possible contaminates are converted into ranges in the emulsion and plotted as a function of angle, the effect of the aluminum foil is apparent. Figure 5 shows such a plot. From this it may be seen that the only particles with sufficient energy to penetrate the foil would be deuterons scattered from some heavy nucleus such as carbon, nitrogen, or oxygen. However, the low rate of rise of pressure in the closed-off system combined with $p-p$ scattering[9] data which showed no appreciable scattering from such heavy nuclei would seem to indicate that contamination was quite small. This same plot also shows that the amount of hydrogen contamination of the deuterium is not important as far as its contribution to scattered protons is concerned, but of course must be known in order to calculate the correct number of target nuclei per cm^3.

According to theoretical predictions, the cross section of the center-of-mass system should obey an expression of the form $\sigma(\alpha)=A+B\cos^2\alpha+\cos^4\alpha+\cdots$. Attempts to fit the experimental data with terms up to $\cos^6\alpha$ were unsuccessful; however, a reasonable fit was obtained with $\cos^8\alpha$ terms. These coefficients are given in Table IV. A different set would be obtained if slightly different behavior were assumed about the 90° region.

The total cross section for the production of protons

TABLE IV. Coefficients for fit of data to the series.

$\sigma(\alpha)=A+B\cos^2\alpha+C\cos^4\alpha+D\cos^6\alpha+E\cos^8\alpha$
$A=+0.442\times10^{-26}$ cm²
$B=+0.708\times10^{-26}$ cm²
$C=-5.238\times10^{-26}$ cm²
$D=+5.122\times10^{-26}$ cm²
$E=+1.942\times10^{-26}$ cm²

was obtained from the above values by computing the sum

$$\sigma_T=4\pi\sum_{\alpha=0}^{\alpha=90}\sigma(\alpha)\sin\alpha\Delta\alpha$$

for $\Delta\alpha$ taken in 10° steps. The value so obtained was $\sigma_T=7.3\times10^{-26}$ cm².

VII. ACKNOWLEDGMENT

The authors wish to express their thanks to Professors G. F. Tape and A. O. Hanson for the many helpful discussions of the problems associated with this experiment; to Professor R. E. Meagher for his cooperation throughout the experiment and help in making many of the measurements; and to the cyclotron staff for their assistance in carrying out the numerous tasks connected with the construction and operation of the equipment.

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Proton-Proton Scattering at 5 Mev

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The differential cross section for the scattering of protons by protons has been measured at an energy of 4.96±0.08 Mev, at angles from 12.5° to 55° in the laboratory system. The scattering medium was hydrogen gas at a pressure of approximately 5 cm of Hg. The scattered or recoil protons were detected by nuclear track photographic plates. Approximately 10,000 proton tracks were counted at each angle of observation. An estimate indicates the error of each individual cross section to be about two percent. Because the observations at each angle are made simultaneously, the relative accuracy from one angle to another is smaller and amounts to about 1.3 percent.

I. INTRODUCTION

THE scattering of protons by protons has been studied at a number of energies between 0.5 and 4.2 Mev[1–4] and at 7, 8, 10, and 14.5 Mev.[5–8] This experiment provides data at 5 Mev with hydrogen gas as the scattering medium and photographic plates for the detection of scattered protons in much the same way as the scattering experiment which was done at 7 Mev by Dearnley, Oxley, and Perry.[5] However, the details of the slit system differ from those reported by them. The photographic method of detection was selected because it provided a convenient method of rejecting background counts, otherwise troublesome in electrical systems, and because it permitted the recording of a large amount of data in a relatively short cyclotron operating time.

The scattering chamber described by Rodgers, Leiter and Kruger[9] was used to carry out this experiment.

* Assisted by the Joint Program of the ONR and AEC.

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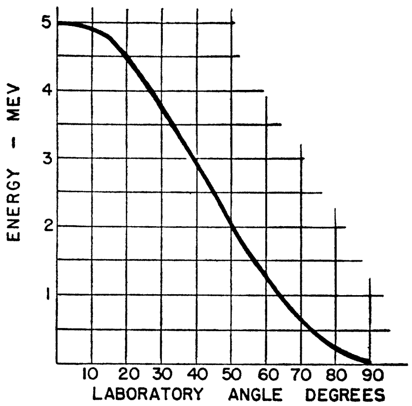

FIG. 1. Energy of scattered protons *vs*. laboratory scattering angle.

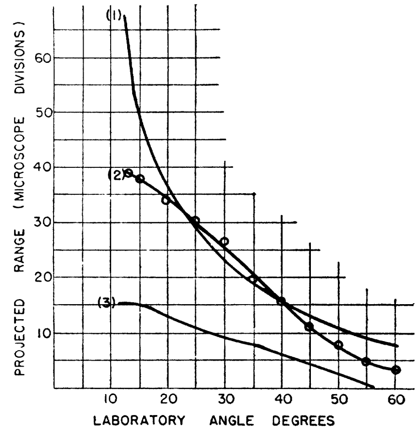

FIG. 2. (1) The projected range available *vs*. angle for an emulsion thickness of 50μ. (2) The observed maximum projected range of protons *vs*. angle. In the region between 25° and 38° a large fraction of the protons would actually hit the glass backing of the photographic plate. (3) The minimum projected range considered acceptable in counting tracks.

The incident beam of protons required for this experiment was obtained by accelerating singly charged molecular hydrogen ions (H_2^+).

II. SLIT SYSTEM

The equations relating the constants of the apparatus to the differential cross section are given elsewhere,[10] while the numerical values of the slit constants are given in reference 9.

In this experiment the length of the swath, either 1 cm or 1.5 cm, and the number of swaths counted for each observed angle of scattering were adjusted so that the number of tracks counted was between 5000 and 6000 per exposure per angle. A total of about 10,000 tracks were counted in two exposures for each angle. These exposures were called Run 8 and Run 10 and the final equations giving the cross section as a function of angle for these exposures are:

Run 8

$$\sigma_{\text{lab}} = (\text{counts per 1-cm swath}) \times (1.205^4 \times 10^{-4})/\sin^2\theta \sin(\theta+\phi) \text{ barns.}$$

Run 10

$$\sigma_{\text{lab}} = (\text{counts per 1-cm swath}) \times (1.239^4 \times 10^{-4})/\sin^2\theta \sin(\theta+\phi) \text{ barns,}$$

where a barn is 10^{-24} cm^2.

The penetration correction has been discussed for this apparatus in reference 9. In this experiment the fractional increase in slit width is 0.03, 0.03, 0.04, 0.03, 0.03, 0.02, 0.02, 0.01, 0.01, and 0.01, for laboratory angles of 12.5°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°, and 55°, respectively.

A simple experiment was carried out with slits which were approximately just closed to verify the correctness of the slit penetration correction. Within statistical errors the calculated corrections were verified.

A run was carried out which indicated that the slit scattering in this experiment amounted to approximately 0.2 percent of a normal run at 15° and about 0.02 percent of a normal run at 45°. Corrections corresponding to these figures were used in obtaining the final cross sections.

The angular resolution varied from about ±1° at 12.5° to ±2° at 55°.

III. DETECTION OF PROTONS

Just as in references 9 and 10, the plates were designated C-2 and manufactured by the Ilford Company. The energy of the protons as a function of angle which the plates must register is easily calculated from the equations for conservation of energy and momentum. A graph of proton energy as a function of angle is given in Fig. 1.

The Counting of Proton Tracks

In observing the proton tracks at a given angle, the projected length varies from one track to another. The expected length of a track can be plotted and is shown in Fig. 2 as curve (2). Any given track may appear longer than that shown by a small amount because the proton may be scattered so that it makes a smaller angle with the plate than an unscattered proton. Any given track may be shorter than the curve (2) because of angular scattering, because it hits the glass backing (between 24° and 38°), because it has penetrated the slit, or because it results from some undesirable reaction unrelated to p–p scattering. Most of the tracks do fall into one large group. This can be seen by a range analysis as shown in Fig. 3.

It is not desirable to count the very short tracks because they do not represent p–p scattering (unless they penetrate the slit edge). On the other hand, relatively small amounts of scattering in angle can considerably reduce the projected range. It is, therefore, necessary to count tracks considerably shorter than the

[10] Leiter, Rodgers, and Kruger, Phys. Rev. **78**, 663 (1950).

normal length. The criterion selected for counting was such that the minimum length had to be greater than 40 percent of the normal length [Fig. 2, curve (3)].

The counting was carried out by three persons.[11] After an initial practice period it was found that any two observers agreed on the number of tracks in a given swath to better than 0.5 percent. No observer was consistently high or low. An individual observer can usually repeat his own count of a given swath to somewhat better than 0.5 percent.

IV. MEASUREMENT OF PROTON BEAM CURRENT

Proton Beam and Faraday Cage

The beam used in this experiment was obtained by accelerating molecular hydrogen ions (H_2^+). This has the advantage that the resonance and focusing conditions are substantially the same as for deuterons for which the cyclotron had been adjusted. The two protons in such an ion are, of course, bound by only a few volts so that there should be no appreciable effect on the scattering or the measurement of current since the beam must pass through a Nylon foil and a dural foil, each of which absorbs energy.

In order to show that the use of H_2^+ ions did not affect the beam current measurement, a simple experiment was performed by noting the size of the current received at the Faraday cage with and without the presence of the Nylon and aluminum foils. This experiment indicated that the current received at the Faraday cage was larger by the factor 2.03 ± 0.03 than when no foils were present. In the measurements made of proton current no error has been listed due to this cause.

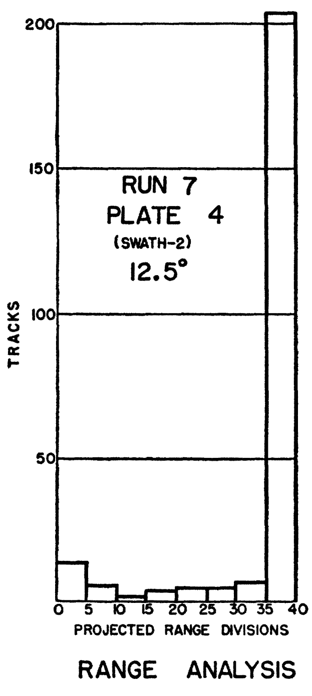

FIG. 3. A typical histogram of the tracks observed with a normal slit.

[11] H. A. Leiter, F. A. Rodgers, and the author.

TABLE I.

Faraday cage volts with respect to ground	Main cyclotron target beam current in μa	Faraday cage beam current in amp. No magnet	Magnet
+45	14 ± 1	6×10^{-11}	10.0×10^{-11}
0	14 ± 1	10×10^{-11}	10.1×10^{-11}
−45	14 ± 1	15×10^{-11}	10.0×10^{-11}

An appreciable number of secondary electrons are associated with any surface struck by the beam. A permanent magnet with a field of about 2000 oersteds was placed so that its field was perpendicular to the beam at the entrance to the Faraday cage to prevent these electrons from invalidating the readings. Again a simple experiment was performed to check this. An arrangement was made to change the potential of the Faraday cage with respect to ground (the outside vacuum-tight shell) and at the same time to read current. The current at the main cyclotron target was held as constant as possible. If secondary electrons were causing any current to be registered then the accelerating voltage (45 v) would tend to change the measured value of the beam. The results are shown in Table I. Although the readings were taken rapidly so that adjustments could not drift very much, the experiment cannot be considered precise. Nevertheless, it is clear that any secondary electron effect (or leakage due to gas ionization current) must be fairly small. In measurements the effect was assumed to be negligible.

The pressure in the Faraday cage during runs was 2×10^{-5} mm of Hg as measured on an ion gauge.

Charge Measurement

An electronic circuit, briefly referred to in this paper as a "*Q*-meter," was used to integrate over time the current passing to the Faraday cage. Its operation was fundamentally very simple: The current from the proton beam caused a condenser of 33 $\mu\mu$f to charge up in the positive direction with respect to ground. When the potential on the condenser changed by 3 v the electronic circuit automatically shorted the condenser back to its starting potential and a total charge of about 10^{-10} coulomb was registered as a count. The process was repeated indefinitely and the number of counts was observed by a suitable scalar and register circuit.

Because the currents measured were of the order of 10^{-9} amp. electrometer type tubes were used in the input to the circuit.

The condenser of 33 $\mu\mu$f was made up mainly of the capacity of the Faraday cage to ground and to a less extent of the connecting lead wires and the electrometer tubes. Except for the actual input circuit to which the Faraday cage was connected, the circuit was a.c. coupled so that changes of d.c. potential associated with electrometer tubes could not affect the calibration. It was therefore the change in voltage of three volts

TABLE II. Run constants.

	Accepted	Closed slit run	Accepted
Run No.	8	9	10
Date of run (1948)	Nov. 18	Nov. 22	Nov. 23
Faraday cage pressure (10^{-5} mm Hg)	1.5	2.5	2
Vacuum in chamber (10^{-5} mm Hg)	5	2.0	4
Rate of rise (10^{-4} mm Hg per min.)	1.25	0.5	1.6
Beginning pressure cm of oil	75.920	74.39	73.040
Beginning temperature of oil in °C	23.7	22.6	21.5
Beginning temperature of H_2 in °C	25.0	24.8	22.9
Final pressure cm of oil	76.405	75.110	73.742
Final temperature of oil in °C	24.2	23.3	21.8
Final temperature of H_2 in °C	25.9	25.8	23.5
Total Q counts	19,204	19,202	19,201
Time of run in sec.	1632		2802
Acceleration of gravity = 980.14 cm/sec.^{2}			

across the condenser which was held constant rather than the absolute d.c. values of the voltage across the condenser.

Calibration of the instrument was accomplished by using a high resistance and knowing the average voltage across the resistor. The voltage was read on two voltmeters, one at each end of the resistor measuring the respective potentials to ground. One of the voltmeters was of necessity a vacuum tube voltmeter. Both voltmeters were originally calibrated against a standard cell but their panel type movements permit reading the voltage to only about ±0.5 percent.

The resistors were calibrated against a Victoreen resistor labeled 2540 megohms and calibrated by the Bureau of Standards to be 2450 megohms at 25°C and 2440 megohms at 30°C with a stated accuracy of ±0.5 percent. The resistances calibrated were from 5000 to 100,000 megohms. A simple shielded Wheatstone bridge was set up with the calibrated resistor as standard. The detector for null was a polystyrene dielectric condenser of 0.1 μf. Time was allowed for the condenser to charge and then its potential was noted by the size and polarity of the signal as observed in an oscilloscope when the condenser was discharged through a 10-megohm resistor. Since this method is subject to some systematic errors a considerable error must be assigned to this measurement, especially for resistors higher than 5000 megohms. An error of one percent has been assigned to this. Considering these three errors to be 0.5, 0.5 and, one percent and combining them,[12] one obtains 1.22 percent as the error in charge measurement.

The calibrations for run 8 and run 10 made immediately preceding the runs is shown in Fig. 4.[13]

It may be noted that at low currents the "Q-meter" calibration constant increased but the instrument was generally used considerably above this region. It may be noted that the shape of the calibration curve had changed from run 8 to run 10. No explanation of this can be given though it is not reasonable that the shape should change. Accordingly, a sort of average line has been drawn for both calibrations and an error of perhaps 0.5 percent must be considered in ability to repeat measurements. This brings the combined error for charge measurement up to ±1.32 percent.

There was some evidence that violent arcs of the cyclotron power oscillator affected the Q-meter by the insertion of extra counts. On a single such arc-back as many as 20 counts were noted. Out of a normal 19,200 counts for a run this is seen to be a possible 0.1 percent per arc-back. However, "violent" arc-backs were eliminated by operating the power oscillator on a lower step but some smaller arcs almost always occur during a run of 30 min. In trying to simulate the effect during a calibration the effect was small compared to 0.5 percent.

V. MEASUREMENT OF PROTON ENERGY

The measurement of proton energy was made by a simple range measurement. With the chamber evacuated and the Faraday cage assembly removed, the beam passed through a single dural foil about 0.001 in. thick. The maximum range of the protons was then noted on a fluorescent screen while the analyzer current was close to 28.8 amp. which was the normal value of the analyzer current during the runs. The measurements made were: "maximum" range in air = 30.4±0.5 cm, atmospheric pressure = 74.79 cm of Hg, temperature = 20°C.

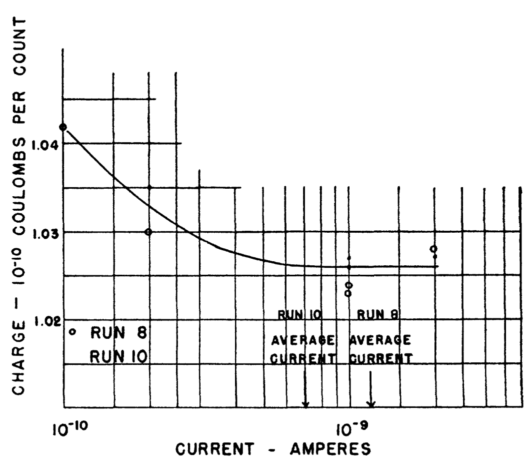

FIG. 4. Calibration of the Q-meter circuit made immediately preceding run 8 and run 10. Check points are shown taken at 10^{-9} amp. immediately following each run.

[12] R. T. Birge, Am. Phys. T. 7, 352 (1939).

[13] A second method of calibration of the Q-meter was developed by G. F. Tape and J. G. Cottingham after the completion of the experiment. The charge required for calibration was supplied (through one of the calibration resistors) by a standard condenser. The amount of the charge was determined by the measurement of a potential difference (measured with a potentiometer and standard cell with a vacuum tube circuit as detector) and a standard capacitance calibrated by the Bureau of Standards. A comparison of the two methods, resistor and capacitor, showed a difference of 1.3 percent, the capacitor method giving the lower value of coulombs per count. It is believed that the capacitor method is the more accurate of the two. Consequently, the quoted cross sections may be too low by 1.3 percent in absolute value.

TABLE III. Results of runs 8 and 10.

θ	Number of 1-cm swaths per run	Run 8 Total counts	Run 8 σ_{lab} (uncorrected) barns	Run 10 Total counts	Run 10 σ_{lab} (uncorrected) barns	σ_{lab} average (barns)	σ_{lab} corrected for slit penetration (barns)	σ_{lab} corrected for vacuum run (barns)	$\sigma_{C.M.}$ (barns)
12.5°	36	5763	0.4881	5474	0.4767	0.4824	0.4686	0.4679	0.1198
15	27	5150	0.3960	4780	0.3780	0.3870	0.3750	0.3741	0.0968
20	18	5747	0.3629	5455	0.3541	0.3585	0.3465	0.3463	0.0921
25	12	5864	0.3509	5625	0.3461	0.3485	0.3370	0.3369	0.0929
30	9	6196	0.3435	6033	0.3440	0.3438	0.3334	0.3333	0.0962
35	6	5265	0.3264	5285	0.3369	0.3317	0.3242	0.3241	0.0989
37.5				5750	0.3232	0.3232	0.3163	0.3163	0.0997
40	6	6537	0.3190	6064	0.3043	0.3117	0.3054	0.3054	0.0997
40.5				6301	0.3095	0.3095	0.3034	0.3034	0.0998
45	1183	7102	0.2852	6988	0.2888	0.2870	0.2834	0.2833	0.1002
50				7512	0.2655	0.2655	0.2628	0.2628	0.1022
55				7304	0.2284	0.2284	0.2267	0.2267	0.0988

TABLE IV. Other corrections and final cross sections.

θ	Number of swaths counted per plate	Length of swath cm	$\frac{\Delta b \sin 2\theta}{2(b-a)}\frac{180}{\pi}$	% per degree correction	% correction due to angle	% correction due to $(b-a)$	% correction due to b	All Table IV corrections in percent	$\sigma_{C.M.}$ from Table III (barns)	$\sigma_{C.M.}$ after all corrections (barns)
12.5°	4	1.5	−0.147	+4.76	−0.70	+1.14	+0.80	+1.24	0.1198	0.1213
15	3	1.5	−0.115	+5.71	−0.65	+0.81	+0.54	+0.70	0.0968	0.0975
20	2	1.5	−0.067	+9.00	−0.60	+0.37	+0.25	+0.02	0.0921	0.0922
25	2	1.0	+0.0069	+7.88	+0.05	−0.03	−0.02	0	0.0929	0.0929
30	1	1.5	+0.022	+5.71	+0.13	−0.09	−0.06	−0.02	0.0962	0.0962
35	1	1.0	+0.097	+4.25	+0.41	−0.34	−0.23	−0.16	0.0989	0.0988
37.5	1	1.0						−0.27	0.0997	0.0994
40	1	1.0	+0.129	+2.82	+0.36	−0.45	−0.30	−0.39	0.0997	0.0993
40.5	1	1.0						−0.41	0.0998	0.0993
45	1	1.0	+0.156	+1.90	+0.30	−0.53	−0.36	−0.59	0.1002	0.0996
50	1	1.0	+0.173	0	0	−0.62	−0.42	−1.04	0.1022	0.0992
55	1	1.0	+0.180	−1.23	−0.22	−0.67	−0.45	−1.34	0.0988	0.0975

Correcting the maximum range to an atmosphere of 76 cm of Hg and 15°C one gets 29.4±0.5 cm as the maximum range. Since the mean range is about 2.7 percent less than the maximum range observed in this way,[14] the mean range is 28.6±0.5 cm. This gives an energy of 4.53±0.05 Mev. To this energy must be added the energy lost in passing through the dural foil.

From Smith[15] the figure of 8.2 mg/cm^2 of Al is found equivalent to 0.5 Mev of protons between 4.5 Mev and 5.0 Mev. From Aron[16] 2.00 mg/cm^2 of Cu is equivalent to 1 cm of air for 5.0-Mev proton, or since about 5.6 cm of air is equivalent to 0.5 Mev of protons then $2\times 5.6 = 11.2$ mg/cm^2 of Cu is equivalent to 0.5 Mev of protons.

Since dural is approximately 95 percent aluminum and five percent copper, then $8.2\times 0.95 + 11.2\times 0.05 = 8.35$ mg/cm^2 of dural is equivalent to 0.5 Mev of protons. Two samples of dural foil from the same sheet as the piece used at the entrance to the Faraday cage gave 7.50 mg/cm^2 and 7.55 mg/cm^2. Taking the average, one gets for the foil thickness $(7.525/8.35)\times 0.5 = 0.45$ Mev. The error in this figure is a little difficult to assess.

The energy of the beam in the scattering chamber without hydrogen was then 4.98±0.05 Mev. The beam passed through about 20 cm of hydrogen gas at a

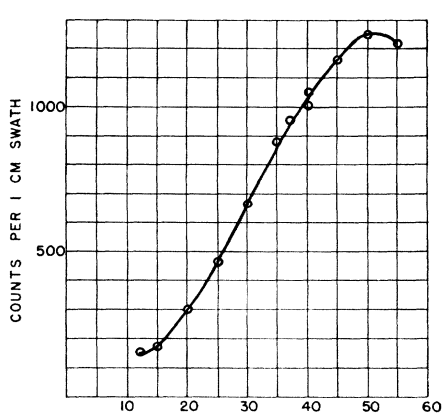

FIG. 5. The density of tracks in counts per swath area 1 cm ×0.01869 cm taken from the results of both run 8 and run 10.

[14] In an experiment which was carried out at a later date on a 10-Mev deuteron beam measuring the ionization current as a function of range Mr. C. J. Taylor has found that the mean range is about 2.7 percent less than the "maximum" range which probably would be observed by using a fluorescent screen. This same experiment would give the energy of the proton beam in the scattering chamber to be 4.93±0.05 Mev if the proton beam were assumed to have half of the energy of the deuteron beam.

[15] J. H. Smith, Phys. Rev. **71**, 32 (1947).

[16] *Handbook of Radioactivity and Tracer Methodology* (1948), Air Force Technical Report No. 5669, p. 183.

pressure of 5 cm of Hg before it reached the center of scattering volume during a normal experiment. This reduced the energy to about 4.96±0.05 Mev.

An additional uncertainty in energy was introduced by the fact that the analyzer was adjusted during a run in such a way as to keep the beam in the Faraday cage a maximum. The analyzer current was thus not strictly a constant and its variation may have been due to iron hysteresis or beam energy or a combination of the two. The average variation during a run did not amount to more than ±0.2 amp. The normal current was 28.8 amp. Since the field H is proportional to the current at the low values of induction used, and the energy is proportional to the square of the field, the energy uncertainty corresponding to ±0.2 ampere is about ±1.4 percent.

Combining these errors one obtains the energy as 4.96±0.08 Mev.

VI. SCATTERING RUNS

Procedure

The filling of the main scattering chamber with hydrogen has been discussed in reference 9. Similarly, the procedure carried out in taking the data was the same. The important constants associated with the three runs from which the data were analyzed are given in Table II.

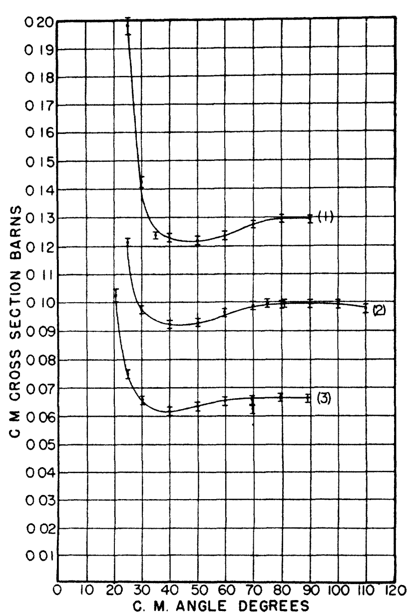

FIG. 6. Cross section *vs.* angle. (2) 4.96±0.08 Mev from Table IV; for comparison with the results published at other energies (1) shows 3.53±0.02 Mev from Blair, Freier, Lampi, Sleator, and Williams (reference 3) and (3) shows 7.03±0.06 Mev from Dearnley, Oxley, and Perry (reference 5). The errors drawn for (1) are the uncertainties listed by the authors in reproducing the cross sections from week to week, while those drawn in (2) and (3) are the relative errors from one angle to another.

Results

The results of runs 8 and 10 are shown in Tables II–IV.

A simple statistical analysis has been made of run 10. It shows that the number of counts N on 39 percent of the points on each plate deviate by more than $(N)^{\frac{1}{2}}$ with respect to the average N for the corresponding angle on all plates. Also, totaling all the counts on each plate shows that two out of the six plates deviate by more than $(N)^{\frac{1}{2}}$ from the average for all six plates.

The total number of tracks counted at each 5° angle is somewhat over 10,000 except at 50° and 55°. The total number of tracks recorded for these data is 120,175.

The conversion from laboratory cross section to center-of-mass cross section is carried out by noting that $\sigma_{\text{C.M.}}=\sigma_{\text{lab}}/4\cos\theta$ where θ is the angle in the laboratory system.

Figure 5 gives the number of tracks per 1-cm swath area (0.01869 cm^2) on a plate as function of angle.

Because the 40° position on run 10 gave a low cross section, counts were made at 37.5° and 40.5°.

Corrections

Measurements of the photographic plate positions indicated that the photographic plates were not parallel to the beam. The diameter $(2b)$ was measured at the low angle end and the high angle end of each pair of plates. The measurements are shown in Table V.

As a consequence of this the photographic plates were 0.0147 cm farther from the beam at the low angle end than normal and 0.0193 cm closer to the beam at the high angle end than normal. This causes certain errors, which together with an error due to the fact that the photographic plates are flat instead of curved need to be considered before obtaining the final cross sections. These corrections are given in Table IV and will now be considered.

The angle θ at which each plate was counted was not the even angle listed in the tables because b was not the normal value. The radius b is in error both because the radius changes slightly with position on the plate and because the plate is flat. The first of these errors is obtained directly from the plate position error, while the second requires finding the average angle which results from using a straight swath rather than a curved swath. The error in angle $\Delta\theta$ from these two effects is

$$\Delta\theta=\frac{\Delta b\sin2\theta}{2(b-a)}\frac{180}{\pi}\text{ degrees.}$$

The error in angle $\Delta\theta$ together with Fig. 5 makes possible the correction in percent due to the angle error.

An error in $(b-a)$, the radial distance from the slit to the plate, is a simple geometrical error due to the error in b.

The error in b is the result of the fact that the azimuthal angle is in error. The azimuthal angle in the derivation has been taken to be $2(c/b)$ radians where c is one-half the swath length. On the other hand, the exact azimuthal angle, since the photographic plate is flat is

$$2 \text{ arc } \tan\frac{c}{b} = 2\left[\frac{c}{b} - \frac{c^3}{3b^3} + \cdots\right]$$

so that a correction of $-\frac{1}{3}c^2/b^2$ is necessary.

The last column of Table IV gives the final center-of-mass cross sections for each angle.[17] These are plotted in Fig. 6.

ERRORS

An attempt is made to assign errors to each quantity entering into the final cross sections. The point of view chosen will be that of the assessment of errors for a given point on an absolute basis. Clearly some of the errors could be omitted if relative errors only, for example from one angle to another, were considered. It is believed that the errors listed below are r.m.s. errors. They will be combined by taking the square root of the sum of the squares.[12] In each case in the final table the actual error of the quantity is not necessarily listed but rather its effect upon the cross section.

Tabulation of errors.—Number of counts: human counting error, 0.5 percent; statistics, 1.0 percent.

Incident beam charge (Q-meter), 1.32 percent.

[17] The S-wave phase shifts corresponding to these data are given by J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. 22, 77 (1950).

TABLE V.

Plates	High angle end b	Low angle end b
1–4	4.752 cm	4.795 cm
2–5	4.775	4.797
3–6	4.757	4.795
Average	4.761	4.795

Atoms per cm^3 of gas: height of oil column, 0.05 percent; temperature, 0.3 percent; oil density, 0.1 percent.

Solid angle: swath width, 0.3 percent; swath length, 0.4 percent; b (radius to plate), 0.1 percent.

Length of target: W (slit width), 0.1 percent; $(b-a)$ 0.4 percent.

Slit penetration correction, 0.5 percent; slit angle θ, 0.1 percent.

Square root of the sum of the squares of the preceding errors, 1.94 percent.

Energy, 1.7 percent.

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