Letters to the Editor

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A Suggestion Regarding the Spectral Density of Flicker Noise

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AN explanation of the flicker noise in oxide cathodes was first given by Schottky.[1] He assumed that the work function is influenced by the presence on the outer surface of impurity atoms, the density of which was subject to fluctuation because of diffusion effects. The expression for the mean square noise current per unit frequency interval based on this assumption is

$$\langle i^2\rangle_{\text{Av}}\sim\tau I^2/(1+\tau^2\omega^2), \tag{1}$$

where I is the average current and τ is the average time spent by an atom on the surface. Assuming $\tau\omega\gg1$ we find

$$\langle i^2\rangle_{\text{Av}}\sim I^2/\tau\omega^2. \tag{2}$$

The experimental results, however, follow more closely the law $\langle i^2\rangle_{\text{Av}}\sim I^2/\omega^{1.25}$. Similar discrepancies[2] are found in other cases where the same assumptions might be used to explain a fluctuating barrier height e.g., metal semiconductor rectifiers, lead sulfide cells and carbon microphones. The frequency dependence may be close to ω^{-1}. Also the dependence on the current may differ somewhat from the square law, but this may be due to slight deformations caused by changing the applied electric field, as was suggested by Christensen and Pearson.[3]

Recently, MacFarlane[4] has extended and modified Schottky's theory in order to obtain the correct frequency dependence. His result is

$$\langle i^2\rangle_{\text{Av}}\sim I^{2m+2}/\omega^{2m+1},\quad\text{with}\quad 0<m<\tfrac{1}{2}. \tag{3}$$

Although this improves the agreement with experiment considerably, neither theory explains the relative insensitivity of the flicker noise to the temperature. This is so because in Schottky's expression (2) the relaxation time for the diffusion will depend on the temperature according to

$$\tau=\tau_0\exp(Q/kT),$$

and because in (3) the proportionality factor depends strongly on T.

More recently Richardson[5] has obtained a $1/\omega$ dependence without a strong temperature influence by considering the contact between two rough surfaces and making rather special assumptions about the statistical properties of the surface irregularities.

It is our aim to show that a slight modification of Schottky's theory will describe both the frequency and the temperature dependence correctly. Our fundamental hypothesis is that there will be a certain spread in the values of the diffusion activation energy, Q, at different points of the barrier surface. More precisely we suppose that the fraction of the barrier surface having a value of Q between Q and $Q+dQ$ is given by $f(Q)dQ$. This leads immediately instead of (1) to

$$\langle i^2\rangle_{\text{Av}}\sim\int_0^\infty dQ f(Q)\tau I^2/(1+\tau^2\omega^2)=(I^2/\omega)\int_0^\infty dQ f(Q)\tau\omega/(1+\tau^2\omega^2).$$

The expression $\tau\omega/(1+\tau^2\omega^2)$ as a function of Q, will show a sharp maximum at $Q_m=-kT\ln(\tau_0\omega)$, corresponding to the value τ_m of τ determined by $\tau_m\omega=1$. Assuming that $f(Q)$ varies relatively little in a range of the order of kT, we can also write

$$\langle i^2\rangle_{\text{Av}}\sim(I^2/\omega)f(Q_m)\int_0^\infty dQ\tau\omega/(1+\tau^2\omega^2).$$

The integral can be evaluated by taking as a new variable $z=\tau/\tau_m$ and using $\tau_m\omega=1$. This leads to

$$\int_0^\infty dQ\tau\omega/(1+\tau^2\omega^2)=kT\int_\delta^\infty dz/(1+z^2),\quad\text{where}\quad\delta=\exp(-Q_m/kT).$$

As we must expect that $Q_m\gg kT$ we can integrate from zero to infinity and obtain $kT\pi/2$. Therefore

$$\langle i^2\rangle_{\text{Av}}\sim(I^2/\omega)Tf[-kT\ln(\tau_0\omega)]. \tag{4}$$

We will obtain the often observed proportionality with ω^{-1} when $f(Q)$ in a certain range does not depend on Q. We wish to emphasize that in order to have the ω^{-1} law from $\omega=10^7$ to $\omega=1$ sec.$^{-1}$ we only need a "homogeneous" spread in Q of about 0.25 ev. For according to (4) the extreme values of Q that we have to consider are $-kT\ln(10^7\tau_0)$ and $-kT\ln(\tau_0)$. Their difference is $kT\ln(10^7)=0.25$ ev. It is known that such a variation in Q may occur under the influence of inhomogeneous strains.

Using results reported by Miller,[6] we find it possible to describe the noise spectrum of a silicon rectifier at room temperature and at liquid air temperature, in the range $\omega=10^6$ to $\omega=10^2$ sec.$^{-1}$, by having $f(Q)$ vary gradually in a range of Q values about equal to the one mentioned.

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[2] D. K. C. MacDonald, Report on Progress in Physics **XII**, 56 (1949).
[3] D. J. Christensen and G. L. Pearson, Bell Sys. Tech. J. **15**, 197 (1936).
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[5] J. M. Richardson, Bell Sys. Tech. J. **29**, 117 (1950).
[6] S. J. Angello, Elec. Eng. **68**, 865 (1949).

Nuclear Magnetic Moments of the Gallium Isotopes

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IN a recent letter with the above title, Béné, Denis, and Extermann[1] discuss an interpretation of the observed discrepancy between the moments of the isotopes of gallium as determined from magnetic resonance experiments and from observations on the Zeeman effect of the hyperfine structure of gallium in the $^2P_{\frac{1}{2}}$ state. The latter of these observations yields both the h.f.s. splitting at zero field ($\Delta\nu$) and the ratio of the nuclear g_I to the electronic g_J. Béné *et al.* apply corrections arising from the finite nuclear size to the value of the magnetic moment as calculated from $\Delta\nu$. Unfortunately, the calculation of a magnetic moment from $\Delta\nu$ is subject to rather considerable uncertainties and a discrepancy of 0.65 percent between the directly measured moment and the calculated one cannot be considered as evidence of the existence of an effect arising from finite nuclear size. However, the g values, obtained from h.f.s. measurements, given by Becker and Kusch[2] are obtained directly from the observed ratio g_I/g_J, and the correction given by Béné *et al.* is entirely meaningless when applied to these experimental quantities.

The nuclear resonance experiments and the h.f.s. experiments determine, in each case, the quantity $g_I\mu_0H'/h$ where H' is the external applied magnetic field, H_0, modified by diamagnetic, or possibly paramagnetic, effects of the electron configuration. A different g_I can be obtained in the two methods, only if H'/H_0 is different in the molecule studied in the nuclear resonance experiments ($GaCl_3$) and in the atoms used in h.f.s measurements. In the nuclear resonance method, large shifts have been observed[3] in the resonance frequency of a nucleus in a metal and in a salt. These effects have been discussed[4] in terms of a paramagnetic field at the nucleus due to conduction electrons in a metal.