

The Angular Distribution of Protons from the D—D Reaction at 10 Mev*

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 (Received November 7, 1949)

The gas-filled scattering chamber has been applied to the study of the distribution in angle of the protons from the D—D reaction with 9.94 ± 0.08 -Mev deuterons from the cyclotron. Photographic plates were used as detectors. The other charged particles produced in the reaction and from scattering processes were prevented from reaching the emulsion by covering the photographic plates with a 53.2-mg/cm^2 aluminum foil. Differential cross sections were determined at 10° intervals from 15° to 65° in the laboratory system. Background runs were made to determine the correction necessary because of the flux of neutrons in the chamber. Additional corrections were made for the penetration of the protons through the edges of the defining slit and for geometrical errors. The total cross section for the production of protons was calculated to be $7.3 \times 10^{-26} \text{ cm}^2$.

I. INTRODUCTION

THE nuclear reactions

$$\text{H}^2 + \text{H}^2 \rightarrow \text{H}^3 + \text{H}^1 + Q_1 \quad (1)$$

$$\rightarrow \text{He}^3 + n^1 + Q_2 \quad (2)$$

have been known for many years and have been the subject of many papers.¹ The angular distribution of the disintegration products has been extensively studied² also.

It is the purpose of this paper to report on the angular distribution of the protons in reaction (1) when the energy of the incident deuteron is 10 Mev. A preliminary report³ has been given earlier.

II. EXPERIMENTAL PROCEDURE

The apparatus used in this experiment was that used for measurements on deuteron-proton scattering⁴ and proton-proton scattering⁵ in this laboratory and has been described in detail in those papers.

* This work has been supported jointly by the ONR and the AEC.

† Submitted in partial fulfillment of the requirements for the Ph.D. at the University of Illinois. Now at Westinghouse Research Laboratories, East Pittsburgh, Pennsylvania.

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¹ Oliphant, Harteck, and Rutherford, Proc. Roy. Soc. A144, 692 (1934); R. Ladenburg and M. H. Kanner, Phys. Rev. 52, 911 (1937); R. B. Roberts, Phys. Rev. 51, 810 (1937); Amaldi, Hafstad, and Tuve, Phys. Rev. 51, 896 (1937); L. I. Schiff, Phys. Rev. 51, 783 (1937); R. B. Myers, Phys. Rev. 54, 361 (1938); Haxby, Allen, and Williams, Phys. Rev. 55, 140 (1939).

² P. I. Dee, Nature 133, 564 (1934); Kempton, Brown, and Maasdrorp, Proc. Roy. Soc. A157, 386 (1936); H. Neuert, Physik. Zeits. 38, 122 (1937); Huntoon, Ellett, Bayley, and Van Allen, Phys. Rev. 58, 97 (1940); Manning, Huntoon, Myers, and Young, Phys. Rev. 61, 371 (1942); Bennett, Mandeville, and Richards, Phys. Rev. 69, 418 (1946); Bretscher, French, and Seidl, Phys. Rev. 73, 815 (1948); E. J. Konopinski and E. Teller, Phys. Rev. 73, 822 (1948); Blair, Freier, Lampi, Sletor, and Williams, Phys. Rev. 74, 1599 (1948); Rosen, Tallmadge, and Williams, Phys. Rev. 75, 1632 (1949).

³ Leiter, Meagher, Rodgers, and Kruger, Phys. Rev. 76, 167(A) (1949).

⁴ Rodgers, Leiter and Kruger, Phys. Rev., 78, 656 (1950).

⁵ R. E. Meagher, Phys. Rev. 78, 667 (1950).

A preliminary analysis⁶ of the two reactions and the elastic scattering of deuterons by deuterons based on the energy and momentum conservation laws yielded the results shown in Fig. 1 for the ranges of the charged particles in the emulsion. The calculations assumed $Q_1 = 3.98$ Mev, for reaction 1, $Q_2 = 3.25$ for reaction 2, $E = 10$ Mev, and a stopping power of 2000 for protons in the emulsion. Also shown in Fig. 1 is a curve giving the equivalent range in an 8-mil aluminum foil. It is seen that this thickness of foil is sufficient to stop the nuclei H^3 and He^3 and elastically scattered deuterons, while the protons would have enough energy to penetrate the foil and register in the emulsion. The neutrons from the reaction and those produced in other (d, n) processes would cause recoil protons to be produced in

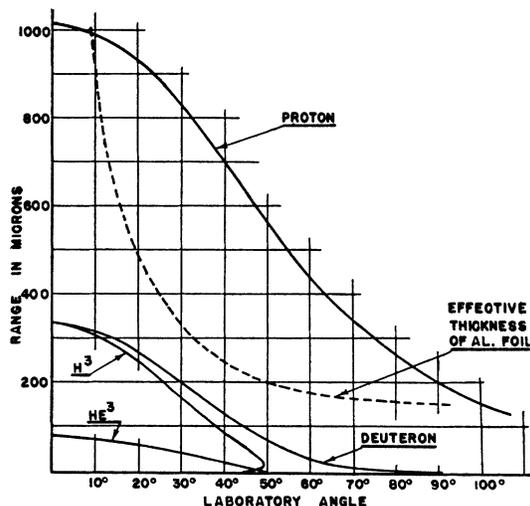


FIG. 1. Ranges of charged particles emitted in the D-D reaction and deuterons scattered by deuterons in Ilford C-2 emulsion. Dashed lines show effect of 8-mil aluminum foil covering emulsion.

⁶ Treatments of the mechanics involved in nuclear reactions may be found in M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 276 (1937); C. E. Mandeville, J. Frank. Inst. 244, 385 (1947); L. I. Schiff, Quantum Mechanics (McGraw-Hill Book Company, Inc., New York, 1949), 96; A. O. Hanson and R. F. Taschek, Preliminary Report No. 4, Nuclear Science Series, National Research Council.

the emulsion while the high energy neutrons might also produce deuteron recoils of sufficient energy to penetrate the foil. These effects would have to be determined by background runs in which the protons would be prevented from reaching the emulsion.

The procedure followed for the experimental runs was identical with that described in reference 4. For the final data two runs and two background runs were made, exposing six plates in each run. One background run was made with a hollow brass tube (of a thickness sufficient to stop the protons) extending from the plate holder to the front of the chamber, while the second run was made with a brass plate covering the annular defining slit.

III. CALCULATIONS

The differential cross section in the laboratory system for the reaction can be expressed by the equation

$$\sigma(\theta) = \frac{\text{No. of protons per unit solid angle}}{\text{No. of incident deuterons}} \cdot \frac{1}{\text{No. of target nuclei per cm}^2}. \quad (3)$$

For convenience the factor giving the number of target nuclei per cm^2 is split into two parts: the number of atoms per cm^3 , which may be easily calculated from the measured temperature and pressure of the gas in the chamber, and the effective length of the beam from which the protons can reach the photographic plate, which may be calculated from the geometry involved.⁷ Thus, the above equation becomes

$$\sigma(\theta) = N_0(\theta)/n_i n_t L(\theta) \delta\Omega, \quad (4)$$

where $\sigma(\theta)$ is the laboratory cross section per unit solid angle, $N_0(\theta)$ is the number of protons projected into a solid angle at an angle θ with respect to the

deuteron beam, n_i is the number of incident deuterons in the beam, n_t is the number of target nuclei per cm^3 , and $L(\theta)$ is the effective target thickness.

The quantities $\delta\Omega$ and $L(\theta)$ depend on the geometry of the camera and the area in which the tracks are counted. From Fig. 2 the solid angle $\delta\Omega$ for a rectangular swath of width w across one of the plates at a radius b from the beam axis is seen to be

$$\delta\Omega = \frac{w\gamma \sin^2\theta}{b}, \quad (5)$$

where γ is the angle at the axis subtended by the length of the swath. From Fig. 3 the effective target thickness is

$$L(\theta) = \frac{W \sin(\theta + \phi)}{\sin\theta} \cdot \frac{b}{b-a}, \quad (6)$$

where W is the optical width of the slit, ϕ is the angle of inclination of W with respect to the beam axis, b is the radial distance from the axis to the photographic plate, and a is the radial distance from the axis to the center of the slit. A small correction is necessary for the finite width of the swath.

To find the cross section in the center-of-mass system use is made of the definition

$$\sigma_{C.M.}(\alpha) d\Omega_\alpha = \sigma_{\text{lab}}(\theta) d\Omega_\theta \quad (7)$$

where α is the angle of emission of the proton in the center-of-mass system. The ratio of the solid angles may be calculated from

$$g(\alpha, \theta) = d\Omega_\theta / d\Omega_\alpha = \sin\theta d\theta / \sin\alpha d\alpha \quad (8)$$

which can be put into the form

$$g(\alpha, \theta) = \cos(\alpha - \theta) \sin^2\theta / \sin^2\alpha \quad (9)$$

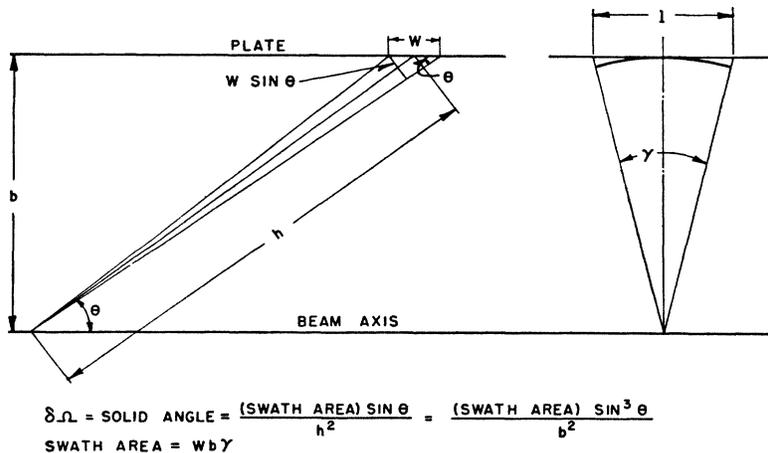


FIG. 2. Geometry for the calculation of the solid angle defined by the swath area in which tracks are counted.

⁷ The viewpoint adopted here differs slightly from that in reference 4 with respect to the factor $\theta\delta L(\theta)$, following more closely that of Herb, Kerst, Parkinson, and Plain, Phys. Rev. 55, 998 (1939).

TABLE I. Constants of the runs.

	Run A	Run B
D ₂ pressure	74.79±0.02 cm of oil	76.54±0.02 cm of oil
D ₂ temperature	24.0±1°C	25.9±1°C
Oil temperature	23.0±1°C	24.1±1°C
Oil density	0.9118 g/cm ³ ±0.1%	0.9110 g/cm ³ ±0.1%
Q-meter counts	192,003	136,929
Coulombs/count	1.036×10 ⁻¹⁰ ±2%	1.032×10 ⁻¹⁰ ±2%
Coulombs	1.989×10 ⁻⁵ ±2%	1.413×10 ⁻⁵ ±2%
n _i	1.242×10 ¹⁴ ±2%	0.882×10 ¹⁴ ±2%
n _i (for 99.5% D ₂)	3.243×10 ¹⁸ ±0.4% atoms/cm ³	3.294×10 ¹⁸ ±0.4% atoms/cm ³

by making use of the relations

$\sin(\alpha - \theta) = \text{vel. of center of mass/vel. of proton}$
in center-of-mass system $\sin\theta$ and

$$v_{C.M.}/v_{p,C.M.} = (E/3(E+2Q_1))^{1/2} \quad (11)$$

where E is the energy of the incident deuteron in the laboratory system. Thus the cross section in the center-of-mass system is obtained by multiplying the laboratory cross section by $g(\alpha, \theta)$.

IV. EXPERIMENTAL DATA

The run constants for the two experimental runs are given in Table I. The geometrical constants associated with the apparatus are given in Table I of reference 4.

V. CORRECTIONS

Because of the high energy of the protons from the reaction the correction due to protons which penetrate the edges of the defining slit should be considerable. Following the procedure outlined in reference 4 [see Fig. 7 and Eq. (10) of that paper], the ratio of the penetration tracks to the open slit tracks was calculated. This was done by applying the integral stopping power curves for the emulsion⁵ to the minimum range of track accepted in counting to find the energy of the proton entering the emulsion. A similar calculation involving the stopping power of the aluminum foil⁸ gave the energy of the proton entering the foil. This energy was converted into range in copper⁹ and compared with the range in copper that a proton passing through the open slit would have. The difference in these two ranges gave a value for the penetration thickness R from which the correction was then calculated. The results of this calculation are given in Table II. It is seen that the maximum value of the correction is about 11 percent, the values decreasing toward larger and smaller angles as would be expected from the shape of the slit edges. The limits on the corrections are based on the probable errors in the stopping powers of the emulsion, aluminum foil, copper, energy of the particles, and statistics of the penetration counts.

A second large correction in this experiment is necessary as a result of the flux of neutrons in the chamber. These neutrons can produce recoil protons in the emul-

⁸ J. H. Smith, Phys. Rev. 71, 32 (1947).

⁹ *Handbook of Radioactivity and Tracer Methodology* (1948), Air Force Technical Report No. 5669, p. 183.

TABLE II. Penetration and background corrections.

θ	N_p/N_0 in percent	Background correction in percent
15°	3.2±0.6	0.9±0.1
20°	7.0±0.8	1.8±0.1
25°	10.1±0.9	2.8±0.2
30°	11.5±0.9	3.5±0.3
35°	11.4±0.9	4.1±0.3
40°	11.0±0.9	4.1±0.3
45°	9.7±0.9	3.8±0.2
50°	7.9±0.6	3.2±0.2
55°	6.2±0.5	6.3±0.4
60°	4.3±0.2	6.7±0.4
65°	2.7±0.1	7.9±0.4

sion which are hard to separate from good tracks because the finite depth of focus of the microscope would not allow the observer to tell at a glance whether the track started in the surface of the emulsion or just below the surface. In addition, the high energy neutrons could impart sufficient energy to a deuteron to cause it to penetrate the aluminum foil covering the plates. These could not of course be distinguished from the good tracks.

On comparing the results of the two background runs it was found that the first run with a long tube projecting to the front of the chamber gave on the average 0.5 percent more tracks than the second run which was made with a plate over the slit. The two runs were combined, account taken of the difference, normalized to run A, and a smooth curve drawn from which the correction values were taken. These values are given in Table II.

It is seen that there is a sudden jump in the correction value at 55°. This is the result of the change in criterion of minimum acceptable track length to quite small values at 55°, 60°, and 65°. Here the tracks were so short because of the high angle of incidence that it was much more difficult to observe the direction of the tracks. This increase in the number of short tracks made the correction quite large. Consequently, the authors do not feel that too much significance should be attached to the shape of the cross-section curve at these angles. In view of this effect it is advisable whenever possible to design the camera so that reasonably long tracks may be obtained.

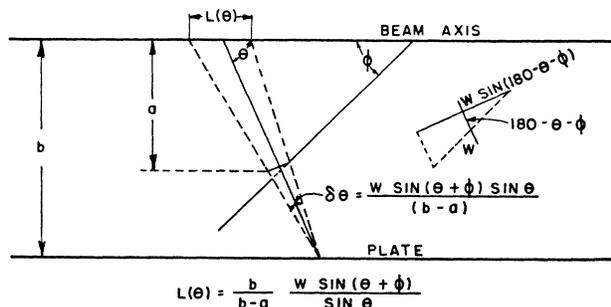


FIG. 3. Geometry for the calculation of the effective target length $L(\theta)$. Effect of finite swath width is not shown.

TABLE III. Values of cross sections.

θ_{lab}	Total No. of uncorrected tracks	$\sigma(\theta)_{\text{lab}}$	$\alpha_{\text{C.M.}}$	$\sigma(\alpha)_{\text{C.M.}}^a$	Probable error
15°	4845	$3.24 \times 10^{-26} \text{ cm}^2$	21.5°	1.62	1.9 percent
20°	7211	1.90	28.5	0.968	1.8
25°	6252	0.867	35.5	0.452	2.0
30°	3584	0.511	42.4	0.274	2.2
35°	4327	0.464	49.4	0.258	2.1
40°	4392	0.573	56.1	0.331	2.1
45°	4683	0.664	62.8	0.400	2.1
50°	5896	0.723	69.4	0.458	1.9
55°	4290	0.687	75.8	0.460	2.0
60°	3961	0.644	81.9	0.458	2.0
65°	3148	0.588	88.0	0.446	2.1

^a See footnote 13 in reference 5.

Other small corrections of less than one percent at all angles were necessary because of the lack of parallelism of the plate with the beam, and because rectangular instead of parabolic swaths were used (see reference 4).

The angular resolution varied from about 1° at $\theta = 12.5^\circ$ to about 2° for $\theta = 65^\circ$.

VI. EXPERIMENTAL RESULTS

The results of the two experimental runs are given in Table III. The total uncorrected number of tracks

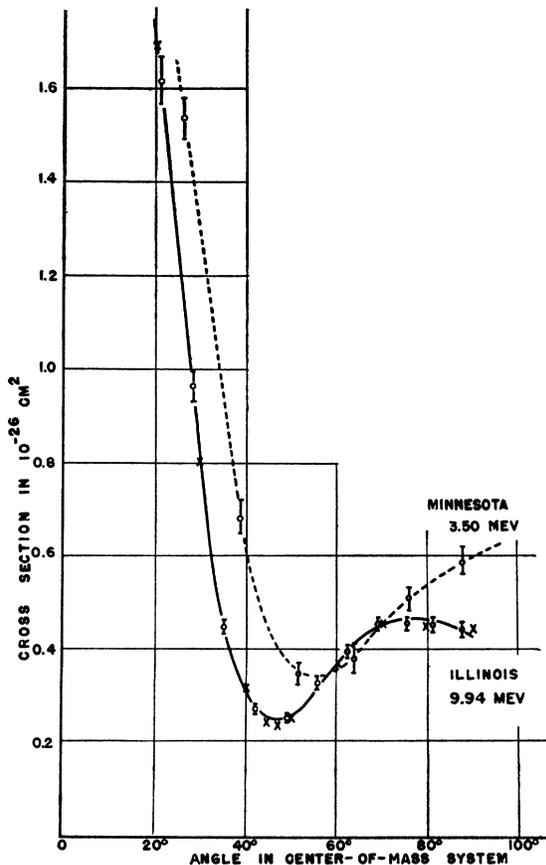


FIG. 4. Differential cross sections in center-of-mass system for the production of protons from D-D. Dashed curve shows results obtained by Blair *et al.* (reference 10). Crosses indicate fit to series with coefficients given in Table IV.

is given for each angle. The cross sections given have been corrected as described above. These values have been plotted in Fig. 4 with the results obtained by the Minnesota group at 3.5 Mev for comparison.¹⁰

The probable errors for each angle are given in Table III. These include the statistical error in the net number of tracks after subtracting the background tracks and the penetration tracks. Other errors are the same as those listed in reference 4, except for the human error which has been increased to 1.6 percent and the penetration error which has been treated separately for each angle according to the data given in Table II. The human error was increased to 1.6 percent because it was found that the neutron background tracks made accurate counting more difficult than for the previous work. This was deduced from the number of tracks counted on several different swaths by three different observers. The error in the deuteron energy (± 0.08 Mev), 0.8 percent has not been included in the values in Table III. The determination of the deuteron energy is discussed in reference 4.

A possible source of uncertainty in this experiment is the contamination of the gas in the chamber or in the beam. For the latter, the presence of 5-Mev protons would be the only possibility since molecular hydrogen could be accelerated along with the deuterons but would break up on going through the Nylon foil. This would affect the Q -meter readings, giving too large a value for the number of incident deuterons. Contaminates in the chamber would be a different matter. If the energies of the various particles from the reaction and from the scattering of 10-Mev deuterons and 5-Mev protons from

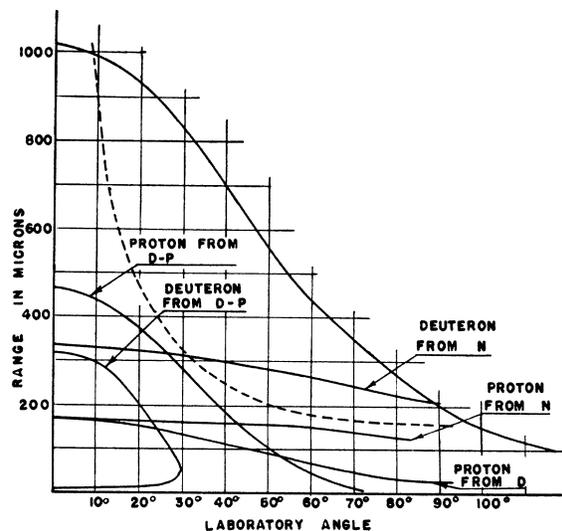


FIG. 5. Effect of contaminants. Curves show ranges in emulsion of 10-Mev deuterons and 5-Mev protons scattered from hydrogen, deuterium, and nitrogen. Upper solid line gives range of proton from D-D and dashed line is equivalent thickness of 8-mil aluminum foil.

¹⁰ Blair, Freier, Lampi, Sleator, and Williams, Phys. Rev. 74, 1599 (1948).

possible contaminates are converted into ranges in the emulsion and plotted as a function of angle, the effect of the aluminum foil is apparent. Figure 5 shows such a plot. From this it may be seen that the only particles with sufficient energy to penetrate the foil would be deuterons scattered from some heavy nucleus such as carbon, nitrogen, or oxygen. However, the low rate of rise of pressure in the closed-off system combined with $p-p$ scattering⁹ data which showed no appreciable scattering from such heavy nuclei would seem to indicate that contamination was quite small. This same plot also shows that the amount of hydrogen contamination of the deuterium is not important as far as its contribution to scattered protons is concerned, but of course must be known in order to calculate the correct number of target nuclei per cm².

According to theoretical predictions, the cross section of the center-of-mass system should obey an expression of the form $\sigma(\alpha) = A + B \cos^2\alpha + C \cos^4\alpha + \dots$. Attempts to fit the experimental data with terms up to $\cos^6\alpha$ were unsuccessful; however, a reasonable fit was obtained with $\cos^8\alpha$ terms. These coefficients are given in Table IV. A different set would be obtained if slightly different behavior were assumed about the 90° region.

The total cross section for the production of protons

TABLE IV. Coefficients for fit of data to the series.

$\sigma(\alpha) = A + B \cos^2\alpha + C \cos^4\alpha + D \cos^6\alpha + E \cos^8\alpha$	
A	$= +0.442 \times 10^{-26} \text{ cm}^2$
B	$= +0.708 \times 10^{-26} \text{ cm}^2$
C	$= -5.238 \times 10^{-26} \text{ cm}^2$
D	$= +5.122 \times 10^{-26} \text{ cm}^2$
E	$= +1.942 \times 10^{-26} \text{ cm}^2$

was obtained from the above values by computing the sum

$$\sigma_T = 4\pi \sum_{\alpha=0}^{\alpha=90} \sigma(\alpha) \sin\alpha \Delta\alpha$$

for $\Delta\alpha$ taken in 10° steps. The value so obtained was $\sigma_T = 7.3 \times 10^{-26} \text{ cm}^2$.

VII. ACKNOWLEDGMENT

The authors wish to express their thanks to Professors G. F. Tape and A. O. Hanson for the many helpful discussions of the problems associated with this experiment; to Professor R. E. Meagher for his cooperation throughout the experiment and help in making many of the measurements; and to the cyclotron staff for their assistance in carrying out the numerous tasks connected with the construction and operation of the equipment.

Proton-Proton Scattering at 5 Mev

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(Received November 7, 1949)

The differential cross section for the scattering of protons by protons has been measured at an energy of 4.96 ± 0.08 Mev, at angles from 12.5° to 55° in the laboratory system. The scattering medium was hydrogen gas at a pressure of approximately 5 cm of Hg. The scattered or recoil protons were detected by nuclear track photographic plates. Approximately 10,000 proton tracks were counted at each angle of observation. An estimate indicates the error of each individual cross section to be about two percent. Because the observations at each angle are made simultaneously, the relative accuracy from one angle to another is smaller and amounts to about 1.3 percent.

I. INTRODUCTION

THE scattering of protons by protons has been studied at a number of energies between 0.5 and 4.2 Mev¹⁻⁴ and at 7, 8, 10, and 14.5 Mev.⁵⁻⁸ This experi-

ment provides data at 5 Mev with hydrogen gas as the scattering medium and photographic plates for the detection of scattered protons in much the same way as the scattering experiment which was done at 7 Mev by Dearnley, Oxley, and Perry.⁵ However, the details of the slit system differ from those reported by them. The photographic method of detection was selected because it provided a convenient method of rejecting background counts, otherwise troublesome in electrical systems, and because it permitted the recording of a large amount of data in a relatively short cyclotron operating time.

The scattering chamber described by Rodgers, Leiter and Kruger⁹ was used to carry out this experiment.

* Assisted by the Joint Program of the ONR and AEC.
¹ Tuve, Heydenburg, and Hafstad, Phys. Rev. **50**, 806 (1936); Hafstad, Heydenburg, and Tuve, Phys. Rev. **53**, 239 (1938); Heydenburg, Hafstad, and Tuve, Phys. Rev. **56**, 1078 (1939).
² Herb, Kerst, Parkinson, and Plain, Phys. Rev. **55**, 998 (1939).
³ Blair, Freier, Lampi, Sleator, and Williams, Phys. Rev. **74**, 553 (1948).
⁴ A. N. May and C. F. Powell, Proc. Roy. Soc. **A190**, 170 (1947).
⁵ Dearnley, Oxley, and Perry, Phys. Rev. **73**, 1290 (1948).
⁶ R. R. Wilson and E. C. Creutz, Phys. Rev. **71**, 339 (1947).
⁷ R. R. Wilson, Phys. Rev. **71**, 384 (1947).
⁸ Wilson, Lofgren, Richardson, Wright, and Shankland, Phys. Rev. **71**, 560 (1947); **72**, 1131 (1947).

⁹ Rodgers, Leiter, and Kruger, Phys. Rev. **78**, 656 (1950).