additional absorption. This drawback could be overcome if instead of crystals one were able to use liquids, which can be applied in large thicknesses without considerable absorption. In the course of the investigation of the fluorescent efficiency of liquid solutions when excited by gamma-radiation and by neutrons, special solutions were used which exhibited a fluorescent efficiency high enough to make them applicable for counting work.

Circuit.—The experimental arrangement used consisted of a multiplier tube of Type 5819. On its light-sensitive top surface a glass or porcelain tube of almost equal diameter was cemented. The glass tube was supplied with a reflecting layer (white paint or metal surface), or it can even be used with transparent walls. The liquid filled this tube and thus contacted the top surface of the multiplier directly. The upper surface of the liquid was covered with a reflecting layer. With the best solutions the following results were obtained.

Gamma-radiation.—With gamma-radiation from a radium source maximum scintillation peaks were obtained which were about five times larger than the maximum noise peaks. This means that these light flashes of maximum intensity released about 30 to 40 primary electrons from the photo-cathode. Also with softer x-ray sources the peaks could easily be detected without using a coincidence arrangement.

Neutron radiation.—With a polonium and beryllium neutron source larger peaks were obtained. The maximum size was about 15 to 20 times larger than the largest noise pulse, corresponding to a primary emission of 100 to 150 electrons per light flash. In the case of neutrons, as well as in that of gamma-radiation, the number of peaks were considerably larger than that obtained with normally available organic crystals.

Alpha-radiation.—Alpha-particles from a polonium source bombarded the surface of a solution of about 1-cm thickness. This means that the alpha-particles hit the surface of the liquid about 1 cm away from the photo-sensitive layer. In this case the alpha-particles gave rise to peaks about five times the size of the maximum noise peaks. Since the alpha-particles have an energy of about five million electron volts their efficiency in producing light emission is about three times smaller than that of gamma-radiation (gamma-radiation of about two million volts gives the same peak height). When the thickness of the solution was increased to about 6 cm the alpha-peak intensity decreased to approximately three times the size of the maximum noise peaks. Since in this case the light flashes are only produced at the upper surface, the solid angle subtended by the light at the photo-cathode is much smaller. A considerable part of the light only hits the photo-cathode after one or even several reflections at the side walls and it is quite probable that a part of the loss in intensity with increasing thickness of solution may be due to losses connected with the reflection at the walls. It may be that in large thicknesses of more than 10 cm a certain amount of absorption in the solution is already taking place.

General considerations.—Similar experiments in glass tubes without a reflecting layer at the walls gave intensities about 25 percent less than those with reflecting walls. This indicates that the walls of the tube partly operate as reflector by total reflection.

When all radiation sources are removed, a certain number of very large peaks were observed which sometimes were larger than 50 times the maximum noise pulse peaks, which means the emission of several hundreds primary electrons from the photo-cathode. These pulses were characteristic of the solution and disappeared when the solution was removed. It is supposed that these large peaks originated in cosmic-ray particles crossing the solution. Their frequency amounted to three peaks per minute which, with a solution of 50-mm thickness, would be a reasonable number of peaks to be induced by cosmic radiation.

Time constants of the solutions.—Very preliminary and rough checks of the time constants of these solutions were made. The electric circuit contained a resistor of 50,000 and in some cases of 10,000 ohms. In these cases the decay time of the observed pulses were those caused by the time constant of the electric circuit. The rise times of the peaks were practically that of the operating amplifier. This means that a very considerable part of the light emission of these solutions takes place within a period of time smaller than 10^{-7} sec.

The solutions.—A variety of solutions were found applicable for counting work. Solutions of toluene and xylene with fluorene, carbazole, phenanthrene, and anthracene prove to be nearly equally successful. Also mixtures of these solutions could be used advantageously. All solutions exhibited a maximum concentration for light efficiency, which in the case of carbazole was as low as 0.2 gram per liter. With phenanthrene the maximum concentration came out to be 8 gram per liter. The light efficiency was also increased when paraffin oil was added to the solution. Another type of equally good fluorescent solution was paraffin oil with dissolved xylene and with the addition of small amounts of anthracene, fluorene, phenanthrene, and carbazole. It is most essential for those experiments to work with pure substances, since very small amounts of contamination give rise to quenching effects. Thus, most of the substances used exhibited their maximum light efficiency only after being purified in our laboratory.

I was greatly assisted in these experiments by Mr. Milton Furst and Miss Miriam Sidran.

* This work was sponsored by the Signal Corps Engineering Laboratory, Fort Monmouth, New Jersey. Contract No. DA 36-039 sc-35.

Nuclear Coupling and Shell Model

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April 14, 1950

In a recent paper[1] Feenberg examined some general aspects of nuclear structure from the point of view of strong spin-orbit coupling, and we wish to add some remarks to his considerations.

Feenberg considered as a "reasonable supposition" that the statistical weight of the singlet component for states in the antisymmetric j^2 function space attains a maximum at $I=0$. With the aid of Dirac's vector model it is very easy to prove this supposition and also to extend the calculation to the antisymmetrical j^n function space; i.e., to n-like particles in equivalent orbits.

The statistical weight of the singlet component in the coupling of two particles is given by the expectation value of

$$S_{12}=\tfrac{1}{4}-(\mathbf{s}_1\cdot\mathbf{s}_2), \tag{1}$$

and in extreme jj coupling this expectation value is

$$\begin{aligned}\langle S_{12}\rangle_{\mathrm{Av}}&=\tfrac{1}{4}-(\mathbf{s}_1\cdot\mathbf{j}_1)(\mathbf{s}_2\cdot\mathbf{j}_2)(\mathbf{j}_1\cdot\mathbf{j}_2)/[j(j+1)]^2\\&=\tfrac{1}{4}-(\mathbf{j}_1\cdot\mathbf{j}_2)/(2l+1)^2\\&=\tfrac{1}{4}+[2j(j+1)-I(I+1)]/2(2l+1)^2,\end{aligned} \tag{2}$$

and attains therefore a maximum for $I=0$, in agreement with Feenberg's supposition.

For n-like particles in equivalent orbits the weight of the singlet couplings is

$$\langle\Sigma S_{ik}\rangle_{\mathrm{Av}}=\tfrac{1}{8}n(n-1)+[nj(j+1)-I(I+1)]/2(2l+1)^2 \tag{3}$$

and attains a maximum for the minimal I allowed by the exclusion principle.

If now strong Majorana forces coexist with strong spin-orbit forces, the spin I of the ground state of an odd nucleus should equal the j of the uncoupled particle only in the trivial cases of one particle in an empty shell or one hole in a filled shell, and should be $\tfrac{3}{2}$ in the cases of three particles in an empty shell or three holes in a filled shell, and $\tfrac{1}{2}$ in any other case.

The situation is different in the space orbital approximation. In this case the weight of the singlet couplings,

$$\Sigma S_{ik}=\tfrac{1}{8}n(n+2)-\tfrac{1}{2}S(S+1), \tag{4}$$

is not a sufficient criterion for determining the lowest state, as it has the same value for all terms with the same S, and the statistical weight of the ^1S couplings should be considered. This weight is proportional to the eigen value of

$$Q=\Sigma q_{ik}, \quad (5)$$

where q_{ik} is defined as an operator which has the eigenvalue $2l+1$ when the two equivalent particles i and k are coupled in a ^1S state, and the eigenvalue 0 in any other case.

It has been shown elsewhere[2] that for a given configuration the eigenvalues of Q depend only on the "seniority number," v, of the term; i.e., on the number of particles of the configuration l^v in which the term appeared for the first time. As

$$Q=\tfrac{1}{4}(n-v)(4l+4-n-v), \quad (6)$$

the terms with maximal statistical weight of ^1S couplings are those with minimal v; i.e., a ^1S $(v=0)$ for n even, and a ^2l $(v=1)$ for n odd.

The experimental fact that the L of an odd nucleus seems to equal the l of the uncoupled particle is therefore an argument in favor of the LS coupling against the jj coupling model.

Owing to the fact that the minimal value of v is zero for n even, and unity for n odd, Eq. (6) gives also a quantitative expression for the odd-even structure of the energy surface.

It may also be pointed out that the fact that $Q(n, v)$ reaches its maximal value exactly for $n=2l+2$ may perhaps give the possibility of explaining the magic numbers also without assuming strong jj coupling.

[1] E. Feenberg, Phys. Rev. **76**, 1275 (1949).
[2] G. Racah, Phys. Rev. **63**, 367 (1943), Eq. (50).

On the Detection of γ-Ray Polarization by Pair Production

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April 13, 1950

THE measurement of the polarization of γ-radiation can be important in that it would allow a determination of the change in parity between states involved in a radiative transition.[1] Although Compton scattering is a means of detecting a preferred orientation of polarization, it begins to fail at high energies because the polarization correlation of the scattered photon decreases with increasing energy, and the cross section itself decreases. For the high energy range, we believe that pair production may provide a useful technique. This conclusion has also been reached by Yang, who recently discussed the possibility of determining whether the neutral meson is scalar or psuedoscalar.[3]

The differential cross section for pair production is, in Heitler's[4] notation,

$$d\sigma=\frac{Z^2}{137}\frac{e^4}{4\pi^2}\frac{p_+p_-dE_+d\Omega_+d\Omega_-}{k^3q^4}\left\{\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)^2(q^2-4E_-^2)}{(E_+-p_+\cos\theta_+)^2}+\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)^2(q^2-4E_+^2)}{(E_--p_-\cos\theta_-)^2}-\frac{2(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)(q^2+4E_+E_-)}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}+\frac{k^2[p_+^2\sin^2\theta_++p_-^2\sin^2\theta_-+2p_+p_-\sin\theta_+\sin\theta_-\cos(\varphi_+-\varphi_-)]}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}\right\}.$$

$\boldsymbol{\varepsilon}$ is a unit vector in the direction of polarization of the incident photon.

We suppose that the normal to the plane of the pair is perpendicular to $\mathbf{k}$, and we consider only pairs such that $\varphi_-=\varphi$, $\varphi_+=\varphi+\pi$ with φ measured from $\boldsymbol{\varepsilon}$. A cross section determined for a particular experimental arrangement can be written as

$$\Sigma=A+B\cos 2\varphi,$$

where φ is the angle between the plane of polarization and the plane of the pair. The ratio, R, of the cross section for the pair to appear in a plane perpendicular to the plane of polarization $(\varphi=\pi/2)$ to the cross section for the plane of the pair parallel to the plane of polarization $(\varphi=0)$ is

$$R=\Sigma_{\perp}/\Sigma_{\parallel}=(A-B)/(A+B).$$

The cross sections for the experimental arrangements to be described are obtained from the differential cross section by appropriate integrations. We make use of the relativistic and small angle approximations in performing the integrations.

We consider several different cases.

Case I.—The photon is plane polarized. The plane of the pair is determined by a coincidence of two counters placed symmetrically with respect to the axis determined by the incident photon such that $\theta_+=\theta_-=\theta$. It is assumed that the counters will accept all values of the electron or positron energy and that the angle subtended by the counters is small compared with the range of angles in which an appreciable number of pairs are emitted $(\theta\sim m/k)$. The coincidence count is a measure of the number of pairs emitted in a particular plane (φ). With these assumptions, one finds that R ranges from 1.13 for $\theta=1.4(m/k)$, to 6.73 for $\theta=0.4(m/k)$. Outside of this range for θ the cross section falls rapidly. One finds more pairs in the plane perpendicular to the plane of polarization.

Case II.—We consider again a completely polarized photon. The experiment is arranged so that *all* pairs lying in a particular plane (φ) are counted. This can be done by magnetic separation where counters can be arranged to pick up almost all energies of the positron or electron independently of the angle of emission; that is, one can use the properties of semicircular focusing that occur for high energy pairs. The plane of the pair is again determined by coincidence of the electron-positron counters. By rotating the magnet about the symmetry axis, one can determine the counting rate for a given plane. In this case, the expected $R=1.23$.

Case III.—The incident photon is assumed to be plane polarized. We suppose that of all pairs lying in a particular plane (φ) only those pairs are counted for which the electron and positron have equal energy. This can be achieved by magnetic separation of the pair. For this arrangement $R=1.30$.

Case IV.—Assuming the incident photon to be plane polarized, we shall also assume that only those pairs in a particular plane (φ) are counted for which the electron and positron have equal energies and make equal angles θ with $\mathbf{k}$. The cross section for this situation is simply proportional to $\cos^2\varphi$, so that, contrary to the previous cases, more pairs are found in the plane of polarization. Although this case is theoretically the most favorable $(R=0)$, it is of little practical importance because, in order to achieve this result, it is necessary that $|p_+-p_-|<m(m/k)$ and that $|\theta_+-\theta_-|/\theta<(m/k)^2$, where $\theta\sim m/k$.

TABLE I.

Percent polarization	100	90	80	70
R	1.23	1.18	1.13	1.08

For a fixed energy of the incident photon, most of the pairs will be emitted in a narrow cone whose angular spread is approximately m/k. The solid angle which the pair-producing target subtends at the source of the photons must be smaller than this angle, and may introduce difficulties because of intensity considerations. Hence, from an experimental point of view, *Case II* is the most useful since there is no restriction of energy or angle for the electron and positron.[5] The sensitivity of the method outlined in *Case II* is exhibited in Table I where R is given for various states of polarization of the incident photon.

[1] See, for example, D. Hamilton, Phys. Rev. **74**, 782 (1948); D. L. Falkoff, Phys Rev. **73**, 518 (1948).
[2] Private communication.
[3] C. N. Yang, Phys. Rev. **77**, 722 (1950).
[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1936), p. 196.
[5] For the experiment described by Yang (reference 3), *Case II* yields $\beta^2=1/94$.