PHYSICAL REVIEW VOLUME 78, NUMBER 5 JUNE 1, 1950

Symmetrical Tripartition of U^{235} by Thermal Neutrons

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(Received February 23, 1950)

Making use of an ionization chamber divided into three sections and triple coincidence circuit in conjunction with either a gated ten-channel amplitude discriminator or a double coincidence circuit, it has been found possible to determine the frequency of symmetrical triple fissions in U^{235} relative to the frequency of binary fissions. The results of the experiment are that triple fissions occur to the extent of 6.7 ± 3.0 per 10^6 binary fissions in the case where the fragments come off with comparable masses.

1. INTRODUCTION

THE process of binary fission, namely, the splitting of a heavy nucleus into two lighter nuclei, is now well established. In the case of uranium, for example, the maximum energy liberated is in the neighborhood of 200 Mev, approximately 160 Mev of which is evidenced as kinetic energy of the fission fragments with the remainder going into internal excitation of the fission fragments and kinetic energy of the neutrons emitted during fission. The possibility of fission into three charged particles has been predicted from theoretical considerations based on the liquid-drop model of fission.[1-3] In fact, from a calculation of mass differences one might expect a maximum energy liberation in triple fission of 10–20 Mev greater than that for binary fission.

Evidence for triple fission into two heavy particles and one light particle was first published in the literature by San-Tsiang, Zah-Wei, Chastel, and Vigneron.[4-6] The first discovery of this mode of fission was, however, made by Alvarez during the war. Detailed studies of these light particles using coincidence counting methods were conducted by Farwell, Segrè, and Wiegand[7] who determined the frequency with which long-range charged particles come off during the fission process and also established that these were, for the most part, alpha-particles. Prior to the publication of the work of Farwell, Segrè, and Wiegand, a detailed investigation of the particles emitted during fission was independently carried out by Green and Livesey[8, 9] using photographic plate techniques. They concluded that in approximately one percent of fission events a light nucleus of specific ionization similar to that of the alpha-particle is emitted. They also concluded that the average mass number of these light particles is very nearly 4 and that approximately one-fourth of them have a range exceeding that of the fission fragments. These results are in fair agreement with the results of Farwell, Segrè, and Wiegand. Other investigators[10-15] using either coincidence counting or photographic plate techniques verified at least the broader aspects of these conclusions, although Marshall,[15] on the basis of a study of 18,500 fission tracks, takes issue with Green and Livesey[9] on the existence of short-range alpha-particles. She concludes that practically all of the short-range particles are protons or heavier atoms from the photographic emulsion scattered by the fission fragments.

The present paper is concerned with the frequency of tripartition of U^{235} nuclei under thermal neutron bombardment, where the definition of tripartition is here limited to the division of a compound U^{236} nucleus into three fragments of approximately equal masses. This is not to be confused with the emission of low mass particles discussed above. San-Tsiang, Zah-Wei, Chastel, and Vigneron[16] have published a photomicrograph of one case of ternary fission into three heavy fragments. However, no mention is made of the frequency of such events, although the frequency for quadripartition of U^{235} nuclei was given as 0.0003 ± 0.0002 the frequency of bipartition.[17-20] Green and Livesey[5] also attempted, again using photographic plates, to find triple tracks which could be ascribed definitely to ternary fission into three heavy fragments. No such track was found in an examination of 5,000 fission events. These authors point out the difficulty of positively distinguishing

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[†] This document is based on work performed at Los Alamos Scientific Laboratory of the University of California under AEC Contract W-7405-Eng-36.

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between a triple track due to a triple fission event and a triple track due to a binary fission event plus a heavy recoil originating in the emulsion at approximately the point of fission.

2. EXPERIMENTAL METHOD

Since tripartition into heavy fragments was known to occur extremely rarely, if indeed it occurred at all, it was clear that in order to make a determination of the frequency of these events by observing two, or even all three, of the fragments in coincidence, it would be necessary to eliminate most of the accidental coincidences produced by two binary fissions occurring within the resolving time of the coincidence equipment.[21]

A triple ionization chamber, connected to three linear amplifiers and counting circuits, was used as the detector. In conjunction with this was used either a gated ten-channel pulse amplitude discriminator or a double coincidence circuit. Since most of the results were obtained with the arrangement using the double coincidence circuit, the experiment will be discussed in terms of this arrangement, a brief discussion being appended to indicate how the ten-channel pulse amplitude discriminator was utilized instead of the double coincidence circuit during the preliminary phases of this investigation.

A plan view of the triple ionization chamber is shown in Fig. 1. It is composed of three sections, designated as Chamber I, Chamber II, and Chamber III. Pure U^{235} was coated uniformly[22] to an average thickness of 0.003 mg per cm^2 on 0.1 mg per cm^2 Al, which was glued to the high voltage electrode of the chamber over a $\frac{3}{8}$-in. wide slot along a diameter. This electrode was common to all three sections of the chamber. Figure 2 shows a block diagram of the electronic equipment utilized.

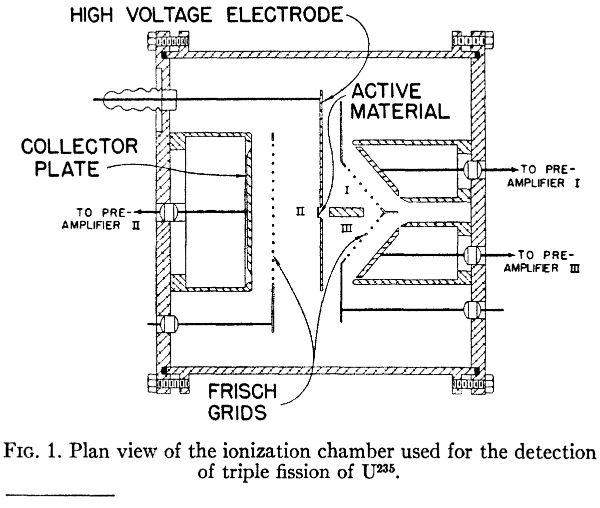

FIG. 1. Plan view of the ionization chamber used for the detection of triple fission of U^{235}.

On filling the chamber with a mixture of argon plus two percent CO_2 and exposing the chamber to a source of thermal neutrons, fission pulses were obtained from all three sections. Each of the three discriminators in the triple coincidence circuit was so adjusted that its channel would respond to a pulse size, from the appropriate section of the chamber, corresponding to a heavy fission particle of initial energy approximately 40 Mev or higher for the case of Chambers I and III, and 48 Mev or higher for the case of Chamber II. The U^{235} was so placed that whenever a binary fission occurred such that one fragment entered either Chamber I or Chamber III, the mate to that fragment, if it was not unduly scattered, was obliged to enter Chamber II. Chamber II is seen to be a standard-type Frisch-Grid counter from which a pulse, under proper conditions, would be proportional to the ionization produced in the chamber and hence to the energy lost by the fission fragment responsible for the ionization.

The triple coincidence circuit registered events occurring simultaneously in each chamber if, of course, sufficient energy was liberated by particles in each of the chambers within the resolving time of the coincidence circuit. Under the conditions of our experiment, a triple coincidence could mean one of three things: (1) A compound U^{236} nucleus actually underwent fission into three comparable masses; (2) Two binary fissions occurred within the resolution time of the triple coincidence circuit; (3) immediately after occurrence of a binary fission, one of the fragments collided with either a U^{235}, aluminum, or argon nucleus in such a manner that the three particles each entered a separate chamber with sufficient energy to activate the coincidence channel with sufficient energy to activate the coincidence channel associated with that chamber. It is shown in Appendix I that the third mechanism could not effect our results. By far the overwhelming majority of the observed triple coincidences resulted by the second mechanism, namely, two binary fissions occurring within the resolving time of the triple coincidence circuit. In order to eliminate most of these, utilization was made of the fact that, under ideal conditions, the sum of the energies of two fission fragments is almost always greater than 100 Mev, which is the average energy of the high energy group of fission particles.

In order to take advantage of the fact that, in almost all cases, the sum of the energies of any two fragments is greater than the energy of any one fragment which

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[21] The resolving time of the coincidence circuit cannot be sensibly less than the rise time of the pulses produced by the fission fragments. For the case of electron collection in a "Frisch Grid" ionization chamber in which the fission particles are completely stopped prior to reaching the grid (a prerequisite if the pulse heights are to be proportional to the energy of the particles) this rise time is of the order of one microsecond, and with such a resolving time accidental coincidences, if not somehow eliminated, would effectively limit the sensitivity of detection of triple fission events to approximately 1 in 50,000 binary fissions. This estimate is based on an experimental running time of approximately two weeks using optimum counting rates.

[22] The U^{235} coating was made by Robert Potter using the "Zapon Technique" in which the uranium as UNO_3 is dissolved in a solution of alcohol and Zapon and this solution is then painted on the aluminum foil.

would result from tripartition into three heavy fragments, the output from the triple coincidence circuit, in addition to going to a scaling circuit, was also fed into one channel of a double coincidence circuit, the discriminator for that channel being set so that the channel was activated by the output pulse from the triple coincidence circuit.[23] Into the second channel of the double coincidence circuit were fed the amplified pulses from Chamber II, the discriminator for this channel being set at such a bias that the channel would only be activated when it received a pulse corresponding to at least the minimum energy (under idealized conditions) liberated by the sum of the energies of two of the fragments from two binary fissions, this being also approximately equal to the maximum energy which one might expect from one fragment resulting from a tripartition into comparable masses. Under the above conditions more than 85 percent of the triple coincidences which resulted from two binary fissions occurring within the resolving time of the triple coincidence circuit were accompanied by a double coincidence. Every triple coincidence not so accompanied represented either, (a) an actual tripartition of a uranium nucleus into three heavy fragments,[24] or (b) two binary fissions occurring within the resolution of the triple coincidence circuit under one of the following conditions: (1) The sum of the fragment energies in Chamber II was less than 100 Mev. (2) One of the fragments which was destined to enter Chamber II was scattered by the Al or U^{235} through such an angle as to make this impossible. (3) One of the binary fissions occurred in Chamber I or III as a result of contamination. It is seen that the triple coincidence counting rate due to factor (b) will vary as the product of the counting rates of Chambers I and III, whereas the triple coincidence counting rate due to factor (a) will be a linear function of only one of those counting rates. The number of triple coincidences due to actual tripartitions could therefore, in principle at least, be separated from the number of triple coincidences due to two binary fissions which gave rise to either conditions (1), (2), or (3) by the well-known technique of plotting the number of triple coincidences per count in Chamber I, for example, as a function of counting rate in Chamber III (see Appendix II).

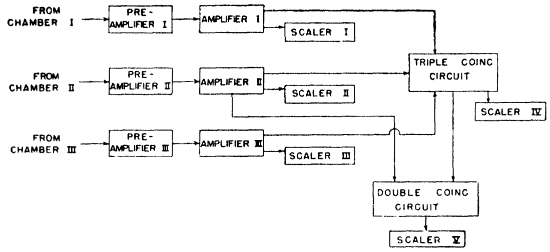

FIG. 2. Block diagram of the electronic equipment.

In order to be certain that the double coincidence circuit received from Chamber II a pulse corresponding to the full sum of any two fragments whose individual mates respectively entered Chambers I and III within the resolving time of the triple coincidence circuit, the clipping time of Amplifier II was made two microseconds, while Amplifiers I and III each had one-microsecond clippers, and the resolving time of the double coincidence circuit was made 2.2 microseconds. The electron collection-time for each of the chambers was approximately 0.8 microsecond. In order that the mate of every fragment which entered Chambers I or III would be certain, if it did not scatter excessively, to enter Chamber II, the active material was masked on the double-chamber side by a disk containing a slot one-half inch shorter and only half as wide as the slot in the plate to which was glued the uranium foil. (This masking plate is not shown in Fig. 1.)

On the basis of scattering considerations, and in the interests of obtaining reasonable pulse height distributions in Chambers I and III, it was deemed desirable to mount the U^{235} foil with the aluminum side facing Chamber II. As a result the pulse height distribution curve for Chamber II was distorted. Furthermore, in order to obtain reasonably meaningful pulse height distributions from Chambers 1 and III, it was found necessary to decrease the pressure in the chamber to such an extent that not all of the fission fragments in Chamber II were now stopped prior to reaching the grid. This, of course, also had an adverse effect on the pulse height distribution curve from this chamber. Figure 3 shows the pulse height distribution due to fission fragments as observed in Chamber II under the actual conditions of the experiment. Figure 4 shows the pulse height distribution as obtained from Chambers I or III. The triple coincidence circuit discriminators for Channels I and III were set so that these channels responded to all pulses from Chambers I and III with energy corresponding to an initial particle energy of greater than 40 Mev as determined from the pulse height distributions from the two chambers and the known average energies of the two groups of particles emitted in binary fission (see Fig. 4). The triple coincidence circuit discriminator for Channel II was set so that this channel responded to all pulses from Chamber II with energy corresponding to a fission fragment energy greater than 48 Mev.

In order to separate accurately triple coincidences due to actual tripartitions from so-called accidental triple

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[23] During the preliminary experiments a gated ten-channel pulse amplitude discriminator was used instead of the double coincidence circuit. Under these conditions the gate was supplied by the output of the triple coincidence circuit and into the amplitude analyzer were fed all the amplified pulses from Chamber II. In this way we obtained the pulse height distribution of the pulses from Chamber II for the pulses occurring simultaneously with triple coincidences. It was then a simple matter to identify most of the triple coincidences which occurred as a result of two binary fissions.

[24] An alpha-particle, no matter what its kinetic energy, could only liberate a maximum of four Mev in any of the chambers. This follows from stopping power and specific ionization considerations.

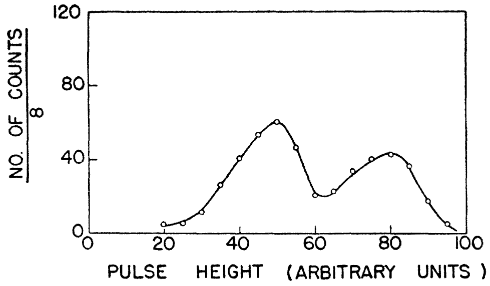

FIG. 3. Pulse height distribution of fission fragments in Chamber II.

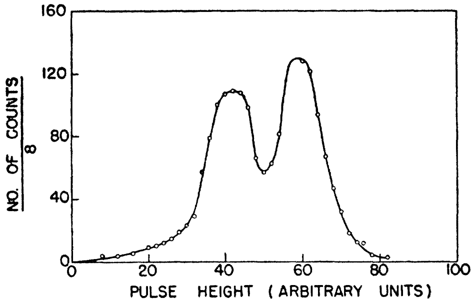

FIG. 4. Pulse height distribution of fission fragments in Chamber I or Chamber III. Identical curves were obtained for these two chambers within the statistical error of the experiment.

coincidences, runs were taken at different counting rates, the thermal column of the Los Alamos slow neutron reactor being used as the neutron source.[25] The power level of the reactor was kept constant to ±0.1 percent during each run. The use of the thermal column and pure U^{235} insured that any effect observed would be due to thermal neutron fission of U^{235}. Appendix II shows the method used to isolate the frequency of "real" triple coincidences from "accidental" triple coincidences.

The procedure used for collecting data was to make a run at a given counting rate for such a time as to obtain approximately 10^6 counts in Chambers I and III. The number of fission fragment pulses in each of the three chambers, the number of triple coincidences and the number of double coincidences were all recorded. Also recorded was the precise duration of the run and, as an additional check, the power level of the reactor. After completing a run, the reactor was turned off and, without moving the chamber, a calibration was made by determining the counting rate of the alpha-particles from the U^{235} foil for given settings of amplifier gains and discriminators. This not only checked the preamplifiers, main amplifiers and counters, but also served as a check on the discriminator and scaling circuits. If the observed counting rate for each chamber did not vary at these conditions, we proceeded to take another run with the reactor operating at a different power level. The fission counting rate from Chambers I and III was varied from 100–350 counts per second. Not less than three runs were taken at each counting rate. The discriminator settings at the inputs to the triple and double coincidence circuits were checked regularly by feeding a pulse from a precision pulser through each channel (from preamplifier to coincidence stage). The pulse size necessary to activate the coincidence stage of any given channel never varied by more than two percent from the predetermined pulse size for a given discriminator setting when the amplifiers were operating properly. As a check on both coincidence circuits and the experimental set-up in general, the discriminator on the double coincidence circuit into which were fed the pulses from Chamber II was periodically adjusted to accept all pulses corresponding to a fission fragment energy of greater than 40 Mev. Under these conditions, virtually every triple coincidence count should be accompanied by a double coincidence count and this was indeed the case when all circuits were operating properly.

3. INSTRUMENTATION DETAILS

The triple ionization chamber was made of dural and all parts were thoroughly decontaminated in nitric acid before assembly. The Frisch Grids, which shielded the collecting electrodes from the positive ions and thus made possible high speed electron collection in such a way that the pulse heights were approximately proportional to the ionization produced in the chamber were made of No. 36 parallel copper wires spaced 0.75 mm apart. The high voltage electrode on which was mounted the fissionable material was kept at minus 2200 volts by a well-filtered r-f type power supply. Additional Π-filter networks were utilized at the input to each chamber. The screens were kept at a constant minus 1000 volts by a second well-filtered r-f power supply. This made the field between screen and collecting electrodes slightly higher than the field outside the screen. However, this was desirable in order to insure that the screen should capture a minimum number of electrons. Without the uranium foil in the chamber, no triple coincidences were observed during an exposure to an integrated thermal neutron flux from the reactor of intensity sufficient to produce 200 triple coincidences at a counting rate from Chamber II of 200 counts per second under normal operating conditions with the active material in the chamber. The chamber was filled with argon plus two percent of CO_2 to a pressure of 50 cm Hg. This was not quite sufficient to stop the highest energy fragments if they traveled in a straight line from fission material to grid. However, as was pointed out above, this was necessary in order to obtain reasonable pulse height distributions from Chambers I and III. The dural plate between Chambers I and III was inserted for the purpose of preventing these two chambers from sharing the ionization produced by any one fission fragment. This plate was at the same poten-

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[25] In the preliminary work when the ten-channel discriminator was used instead of the double coincidence circuit, the Los Alamos cyclotron was utilized as the neutron source.

TABLE I. Combined results of the runs taken at different counting rates. C_3 = counting rate of Chamber III. C_t = triple coincidence counting rate minus double coincidence counting rate. C_1 = counting rate of Chamber I. E = efficiency of chamber for recording a symmetrical tripartition.

C_3 Counts/sec.	C_t/C_1E	Probable error
94.3	3.4×10^{-5}	$\pm0.5\times10^{-5}$
108.0	4.3	0.4
123.8	4.8	0.5
133.0	5.4	1.0
198.6	6.9	0.5
259.2	9.5	0.4
347.3	11.8	0.6

tial as the high voltage electrode. The saturation voltage for each chamber was established by determining the position of the energy peaks of the fission fragment energy spectrum as a function of plate voltage, the grid being maintained at a constant fraction of the plate voltage. It was found that making the plate voltage more negative than minus 1800 volts shifted the positions of the peaks very little. Also, for grid voltages between minus 800 and minus 1300 volts the positions of the peaks showed little change. The chamber pulses were amplified by "Model 100" [26] preamplifiers and main amplifiers. These amplifiers were stabilized by inverse feedback and had a rise time of 0.5 microsecond. By using a shorted delay-line clipping unit, the inverse reflection of the incoming pulse was superimposed upon itself after two microseconds for Amplifier II and after one microsecond for Amplifiers I and III. The amplifiers were shown to be linear to ±1 percent over the amplification interval utilized. The combined amplified noise level due to alpha-particles in the chambers, gamma-rays, amplifier noises, etc., was less than one-fourth of the minimum pulses which could pass the discriminators.

The coincidence circuits, which were designed and built by the Los Alamos electronics group, contained in each channel a blocking oscillator circuit which could produce a square pulse of fixed duration. These pulses were then applied to a coincidence stage. Five Higginbothom-type scalars were used as shown in Fig. 2 to count triple coincidences, double coincidences, and fission pulses from each of the three counters. Double coincidences were counted by a scale-of-eight incorporated into the double coincidence circuit. The triple coincidences were counted by a scale of 32 while the fission pulses were counted by a scale of 128. The triple coincidence circuit had an effective resolving time of approximately 1.0 microsecond. The double coincidence circuit had a resolution of 2.2 microseconds as stated earlier.

4. RESULTS AND DISCUSSION

Table I gives the results of all the runs taken under the conditions previously outlined. These results comprise three complete sets of runs, over a wide range of counting rates.

Figure 5 gives a plot of $C_t/(C_1E)$ as a function of C_3 (see Eq. (8)). From the Y-intercept of the straight line obtained by a least-squares fit and a consideration of the statistical accuracy of each point for which the data are plotted, one arrives at a frequency of ternary fission of 6.7 ± 3.0 per 10^6 binary fissions, assuming that all such fissions occurred symmetrically. (This is the case for which E is calculated; E would be smaller for an asymmetrical division.) Every point in Fig. 5 represents the weighted average of all the data taken at that counting rate for the bias setting prevailing during all the counting rates represented on the curve.

In order to evaluate the validity of the above results it is necessary first to determine to what extent triple coincidences which were due neither to ternary fission events nor to accidental coincidences produced by two or more binary fission events, could occur. In view of the tests carried out on the counters and electronic equipment, the only conceivable cause for such triple coincidence counts is in nuclear collision between fission fragments and U, Al, or A recoils, and this has already been shown to be highly improbable (Appendix I). The second question to be answered is whether or not triple fissions into heavy fragments could have occurred without having been detected with the efficiency which the chamber geometry would permit. Since close attention was paid to the coincidence resolving time, rise times of the pulses produced by the fission fragments, as well as frequency response and time delays in the detector circuits, this possibility must also be ruled out. It must be understood, however, that the frequency of tripartition may be considerably higher than we have given it, if this mode of fission occurs asymmetrically, since our calculation of E is based on a symmetrical division of the U^{236} nucleus, and the efficiency of the chamber for detecting triple fission events diminishes with increasing asymmetry of the disintegration. It is very difficult, however, to imagine any systematic error which would have led us to a lower value for the frequency of tripar-

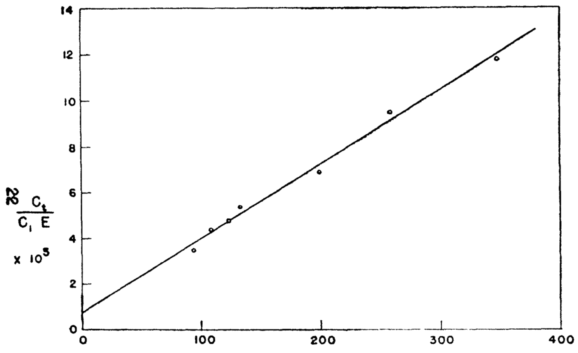

FIG. 5. Plot of $C_t/(C_1E)$ as a function of C_3. The intercept on the vertical axis gives the number of symmetrical triple fissions per binary fission.

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[26] Los Alamos designation.

tition into heavy fragments than the value given by our data. Nevertheless, it is quite obvious that tripartition of uranium does not occur with sufficient frequency to be of practical importance.

APPENDIX I

Let us consider the collision of a fission fragment with another nucleus. Let M = mass of fission fragment, m = mass of recoil nucleus, θ = angle between recoil nucleus and original direction of fission fragment, E = energy of fission fragment before collision, E_1 = energy of recoil nucleus, E_2 = energy of fission fragment after collision, Z = charge of fission fragment, Z' = charge of recoil nucleus, e = electron charge, ϕ = angle through which M is scattered in C.M. system. Then from the law of conservation of momentum and the relationship between energy and momentum we have

$$M(E-E_1)=ME+mE_1-2(mMEE_1)^{\frac{1}{2}}\cos\theta,$$

and

$$E_1=4MmE\cos^2\theta/(M+m)^2. \tag{1}$$

We have seen that the discriminators for Channels I and III are set such that they will pass pulses corresponding to fission fragment energies of greater than 40 Mev, while the discriminator for Channel II is set such that it will pass pulses corresponding to fission fragment energies of greater than 48 Mev. It is apparent, therefore, that if a fission fragment is to project a U, Al, or A nucleus into Chamber II, for example, while it is itself scattered into Chamber I or III, and if the pulse produced by each particle is to be large enough to pass its discriminator, the original fission fragment must have an energy of at least 88 Mev. Now, the maximum energy of a fragment is 105 Mev. Since 40 Mev must go to Chamber I or III, this means that the nucleus entering Chamber II can have any energy from 65 to 48 Mev. It is then calculated from Eq. (1) that the scattering angle θ can have, as a rough approximation, any angle from 25° 16′ to 39° 0′ if uranium is the scattered nucleus and from 24° 47′ to 38° 43′ if aluminum is the scattered nucleus. In obtaining the above values a mass number of 81 and an atomic number of 40 were used to correspond to a fission fragment energy of 105 Mev. We will now calculate the cross section for scattering through these angles for U and Al nuclei.

The cross section for coulomb scattering of M into the ϕ-interval (ϕ_1, ϕ_2) is given by the well-known equation

$$2\pi\int_{\phi_1}^{\phi_2} I(\phi)\sin\phi d\phi, \tag{2}$$

where

$$I(\phi)=\left(\frac{ZZ'e^2}{2E}\right)^2\left(\frac{M+m}{m}\right)^2\frac{1}{4\sin^4(\phi/2)}. \tag{3}$$

The cross section for projecting m into the laboratory angular interval (θ_1, θ_2) is also given by

$$2\pi\int_{\theta_1}^{\theta_2} I(\theta)\sin\theta d\theta, \tag{4}$$

where $I(\theta)$ must be related to $I(\phi)$ by the following:

$$I(\theta)\sin\theta d\theta=I(\phi)\sin\phi d\phi,$$

and

$$\phi=180°-2\theta.$$

After making the indicated transformation the cross section for scattering m into the interval (θ_1, θ_2) is determined to be

$$\pi\left[\left(\frac{M+m}{m}\right)\frac{ZZ'e^2}{2E}\right]^2(\tan^2\theta_2-\tan^2\theta_1). \tag{5}$$

With each value of E is associated an interval (θ_1, θ_2) and hence a cross section for scattering into that interval. We then find that the number of fission fragments scattered in such a manner that the recoil fission fragment has an energy greater than 40 Mev and the knock-on nucleus has an energy greater than 48 Mev is given by

$$\sum_{E=88}^{105}\sigma(E)\cdot\Delta N(E)\cdot\frac{t}{A}\cdot 6.023\times 10^{-23}, \tag{6}$$

where $\sigma(E)$ = average cross section for scattering in the interval (θ_1, θ_2) corresponding to an average fission fragment energy E, $\Delta N(E)$ = fraction of fission fragments in the energy interval E for which the scattering cross section is $\sigma(E)$, t = thickness in g/cm^2 of scattering material, and A = atomic weight of scattering material.

It can be shown that for our geometry we would observe in Chambers I or III less than one scattering from U^{235} per 2×10^6 fissions recorded by those counters, while in the case of Al there would occur less than one such scattering per 10^8 fissions recorded by Chambers I and III. Similar considerations show the possibility of accidental triple coincidences originating from the collision of fission fragments with argon nuclei to be completely negligible. All the approximations made in the above calculations were on the side of making the effect calculated as bad as possible. No account, for example, was taken of the energy lost by both fission fragment and scattered nucleus in the aluminum foil which one or the other must traverse. Also, no account was taken of the screening of the nuclear charge by the electronic field in the calculation of coulomb scattering cross section.

APPENDIX II

Let C_t = triple coincidence counting rate minus double coincidence counting rate, C_1 = counting rate of Chamber I, C_3 = counting rate of Chamber III, τ = resolving time of triple coincidence circuit when it is used as a double coincidence circuit between Channels I and III (Channel II always receives a pulse whenever Channel I or Channel III receives a pulse), N = average number of triple fissions per binary fission, and E = probability, that if a triple fission occurs and one of the fragments enters Chamber I, Chambers II and III will each receive one of the remaining fragments.[27] Then we have the following relation

$$C_t=2C_1C_3\tau+ENC_1, \tag{7}$$

from which

$$C_t/C_1E=(2C_3\tau/E)+N. \tag{8}$$

If one then plots $C_t/(C_1E)$ as a function of C_3, he obtains a straight line for which the Y-intercept will immediately yield the number of ternary fissions per binary fission. It is to be noticed that the standard counting error for each point on the curve is $\pm C_t^{\frac{1}{2}}$. This value is not determined solely by either the triple coincidence counting rate or the double coincidence counting rate. The two counting rates are not independent since every double coincidence must, *a priori*, have been accompanied by a triple coincidence. Therefore, the counting error depends only on the difference of these counting rates.

ACKNOWLEDGMENT

The authors take pleasure in thanking Dr. L. D. P. King for making the Los Alamos slow neutron reactor available for the above experiment and for numerous helpful suggestions during the course of the experiment.

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[27] E was calculated by the Theoretical Division computing group under the supervision of B. Carlson. This calculation was based on the solid angle subtended by the various chambers at the uranium film for the case of symmetrical tripartition.