where $F^{\mu\nu}$ and $F_{\mu\nu}$ are the contravariant and covariant components respectively of the field strengths defined in terms of the vector potentials A_μ in the usual fashion. Four of Maxwell's equations in vacuum follow from

$$\delta\int L\,d^4x=0. \quad (2)$$

To obtain the quantal equations directly it is necessary to replace (1) and (2) by

$$G=-\tfrac{1}{4}(F^{\mu\nu})_{mn}(F_{\mu\nu})_{nm} \quad (3)$$

and

$$\delta G=0, \quad (4)$$

respectively. In (3), $(F^{\mu\nu})_{mn}$ and $(F_{\mu\nu})_{nm}$ are the matrix elements of $F^{\mu\nu}$ and $F_{\mu\nu}$, respectively. Thus, *G is essentially the trace of the classical Lagrangian.* Also in (3)

$$F_{\mu\nu}\equiv i[p_\mu, A_\nu]-i[p_\nu, A_\mu], \quad (5)$$

where the p's are the momentum operators.[1,3] In (4), the variation is taken with respect to the matrix elements of the vector potentials A_ν.

To show that the "quantal" equations follow from the above prescription, we must calculate δG. From (3) and (5)

$$\begin{aligned}\delta G&=-\tfrac{1}{2}(F^{\mu\nu})_{mn}\delta(F_{\mu\nu})_{nm}\\&=-(i/2)(F^{\mu\nu})_{mn}\delta([p_\mu, A_\nu]-[p_\nu, A_\mu])_{nm}\\&=-(i/2)(F^{\mu\nu})_{mn}\delta([p_\mu, A_\nu])_{nm}\\&=-i(F^{\mu\nu})_{mn}\delta(p_\mu A_\nu-A_\nu p_\mu)_{nm}\\&=-i(F^{\mu\nu})_{mn}\delta\{(p_\mu)_{nk}(A_\nu)_{km}-(A_\nu)_{nk}(p_\mu)_{km}\}\\&=-i(F^{\mu\nu})_{mn}\{(p_\mu)_{nk}\delta(A_\nu)_{km}-(p_\mu)_{km}\delta(A_\nu)_{nk}\}\\&=-i\{(F^{\mu\nu}p_\mu)_{mk}\delta(A_\nu)_{km}-(p_\mu F^{\mu\nu})_{kn}(A_\nu)_{nk}\}\\&=+i([p_\mu, F^{\mu\nu}])_{mk}\delta(A_\nu)_{km}.\end{aligned} \quad (6)$$

If δG is to vanish for arbitrary variations of the matrix elements of the A_ν we must have

$$i([p_\mu, F^{\mu\nu}])_{mk}=0,$$

or equivalently

$$i[p_\mu, F^{\mu\nu}]=0, \quad (7)$$

in operator notation. Equation (7) together with the relation

$$i[p_\mu, F_{\nu\sigma}]+i[p_\nu, F_{\sigma\mu}]+i[p_\sigma, F_{\mu\nu}]=0, \quad (8)$$

which is a consequence of the definition (5), are the "quantal" equations for the Maxwell field in vacuum.

I wish to thank the California Institute of Technology for the facilities offered me during my sabbatical leave.

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[2] R. J. Finkelstein, Phys. Rev. **75**, 1079 (1949).
[3] The units are chosen here so that $\hbar=1$.

Angular Correlation in the Reaction $B^{10}(n, \alpha)Li^{7*}(\gamma)Li^7$

B. Rose and A. R. W. Wilson
Atomic Energy Research Establishment, Harwell, Berkshire, England
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The angular correlation between the α-particle and the γ-ray emitted in the slow neutron reaction $B^{10}(n, \alpha)Li^{7*}(\gamma)Li^7$ will yield information on the spin of the excited state of Li^7 involved,[1,2] provided the Li^{7*} does not lose its orientation during the recoil. This is unlikely owing to the relative magnitudes of the mean life of the state[3] (10^{-13} sec.), and of the period associated with the h.f.s. splitting[4] of $Li^7(10^{-9}$ sec.). In particular, a non-zero correlation will exclude the possibility that the excited state has spin $\frac{1}{2}$. The correlation has been measured as follows.

A uniform 1.4-cm square B^{10} foil of thickness ~0.3 mg/cm² was irradiated with neutrons from the Harwell pile, the α-particles emitted at right angles to the neutron beam entering a proportional counter through a 1.4-cm square grid, distant 3.2 cm from the foil. The γ-rays emitted at right angles to the neutron beam were detected in the 37-mm diameter crystal of a scintillation counter which could be rotated about the neutron beam, the distance of the mid-plane of the crystal from the boron film being about 6 cm.

The α-γ-coincidence rate per α-particle, ρ, corrected for random coincidences, was measured at twenty different values of the angle θ between the axes of the counters, and varied by a few percent only (which was also the order of the statistical error of each count), over the angular range $-110°$ to $+110°$. Owing to the extremely high γ-ray background, the genuine coincidence rate was approximately 50 percent of the random rate, and a circuit was therefore devised to monitor continuously the product of the coincidence resolving time and the γ-counting rate, so that the value of the random coincidence rate per α-particle could be determined free from drifts in the resolving time.

Systematic errors due to γ-scattering by the source holder, proportional counter, and adjacent shielding material were measured by replacing the boron film by a thin irradiated gold source of similar dimensions, and measuring the consequent γ-distribution. Corrections of the order of 2 percent were found necessary over a limited range of angles.

The corrected value of ρ was fitted to a curve of the form $a_0+a_1\cos\theta+a_2\cos^2\theta$, the coefficients a_0 and a_2 being determined as functions of a_1 by the least squares method. The parameter a_1 was determined from the eccentricity (~0.010 in.) in the mounting of the boron film and the mean velocity of the Li^7 nuclei at the instant of radiation, which was calculated from the measurements of Elliott and Bell[3] on the mean life of this γ-ray. No correction was applied for the Döppler effect, as the energy dependence of the γ-counter efficiency was small. The resultant value of a_2/a_0, corrected for effects due to counter apertures and the extent of the source, was then the degree of correlation, C, for ideal geometry.

The mean of five determinations of C was -0.9 ± 1.2 percent, the error being the standard error. It should be noted that the range of values of C obtained by allowing V to vary from zero to the full recoil velocity is $+2.6$ to -1.3 percent.

This result rules out all correlations listed by Feld[1] and by Devons[2] with the exception of the spherically symmetrical, and those resulting from mixed transitions over very limited ranges of the mixtures. There is therefore no evidence from this experiment that the spin of the Li^{7*} state involved is not $\frac{1}{2}$.

Further details and discussion will be published elsewhere.

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[2] S. Devons, Proc. Phys. Soc. **62A**, 580 (1949).
[3] L. G. Elliott and R. E. Bell, Phys. Rev. **74**, 1869 (1949).
[4] Kusch, Millman, and Rabi, Phys. Rev. **57**, 765 (1940).

Erratum: Evidence for Multiple Meson and γ-Ray Production in Cosmic-Ray Stars

[Phys. Rev. **76**, 1735 (1949)]

M. F. Kaplon, H. Bradt, and B. Peters
University of Rochester, Rochester, New York

The following misprints occurred in this letter: Line 6, third paragraph should read: ". . . that is 21 more tracks." In the equation, the member $\frac{1}{8}$ should be altered to $1/\bar{\gamma}$.

Production of π-Mesons by High Energy Electrons*

E. Strick and D. ter Haar†
Department of Physics, Purdue University, Lafayette, Indiana
February 16, 1950

In connection with the synchrotron project under way at Purdue University, it was felt useful to make an estimate of the cross section for meson production by high energy electrons. Since it was found in the case of the production of mesons by nucleon bombardment that different methods of calculation might easily lead to different answers, if proper care is not taken to take all important terms into account,[1] we have calculated the cross section for meson production by electrons of extremely high energies by two different methods.

First we used the straightforward third-order perturbation treatment. We used a charged scalar meson field in all our calculations. The calculations are straightforward though tedious, and for the case where the bombarding electron has a very large energy they lead to the following expression for the cross section σ:

$$\sigma_1 = k(g^2/\hbar c)(e^2/\hbar c)^2(\hbar/\mu c)^2(M/\mu)\log(E_0/Mc^2), \qquad (1)$$

where E_0 is the energy of the bombarding electron, μ the mass of the meson, M the mass of the bombarded nucleon, and k a numerical constant of the order of magnitude unity.

The same result was obtained independently by using the impact parameter method, i.e., the so-called Weizsäcker-Williams method.[2,3]

It can be shown that recoil effects can be neglected in the computations leading to Eq. (1). Equation (1) gives only the most important term of a complicated expression and is, therefore, only correct for large values of E_0, i.e., just in the region where the Weizsäcker-Williams method should give reliable results.

For energies just above the threshold, the cross section turns out to be given by

$$\sigma_2 = k(g^2/\hbar c)(e^2/\hbar c)^2(\hbar/\mu c)^2\epsilon^{\frac{1}{2}}, \qquad (2)$$

where ϵ is $(E_0 - E_t)/E_t$, if E_t is the threshold energy. Equation (2), which was obtained by the third-order perturbation method, is the first term in a series in ascending powers of ϵ.

For $E_0 = 300$ Mev, the cross section is about 10^{-31} cm^2 per nucleon.

We can compare the results of Eqs. (1) and (2) with the predictions made by dimensional considerations.[4] For energies just above the threshold energy, dimensional considerations give exactly the same[4] as Eq. (2). For high energies, the matrix element apparently depends on the energy, and the energy dependence of the cross section is different from an inverse square as one should expect at first.[5] The same effect was found to occur in case the bombarding particles were nucleons.[1]

Sneddon and Touschek[6] have calculated the cross section for meson production by electrons, using pseudoscalar fields. The remarkable fact is that they obtain the same cross sections as we do for scalar mesons.[7]

* Partial assistance by ONR is acknowledged.

† Present address: Plastics Institute, Julianalaan 134, Delft, Netherlands.

[1] E. Strick and D. ter Haar, Phys. Rev. **76**, 304 (1949).
[2] C. F. von Weizsäcker, Zeits. f. Physik **88**, 612 (1934).
[3] E. J. Williams, Phys. Rev. **45**, 729 (1934).
[4] The difference between the exponent of Eq. (2) and that found in Eq. (3B) of reference 5 is due to the fact that in our calculations leading to Eq. (2) we have not taken into account the momentum of the nucleon in the Fermi gas inside a nucleus. If a single proton is bombarded, the cross section is given by Eq. (2), but if nuclei are bombarded, the calculations are more complicated because of the zero-point energy of the nucleons inside the nucleus. The same difference occurs in the case of the photomesic effect and was discussed in reference 5.
[5] D. ter Haar, Science **108**, 57 (1948).
[6] I. N. Sneddon and B. F. Touschek, Proc. Roy. Soc. **199A**, 352 (1949).
[7] The occurrence of an extra factor $\log\epsilon$ in their expression for σ_2 is, as far as we can see, only apparent.

On the Ratio of Positive and Negative Photo-Mesons

OTTO HALPERN
University of Southern California, Los Angeles, California
February 14, 1950

A REPORT[1] was made recently of calculations which had been carried out to arrive at a theoretical explanation of the ratio of positive and negative photo-mesons observed. Our own unpublished calculations, although not quite as general as those reported in reference 1, have led to similar results with a characteristic difference which can perhaps be best illustrated by referring to the case of scalar coupling between π-mesons and nucleons, these last being treated as heavy Dirac particles. Here the formula of Brueckner and Goldberger[1] does not seem to take into account the processes in which the spin of the proton changes while the incident quantum is absorbed. The matrix element associated with these electromagnetic processes retains a finite value at the threshold; it occurs symmetrically (neglecting relativistic terms for nucleons) in the formulas for the cross sections of the production of mesons of both signs. On the other hand, the matrix element associated with absorption of a quantum by the proton without change of its spin differs from zero only in the production of the *negative mesons where it gives rise to the asymmetry discussed by Brueckner and Goldberger*, and there goes to zero at the threshold energy.

It is true that the matrix element referring to spin change is considerably smaller than the other, for higher energies; its observation near the threshold may become more difficult due to the capture of slow negative mesons by nuclei.

Furthermore, we encounter here a characteristic limitation of the presently accepted theory of elementary particles which has its origin in the anomalous moment of the proton (and neutron). Theory at present allows us to make quantitative predictions if the proton is treated as a Dirac particle and to make reasonable guesses if one attempts to account for the anomalous moments. Experiments near the threshold can be expected to furnish important information concerning the behavior of the anomalous moment in relativistic processes. Here the theoretical results become dependent upon the type of mesonic interaction chosen.

Summing up the results of our calculations, it can be stated that the threshold value of the cross section for the creation of photo-mesons is finite *as far as the electromagnetic matrix elements are concerned* and (if neutrons are treated as Dirac particles) is independent of the sign of the mesons created. The energy dependence of the ratio of positive to negative mesons created should give information about the anomalous moments and the type of meson interaction.

[1] K. A. Brueckner and M. L. Goldberger, Phys. Rev. **76**, 1725 (1949).

Determination of the Maximum Energy of the β-Rays from 16-Hour Am^{242}

JEAN M. GRUNLUND,* B. G. HARVEY, N. MOSS, AND L. YAFFE
Chemistry Branch, Atomic Energy Project, National Research Council of Canada, Chalk River, Ontario, Canada
February 14, 1950

A SAMPLE of carefully purified Am^{241}, mounted on Al foil, was irradiated in the NRX pile. The active material was dissolved off the foil using 8 M HNO_3 and the solution diluted to a standard volume. An aliquot of the sample was placed on a thin Formvar film ($\sim$50 μg/cm^2) and evaporated so that essentially a weightless point source was formed. Its activity was then measured using a Geiger-Müller counter with an end window (1.9 mg/cm^2).

An absorption curve of the radiation from the irradiated sample of Am^{241} was taken. The technique employed was similar to that used by Yaffe and Justus[1] to determine the maximum energy of S^{35}. The initial sample used was large enough to give a count of about 10,000 counts per minute in our geometry. Various aluminum absorbers were placed in position and the counts recorded until the transmitted intensity was reduced by a factor of 10. An additional sample of americium was then added to the original one so that the count was about 10,000 per minute with the last absorber used in position, and additional absorbers were then added. This procedure was repeated, never counting less than 1000 counts per minute. The samples were then related to