3)$ with some linear combination of spin-angular functions.

The set of functions shown above is classified by definite neutron and proton states. Sets of functions, complete in the same sense as this set and classified differently, might be used just as well. It has been shown[5] that the symmetry under permutations of particles of a nuclear wave function may be significantly related to the nature of nuclear forces. Therefore, the following complete set,[6] in which the symmetry properties are explicitly given, is introduced:

$$\begin{array}{ll} \psi_1={}^{22}P[3] & \psi_5={}^{22}D[2+1] \\ \psi_2={}^{22}P[2+1] & \psi_6={}^{24}D[2+1] \\ \psi_3={}^{24}P[2+1] & \psi_7={}^{42}D[2+1] \\ \psi_4={}^{42}P[2+1] & \psi_8={}^{44}S[1+1+1]. \end{array} \tag{2}$$

The notation is ${}^{(2S+1)(2T+1)}L[\lambda_3+\lambda_2+\lambda_1]$ where $L=$ total orbital angular momentum, $S=$ total spin angular momentum, $T=$ total isotopic spin. The bracket symbol specifies the partition (in the standard terminology for the representations of the symmetric group[5]) to which the function belongs, and characterizes the symmetry properties of the wave function. The orbital part of ψ_1 is completely symmetric under permutations of the three p-particles and that of

[5] E. Wigner, Phys. Rev. **51**, 106 (1937).
[6] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1949), p. 204.

ψ_8 is completely antisymmetric. ψ_1 is the wave function predicted for the Li^7 ground state by the theory of reference 3. Each of the ψ_k is understood to contain the radial dependence $f(r_1)f(r_2)f(r_3)$ where $f(r)$ is normalized. The calculations and results below are given in terms of the functions of the set (2). The relationships of these functions with those of set (1) are given in Appendix I.

3. CONDITIONS IMPOSED BY EXPERIMENTAL DATA

The wave function for the ground state of Li^7 is taken to be

$$\Psi_G=\sum_{j=1}^{8} a_j\psi_j,$$

where the a_j satisfy the normalization condition $\sum|a_j|^2=1$.

The Quadrupole Moment

The calculated value of the quadrupole moment is given by

$$Q=\sum_{j,\,k} a_j^* a_k Q_{jk} \tag{3}$$

where

$$Q_{jk}=(\psi_j,\, r_p^2(3\cos^2\theta-1)\psi_k).$$

The ψ_k are used with $M_J=J$, r_p is the distance of the proton from the center of mass of the nucleus, and θ is the angle between r_p and the z (total angular momentum) axis. The calculated values of the Q_{jk} are given in Appendix II. A basic assumption made here is that the quadrupole moment can be calculated from the charge distribution of the protons alone, with no contribution to the charge distribution resulting from the charge-exchange interaction of the various nucleons.

Difficulty arises on two counts in the comparison of the calculated value of the quadrupole moment with the measured value given in Table I. The measurement is by no means a precise one. There is also considerable uncertainty in assigning a value to the quantity $\langle r_p^2\rangle$ (the average of the square of r_p), in terms of which the calculated value is given. The value of $\langle r_p^2\rangle$ is taken to be $(e^2/mc^2)^2$, which is approximately the square of the nuclear radius as given by the formula $R=1.5A^{\frac{1}{3}}10^{-13}$ cm. The value for the quadrupole moment given by ψ_1 is of the right order of magnitude but of the wrong sign. Its cross-terms with the other ψ_k are sufficiently small that no wave function which is predominantly ψ_1 can give agreement with the sign of the measured quadrupole moment.

The Magnetic Moment

The measured value of the magnetic moment given in Table I is to be compared with the calculated value

$$\mu=\sum_{j,\,k} a_j^* a_k \mu_{jk},$$

where

$$\mu_{jk}=(\psi_j, m_{l\ \mathrm{proton}}+\sum_{i=1}^{3}\mu_i\sigma_{zi}\psi_k)$$

ψ_i with $M_J=J$ are used; μ is given in nuclear magnetons when

$\mu_i=\mu_P=+2.79$ if ith particle is proton,
$\mu_i=\mu_N=-1.91$ if ith particle is neutron.

The calculated values of the μ_{jk} are given in Appendix II. The matrix elements can be evaluated exactly but the possible existence of exchange moments[7] may make the result of the calculation inaccurate.

The Be^7 K-Capture Lifetime

It is assumed that nuclear forces are charge-independent to the extent that the wave function for Be^7 may be obtained from the Li^7 wave function by merely interchanging neutrons and protons. On this assumption, the experimental data on the Be^7 K-capture can be used to give an additional condition on the Li^7 wave function. The lifetime of Be^7 against the reaction

$$Be^7+e_K \begin{cases}\nearrow Li^{7*} & 10.7 \text{ percent}\\ \searrow Li^7 & 89.3 \text{ percent}\end{cases} \tag{4}$$

together with the branching ratio, enables one to obtain the partial lifetime of the reaction going to the ground state; this alone is of interest in the determination of the ground state function. The type of β-decay interaction assumed

$$\mathbf{I}=\sum_{i=1}^{3}\tau_{\xi i}\boldsymbol{\sigma}_i, \tag{5}$$

($\tau_{\xi i}$ reverses the charge state of the ith nucleon) is, in the present treatment, consistent with either the axial vector or the tensor formulations of the theory of β-decay, and corresponds to the Gamow-Teller selection rules ($\Delta J=0,\ \pm 1$; $0\rightarrow 0$ forbidden; no parity change). The strength of the interaction is taken to be characterized by the value[8]

$$|\mathbf{I}|^2 ft_{\mathrm{mean}}=4400, \tag{6}$$

where $|\mathbf{I}|^2$ is the absolute square of the matrix element of $\mathbf{I}$ between the two ground states. The value (6) was obtained from the $H^3\rightarrow He^3$ decay and is presumably the same for all allowed β-decay processes. The value of f is given by the K-capture relation

$$f=2\pi(\alpha Z_{\mathrm{eff}})^3 W^2,$$

where $Z_{\mathrm{eff}}=3.7$ for Be^7, $W=1.66=$ total energy available in units of mc^2 and $\alpha=e^2/(\hbar c)=1/137$. The experi-

[7] R. Avery and R. G. Sachs, Phys. Rev. **74**, 1320 (1948).
[8] E. Wigner (private communication).

mental lifetime and (6) yield the requirement $|\mathbf{I}|^2=1.76$. The calculated value is given by

$$|\mathbf{I}|^2=(\sum_{j,k} a_j^* a_k I_{jk})^2.$$

The I_{jk} are to be found in Appendix II. Even though these matrix elements are evaluated exactly, there is considerable uncertainty in the strength of the β-interaction and even in the type of interaction to be used.

4. THE GROUND STATE WAVE FUNCTION

An investigation of the conditions on the a_j imposed by the experimental data shows that one set of values for the a_j gives reasonable agreement with all the data. No qualitatively different set gives anything like agreement. With the number of conditions to be satisfied (four: one normalization and three experimental data, Q, μ-, and β-decay) and the number of free parameters available (sixteen: the real and imaginary parts of each of the a_j) one might have expected that an excellent fit could be found with many qualitatively different choices of the a_j. Had this occurred, no unique wave function would have been determined; it would, nevertheless, have indicated consistency between the experimental data and the assumption of an s^4p^3 configuration. On the other hand, had it occurred that no fit to all the data could be found, even approximately, one might have concluded that an s^4p^3 configuration is inadequate for the description of the Li^7 nucleus.

The wave function which gives best agreement with the data is only qualitatively unique. That is, small changes of the values of the a_j in a range of general agreement with the data have only the effect of improving the agreement with one of the data at the expense of the others. The general nature of the wave function is determined, however, since the permissible variations in the a_j correspond to changes of only a few percent in the probabilities of the states involved. This spread in the values of the coefficients may be characterized by the following two wave functions:

$$\Psi_G=\tfrac{1}{4}[\psi_1-\psi_2+\psi_4+\sqrt{8}\psi_5-\sqrt{5}\psi_7] \tag{7}$$

which gives

$$Q=0.14\langle r^2\rangle,\quad \mu=2.91\text{ n.m.},\quad |\mathbf{I}|^2=1.76$$

and

$$\Psi_G'=(24)^{-\frac{1}{2}}[\psi_1-\sqrt{2}\psi_2+\sqrt{2}\psi_4+\sqrt{9}\psi_5-\sqrt{9}\psi_7+\psi_8] \tag{8}$$

which gives

$$Q=0.22\langle r^2\rangle,\quad \mu=2.70\text{ n.m.},\quad |\mathbf{I}|^2=1.76.$$

These are to be compared with the following values which are presumed to represent the experimental results:

$$Q=0.25\langle r^2\rangle,\quad \mu=3.25\text{ n.m.},\quad |\mathbf{I}|^2=1.76.$$

The differences in the values of the a_j in the two wave functions are for the most part of no interest, since significance is to be attached only to the general nature of the function and not to the *exact* values of the coefficients. However, a possible slight admixture of ψ_8 in the wave function is of interest since that function has total isotopic spin $T=3/2$ while all the other functions in both Ψ_G and Ψ_G' have $T=1/2$. (ψ_3, Ψ_6, and ψ_8 have $T=3/2$, the rest have $T=1/2$). Hence a wave function of the form (7) is consistent with the idea that T is a good quantum number, while a wave function of the form (8) is not. It is difficult to decide which of (7) and (8) gives the better agreement with experiment. The introduction of a slight admixture of ψ_8 gives a value for Q appreciably closer to the measured value, while $a_8=0$ gives the better value for μ. The measurement of Q is a very unprecise one; on the other hand, there is no good reason to expect that exchange moments are negligible in this nucleus. Thus this difficulty cannot be resolved with any certainty. It can be said, however, that the experimental data are *not inconsistent* with a wave function for which $T=1/2$, and thus suggest that T may be a good quantum number.

Since the differences between (7) and (8) are of little consequence (aside from the question of T), the simpler wave function (7) is arbitrarily selected for use in the calculations given hereafter.

An interesting result of the calculation is that the assumption of the equivalence of the Be^7 and Li^7 ground state wave functions is not essential for the determination of the form of the wave function. This form is determined uniquely by the magnetic and quadrupole moments alone.

The ground state function is very different from that given by the theory of reference 3, which is $\Psi_G=\psi_1$. The wave function (7) contains 80 percent D state and 20 percent P state, with about equal amounts of doublet and quartet spin states. The failure of L and S to be good quantum numbers may be interpreted as indicating that spin-orbit coupling cannot be neglected in any useful approximation.

It should be emphasized that the experimental fact that $Q>0$ is responsible for the inadequacy of ψ_1, in the sense that ψ_1, while giving $Q<0$, nevertheless yields entirely reasonable results for the other two data. Thus any strong deviation from simple "additivity" of individual proton charge distributions to give the nuclear quadrupole moment, such as might arise from the interaction of the nucleons, would make the value of Q calculated from (3) incorrect and thus would invalidate the results of this analysis.

Although the wave function designated as the best approximation for the Li^7 ground state is a complicated linear combination of the functions of the set (2), it might be simpler if expanded in some other, more appropriate, basis. That is, there may be good quantum numbers which have been obscured by the present choice of basic functions. The wave function is not simplified when expanded in the functions of set (1).

TABLE II. Barrier penetrabilities for $B^{10}(n,\alpha)Li^7$, Li^{7*}.

L	$E=2.3$ Mev	$E=2.8$ Mev
0	0.77	0.99
1	0.26	0.41
2	0.030	0.070
3	0.0019	0.0051

TABLE III. The ratios W^*/W necessitated by the branching ratio of (10) for the vatious possible values of compound nucleus spin and J^*.

Compound nucleus spin	J^*	L	L^*	W^*/W
5/2	1/2	1	3	2900
5/2	3/2	1	1	21
5/2	5/2	1	1	21
5/2	7/2	1	1	21
7/2	1/2	3	3	36
7/2	3/2	3	3	36
7/2	5/2	3	1	0.26
7/2	7/2	3	1	0.26

Recently published correlations[9] of the "magic numbers" of nuclear shell structure with the degeneracies given by a (jj)-coupling scheme suggest that the wave function might be simple as an expansion in characteristic functions of the individual nucleon total angular momenta (j). Each of the three p-nucleons can have $j=3/2$ or $1/2$. The set of functions, each with total angular momentum 3/2,

$$\begin{array}{ll} \phi_1=[(3/2,\,3/2)0,\,3/2] & \phi_5=[(3/2,\,1/2)1,\,1/2] \\ \phi_2=[(3/2,\,3/2)2,\,3/2] & \phi_6=[(3/2,\,1/2)1,\,3/2] \\ \phi_3=[(3/2,\,3/2)2,\,1/2] & \phi_7=[(3/2,\,1/2)2,\,1/2] \\ \phi_4=[(1/2,\,1/2)0,\,3/2] & \phi_8=[(3/2,\,1/2)2,\,3/2], \end{array} \qquad (9)$$

using the notation

"[(individual neutron j's) total neutron j, proton j],"

is complete in the same sense as are the sets (1) and (2). These functions are given in terms of the functions of the set (2) in Appendix I. Expressed in terms of these functions, Ψ_G and Ψ_G' are,

$$\Psi_G=0.28\phi_1+0.13\phi_2+0.25\phi_3-0.27\phi_4-0.32\phi_5 +0.55\phi_6+0.60\phi_8 \qquad (7')$$

$$\Psi_G'=0.18\phi_1+0.18\phi_2+0.17\phi_3-0.22\phi_4-0.45\phi_5 +0.64\phi_6+0.44\phi_8. \qquad (8')$$

These differ greatly from the wave function, ϕ_1, predicted for the ground state by the spin-orbit coupling model recently suggested by Mayer.

By the assumption used above of the equivalence of the Be^7 and Li^7 ground states, the wave function (7) implies the following properties for Be^7:

$$Q(Be^7)\approx Q(Li^7)$$

$$\mu(Be^7)=-1.69 \text{ n.m.}$$

The approximate equality implied for the quadrupole moments is a detailed result and of no apparent general significance. The value given for the magnetic moment of Be^7 is found from the *measured* moment of Li^7 and the theorem[10] which relates the sum of the moments of two conjugate nuclei to the coefficients in the expansion of their mutual wave function in eigenfunctions of L and S. The coefficients for the ground state function (7) are used. If the reasonable assumption that exchange moments are equal and opposite for conjugate nuclei is valid, then the correctness of this prediction depends only on the correctness of the distribution of angular momentum between spin and orbital motion given by the wave function, regardless of exchange moment contributions.

5. THE EXCITED STATE

Recent experimental evidence suggests that the total angular momentum (J^*) of the excited state is greater than that of the ground state. This conclusion[11] is based on the determination[12] of the spin of B^{10} to be 3. Thus the compound nucleus in the reaction

$$B^{10}+\text{thermal neutron} \begin{array}{l} \nearrow Li^{7*}+\alpha+2.3 \text{ Mev (93 percent)} \\ \searrow Li^{7}\ +\alpha+2.8 \text{ Mev (7 percent)} \end{array} \qquad (10)$$

has spin 5/2 or 7/2. The branching ratio of the reaction is interpreted in terms of a smaller centrifugal barrier to be penetrated by the α-particle associated with Li^{7*} than by the one associated with Li^7. The corresponding smaller orbital angular momentum of the $Li^{7*}+\alpha$-system would suggest a Li^{7*} spin greater than that of Li^7.

This argument cannot be considered conclusive for the following reason.[13] The transition probability from the compound nucleus to each of the Li^7 states can be approximated as the product of a factor W, the "width without penetration," depending on the detailed nature of the nuclear wave functions involved, and a factor P, giving the penetrability of the α-particle through the barrier. The penetrabilities involved, estimated in the usual manner,[14] are listed in Table II. Very little, however, can be said about the factor W. One cannot exclude the possibility that it is very different for the two decay schemes. If it is assumed that the parity of B^{10} is even (corresponding to an s^4p^6 configuration) then the conservation of parity demands that the outgoing orbital angular momenta be limited to odd L. In Table III are listed the ratios W^*/W necessitated by the branching ratio of (10) for the various possible values of the spin of the compound nucleus and of J^*.

[9] M. G. Mayer, Phys. Rev. **75**, 1969 (1949).

[10] R. G. Sachs, Phys. Rev. **69**, 611 (1946).

[11] D. R. Inglis, Phys. Rev. **74**, 1876 (1948).

[12] Gordy, Ring, and Burg, Phys. Rev. **74**, 1191 (1948).

[13] This reinvestigation was undertaken as a result of a suggestion by Professor Fermi.

[14] H. A. Bethe, Rev. Mod. Phys. **9**, 177 (1937).

TABLE IV. Relative values of the transformation coefficients between the functions ψ and ϕ. The normalization factor is $1/(216)^{\frac{1}{2}}$.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8
ψ_1	$+\sqrt{80}$	$+\sqrt{16}$	$-\sqrt{16}$	$+\sqrt{40}$	0	0	$-\sqrt{32}$	$+\sqrt{32}$
ψ_2	$+\sqrt{50}$	$+\sqrt{10}$	$+\sqrt{40}$	$+\sqrt{4}$	$+\sqrt{27}$	0	$+\sqrt{5}$	$-\sqrt{80}$
ψ_3	$+\sqrt{18}$	$-\sqrt{90}$	0	$+\sqrt{36}$	$-\sqrt{27}$	0	$+\sqrt{45}$	0
ψ_4	$-\sqrt{20}$	$-\sqrt{4}$	$+\sqrt{64}$	$+\sqrt{40}$	0	$+\sqrt{54}$	$-\sqrt{32}$	$+\sqrt{2}$
ψ_5	$+\sqrt{10}$	$+\sqrt{2}$	$+\sqrt{8}$	$-\sqrt{20}$	$+\sqrt{15}$	$+\sqrt{48}$	$+\sqrt{49}$	$+\sqrt{64}$
ψ_6	$-\sqrt{10}$	$+\sqrt{50}$	$+\sqrt{32}$	$+\sqrt{20}$	$-\sqrt{15}$	$-\sqrt{48}$	$+\sqrt{25}$	$+\sqrt{16}$
ψ_7	$-\sqrt{20}$	$-\sqrt{4}$	$-\sqrt{16}$	$+\sqrt{40}$	$+\sqrt{120}$	$-\sqrt{6}$	$+\sqrt{8}$	$+\sqrt{2}$
ψ_8	$-\sqrt{8}$	$+\sqrt{40}$	$-\sqrt{40}$	$+\sqrt{16}$	$-\sqrt{12}$	$+\sqrt{60}$	$+\sqrt{20}$	$-\sqrt{20}$

Only the smallest possible odd values for L and L^* are considered. $J^*=1/2$ cannot be excluded since a value W^*/W as high as 2900 might be expected if the wave function of the compound nucleus is approximately the product of a Li^{7*} wave function and a function describing an α-particle moving about the Li^{7*} core. Since it seems more likely on statistical grounds that the wave function of the compound nucleus does not favor such special states of aggregation, one is inclined to believe that W^*/W is of the order of unity. Then values $J^*>3/2$ are favored. If the parity of B^{10} is odd, the outgoing angular momenta are limited to even L, but the argument and the conclusion are essentially unchanged.

The interpretation of the $B^{10}(n, \alpha)$ reaction shows no preference between $J^*=5/2$ and $J^*=7/2$. The reaction (4), however, indicates that, of the two, the choice 5/2 is far the more likely.[15] The lifetime of the reaction and the branching ratio imply that both transitions are allowed. No conventional formulation of β-decay theory permits $|\Delta J|>1$ for an allowed transition. This fact, together with the assumption that the spin of Be^7 is 3/2, argues against $J^*=7/2$ and indicates that the only possible values of J^* are 1/2, 3/2, and 5/2.

The partial lifetime[16] for Be^7 K-capture leading to Li^{7*} and the mean life[17] of Li^{7*} against γ-decay to the ground state have been used to obtain further information concerning J^*. It is assumed that the excited state can be treated as belonging to a definite configuration. To obtain agreement with the experimental data the s^4p^3 configuration (same as ground state) must be assigned to the excited state, because the β-interaction, Eq. (5), has no non-vanishing matrix elements between states belonging to different configurations.

The experimentally determined partial lifetime for K-capture fixes the matrix element of the operator (5) between the Be^7 and Li^{7*} wave function according to

$$|\mathbf{I}^*|^2=|(Li^{7*}|\mathbf{I}|Be^7)|^2=1.10. \tag{11}$$

Similarly the γ-ray lifetime fixes a value for the matrix element between Li^7 and Li^{7*} of some electromagnetic multipole moment of the nucleus. Since the parities of Li^7 and Li^{7*} are both taken to be odd, the γ-ray transition is due either to a magnetic dipole or to an electric quadrupole interaction. The energy available and the dimensions of the nucleus involved show[15] that the magnetic dipole transition will predominate unless accidental cancellation in nuclear matrix elements occurs. The shortness of the experimental lifetime precludes the possibility of cancellation, however, and it is therefore concluded that the γ-ray transition provides a measure of the matrix element of the magnetic dipole moment, $\mathbf{M}$. The experimental data imply

$$20\leqslant|(Li^7|\mathbf{M}|Li^{7*})|^2\leqslant 40. \tag{12}$$

Information concerning J^* may now be obtained by investigation of the possibility of finding agreement with both (11) and (12) for each of the J^* values 5/2, 3/2, and 1/2.

For $J^*=5/2$ the set of functions, complete in angular and spin dependence, is

$$\begin{aligned} &\psi_9={}^{22}D[2+1] \quad \psi_{12}={}^{42}P[2+1] \\ &\psi_{10}={}^{24}D[2+1] \quad \psi_{13}={}^{22}F[3]. \\ &\psi_{11}={}^{42}D[2+1] \end{aligned} \tag{13}$$

To each of these functions is assigned the same radial dependence as that in the ground state. For the ground state of both Be^7 and Li^7, the wave function given by Eq. (7) is used. By appropriately choosing the coefficients in a linear combination of the functions (13) to describe the excited state,

$$\Psi=\sum_{j=9}^{13} a_j\psi_j,$$

it is found possible to obtain a value of the matrix element (11) in agreement with experiment. No linear combination can be found, however, which gives a sufficiently large value for the matrix element (12). The largest value obtainable is $|\mathbf{M}|^2\approx 10$ and this can be obtained simultaneously with $|\mathbf{I}^*|^2=1.10$ in agreement with (11). The linear combination giving this best agreement consists predominantly of

$$\psi_{11}={}^{42}D[2+1].$$

For the case $J^*=3/2$, the complete set (2) is satisfactory, subject to the restriction that the excited state wave function be orthogonal to that of the ground state [given by Eq. (7)]. The best agreement possible is a value for the γ-lifetime $|\mathbf{M}|^2\approx 14$ and a value for the K-capture partial lifetime $|\mathbf{I}^*|^2\approx 0.60$. The wave function for the excited state which gives the best agreement for $J^*=3/2$ is predominantly $\psi_6={}^{24}D[2+1]$.

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[15] S. S. Hanna and D. R. Inglis, Phys. Rev. **75**, 1767 (1949).
[16] See references in Table I.
[17] L. G. Elliott and R. E. Bell, Phys. Rev. **76**, 168 (1949).

For $J^*=1/2$, the complete set of functions is

$$\begin{array}{ll} {}^{22}P[3] & {}^{42}P[2+1] \\ {}^{22}P[2+1] & {}^{42}D[2+1] \\ {}^{24}P[2+1] & {}^{22}S[1+1+1]. \end{array}$$

In this case no good agreement is possible. The best results are obtained with a function predominantly of type ${}^{24}P[2+1]$, which gives $|\mathbf{I}^*|^2\approx 0.16$ (low by a factor of 7) and $|\mathbf{M}|^2\approx 10$. Thus, in the attempt to obtain information concerning J^* from the γ-ray and K-capture lifetimes, all that can be concluded is that $J^*=1/2$ seems to be inconsistent with the K-capture lifetime. It should be emphasized that this conclusion is contingent upon the use of a ground state wave function of the form (7).

This problem was suggested to us by Professor R. G. Sachs, who has contributed valuable advice during the course of the work.

APPENDIX I. THE SET OF GROUND STATE FUNCTIONS

The functions of the set (2) are given here in terms of the functions of the set (1), which can easily be constructed by the standard rules of combining angular momenta.

$$\begin{aligned}
\psi_1 &= 5^{\frac{1}{2}}/3({}^1S^2p)^2P+2/3({}^1D^2p)^2P \\
\psi_2 &= 2^{\frac{1}{2}}/3({}^1S^2p)^2P-10^{\frac{1}{2}}/6({}^1D^2p)^2P+1/2^{\frac{1}{2}}({}^3P^2p)^2P \\
\psi_3 &= 2^{\frac{1}{2}}/3({}^1S^2p)^2P-10^{\frac{1}{2}}/6({}^1D^2p)^2P-1/2^{\frac{1}{2}}({}^3P^2p)^2P \\
\psi_4 &= ({}^3P^2p)^4P \\
\psi_5 &= 1/2^{\frac{1}{2}}({}^1D^2p)^2D+1/2^{\frac{1}{2}}({}^3P^2p)^2D \\
\psi_6 &= 1/2^{\frac{1}{2}}({}^1D^2p)^2D-1/2^{\frac{1}{2}}({}^3P^2p)^2D \\
\psi_7 &= ({}^3P^2p)^4D \\
\psi_8 &= ({}^3P^2p)^4S.
\end{aligned}$$

The choice of phases made for the functions of the set (1) is that obtained by a self-consistent use of the tables of Condon and Shortley[18] for the combination of angular momenta. To introduce the isotopic spin formalism, it is only necessary to multiply each ψ by the isotopic spin wave function, $\tau^+(1)\tau^+(2)\tau^-(3)$, representing a state where particles 1 and 2 are neutrons and 3 is a proton, and then to antisymmetrize the entire wave function with respect to permutations of particles.

The relations between the function of the set (2) and the function of the set (9) are given in Table IV.

APPENDIX II. THE GROUND STATE MATRIX ELEMENTS, WITH RESPECT TO THE FUNCTIONS OF SET (2).

TABLE V. Q_{jk} in units of $(1/25)\langle r^2\rangle$.

	1	2	3	4	5	6	7	8
1	-6							
2	$-\sqrt{10}$	0						
3	$-\sqrt{10}$	-5	0					
4	0	0	0	-4				
5	$+\sqrt{2}$	$-2\sqrt{5}$	$+\sqrt{5}$	0	0			
6	$+\sqrt{2}$	$+\sqrt{5}$	$-2\sqrt{5}$	0	7	0		
7	0	0	0	-6	0	0	0	
8	0	0	0	0	0	0	$-2\sqrt{10}$	0

TABLE VI. μ_{jk} in nuclear magnetons $(e\hbar/2M_{\text{proton}}c)$.

	1	2	3	4	5	6	7	8
1	+3.13							
2	+0.53	+0.01						
3	+0.53	+2.98	+0.01					
4	0	−1.98	+1.98	−0.56				
5	+0.14	0	−0.22	0	+0.81			
6	+0.14	−0.22	0	0	−2.19	+0.81		
7	0	0	0	+0.40	−2.66	+2.66	+0.39	
8	0	0	0	+0.63	0	0	0	−1.03

TABLE VII. $\mathbf{I}_{jk}$.

	1	2	3	4	5	6	7	8
1	$+(5/3)^{\frac{1}{2}}$							
2	0	$-(5/27)^{\frac{1}{2}}$						
3	0	$-(20/27)^{\frac{1}{2}}$	$+(20/27)^{\frac{1}{2}}$					
4	0	$-(32/27)^{\frac{1}{2}}$	$-(8/27)^{\frac{1}{2}}$	$-(121/135)^{\frac{1}{2}}$				
5	0	0	0	0	$+(1/15)^{\frac{1}{2}}$			
6	0	0	0	0	$+(4/15)^{\frac{1}{2}}$	$-(4/15)^{\frac{1}{2}}$		
7	0	0	0	0	$-(32/15)^{\frac{1}{2}}$	$-(8/15)^{\frac{1}{2}}$	$-(1/15)^{\frac{1}{2}}$	
8	0	0	0	0	0	0	0	$+(20/3)^{\frac{1}{2}}$

[18] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (The Macmillan Company, New York, 1935).