

tistical weight of the 1S couplings should be considered. This weight is proportional to the eigen value of

$$Q = \sum q_{ik}, \quad (5)$$

where q_{ik} is defined as an operator which has the eigenvalue $2l+1$ when the two equivalent particles i and k are coupled in a 1S state, and the eigenvalue 0 in any other case.

It has been shown elsewhere² that for a given configuration the eigenvalues of Q depend only on the "seniority number," v , of the term; i.e., on the number of particles of the configuration l^v in which the term appeared for the first time. As

$$Q = \frac{1}{4}(n-v)(4l+4-n-v), \quad (6)$$

the terms with maximal statistical weight of 1S couplings are those with minimal v ; i.e., a 1S ($v=0$) for n even, and a 3P ($v=1$) for n odd.

The experimental fact that the L of an odd nucleus seems to equal the l of the uncoupled particle is therefore an argument in favor of the LS coupling against the jj coupling model.

Owing to the fact that the minimal value of v is zero for n even, and unity for n odd, Eq. (6) gives also a quantitative expression for the odd-even structure of the energy surface.

It may also be pointed out that the fact that $Q(n, v)$ reaches its maximal value exactly for $n=2l+2$ may perhaps give the possibility of explaining the magic numbers also without assuming strong jj coupling.

¹ E. Feenberg, Phys. Rev. **76**, 1275 (1949).

² G. Racah, Phys. Rev. **63**, 367 (1943), Eq. (50).

On the Detection of γ -Ray Polarization by Pair Production

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THE measurement of the polarization of γ -radiation can be important in that it would allow a determination of the change in parity between states involved in a radiative transition.¹ Although Compton scattering is a means of detecting a preferred orientation of polarization, it begins to fail at high energies because the polarization correlation of the scattered photon decreases with increasing energy, and the cross section itself decreases. For the high energy range, we believe that pair production may provide a useful technique. This conclusion has also been reached by Yang, who recently discussed the possibility of determining whether the neutral meson is scalar or pseudoscalar.²

The differential cross section for pair production is, in Heitler's³ notation,

$$d\sigma = \frac{Z^2}{137} \frac{e^4}{4\pi^2} \frac{p_+ p_- dE_+ d\Omega_+ d\Omega_-}{k^2 q^4} \left\{ \frac{(\mathbf{e} \cdot \mathbf{p}_+)^2 (q^2 - 4E_-^2)}{(E_+ - p_+ \cos\theta_+)^2} + \frac{(\mathbf{e} \cdot \mathbf{p}_-)^2 (q^2 - 4E_+^2)}{(E_- - p_- \cos\theta_-)^2} - \frac{2(\mathbf{e} \cdot \mathbf{p}_+)(\mathbf{e} \cdot \mathbf{p}_-)(q^2 + 4E_+ E_-)}{(E_+ - p_+ \cos\theta_+)(E_- - p_- \cos\theta_-)} + \frac{k^2 [p_+^2 \sin^2\theta_+ + p_-^2 \sin^2\theta_- + 2p_+ p_- \sin\theta_+ \sin\theta_- \cos(\varphi_+ - \varphi_-)]}{(E_+ - p_+ \cos\theta_+)(E_- - p_- \cos\theta_-)} \right\}.$$

\mathbf{e} is a unit vector in the direction of polarization of the incident photon.

We suppose that the normal to the plane of the pair is perpendicular to \mathbf{k} , and we consider only pairs such that $\varphi_- = \varphi$, $\varphi_+ = \varphi + \pi$ with φ measured from \mathbf{e} . A cross section determined for a particular experimental arrangement can be written as

$$\Sigma = A + B \cos 2\varphi,$$

where φ is the angle between the plane of polarization and the plane of the pair. The ratio, R , of the cross section for the pair to appear in a plane perpendicular to the plane of polarization

($\varphi = \pi/2$) to the cross section for the plane of the pair parallel to the plane of polarization ($\varphi = 0$) is

$$R = \Sigma_{\perp} / \Sigma_{\parallel} = (A - B) / (A + B).$$

The cross sections for the experimental arrangements to be described are obtained from the differential cross section by appropriate integrations. We make use of the relativistic and small angle approximations in performing the integrations.

We consider several different cases.

Case I.—The photon is plane polarized. The plane of the pair is determined by a coincidence of two counters placed symmetrically with respect to the axis determined by the incident photon such that $\theta_+ = \theta_- = \theta$. It is assumed that the counters will accept all values of the electron or positron energy and that the angle subtended by the counters is small compared with the range of angles in which an appreciable number of pairs are emitted ($\theta \sim m/k$). The coincidence count is a measure of the number of pairs emitted in a particular plane (φ). With these assumptions, one finds that R ranges from 1.13 for $\theta = 1.4(m/k)$, to 6.73 for $\theta = 0.4(m/k)$. Outside of this range for θ the cross section falls rapidly. One finds more pairs in the plane perpendicular to the plane of polarization.

Case II.—We consider again a completely polarized photon. The experiment is arranged so that all pairs lying in a particular plane (φ) are counted. This can be done by magnetic separation where counters can be arranged to pick up almost all energies of the positron or electron independently of the angle of emission; that is, one can use the properties of semicircular focusing that occur for high energy pairs. The plane of the pair is again determined by coincidence of the electron-positron counters. By rotating the magnet about the symmetry axis, one can determine the counting rate for a given plane. In this case, the expected $R = 1.23$.

Case III.—The incident photon is assumed to be plane polarized. We suppose that of all pairs lying in a particular plane (φ) only those pairs are counted for which the electron and positron have equal energy. This can be achieved by magnetic separation of the pair. For this arrangement $R = 1.30$.

Case IV.—Assuming the incident photon to be plane polarized, we shall also assume that only those pairs in a particular plane (φ) are counted for which the electron and positron have equal energies and make equal angles θ with \mathbf{k} . The cross section for this situation is simply proportional to $\cos^2\varphi$, so that, contrary to the previous cases, more pairs are found in the plane of polarization. Although this case is theoretically the most favorable ($R=0$), it is of little practical importance because, in order to achieve this result, it is necessary that $|p_+ - p_-| < m(m/k)$ and that $|\theta_+ - \theta_-| / \theta < (m/k)^2$, where $\theta \sim m/k$.

TABLE I.

Percent polarization R	100	90	80	70
	1.23	1.18	1.13	1.08

For a fixed energy of the incident photon, most of the pairs will be emitted in a narrow cone whose angular spread is approximately m/k . The solid angle which the pair-producing target subtends at the source of the photons must be smaller than this angle, and may introduce difficulties because of intensity considerations. Hence, from an experimental point of view, *Case II* is the most useful since there is no restriction of energy or angle for the electron and positron.⁵ The sensitivity of the method outlined in *Case II* is exhibited in Table I where R is given for various states of polarization of the incident photon.

¹ See, for example, D. Hamilton, Phys. Rev. **74**, 782 (1948); D. L. Falkoff, Phys. Rev. **73**, 518 (1948).

² Private communication.

³ C. N. Yang, Phys. Rev. **77**, 722 (1950).

⁴ W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1936), p. 196.

⁵ For the experiment described by Yang (reference 3), *Case II* yields $\beta^2 = 1/94$.