Magnetic Deflection of Cosmic-Ray Mesons Using Nuclear Plates*

IAN BARBOUR†
University of Chicago, Chicago, Illinois
(Received January 12, 1950)

An experiment has been carried out in which two nuclear plates, secured with an air gap separating their parallel emulsion surfaces in a perpendicular magnetic field, were used to obtain magnetic deflection measurements on cosmic-ray mesons near the top of the atmosphere. Details of the experimental procedure are given and sources of error are discussed. The signs of the charges of particles were determined, and confirmation obtained for the negative charge of σ-(star-producing) mesons, positive charge for π and μ (observed disintegration), while both signs were observed for ρ-mesons (stopping without visible interaction). The relative abundances of the different meson types were investigated. For particles which stopped in the emulsion, range and magnetic curvature measurements allowed mass determinations which in the most favorable case of long path length gave, for an individual meson, a probable error of 15 percent in the mass determination. The values obtained were, for π- and σ-mesons, 270 ± 20; for μ and ρ, 226 ± 15. No evidence for light or very heavy mesons has been found so far.

1. INTRODUCTION

SINCE the definite identification of the tracks of mesons in photographic emulsions,[1] much valuable information on the cosmic radiation has been obtained through the use of nuclear plates. However, most emulsion techniques such as measurement of range, grain density, scattering, etc., yield no direct evidence as to the sign of the charge of any particle, while the individual determination of the mass of a cosmic-ray particle is of relatively low precision; a magnetic deflection measurement therefore may be expected to contribute better information on the charge and mass of the particle. Since in an emulsion the scattering of any low energy meson will obscure its magnetic curvature, with any attainable field strength, curvature measurements can best be made if the particle experiences a magnetic deflection in a medium of low scattering power and low stopping power, such as an air gap between two plates. Such a method of obtaining magnetic deflection measurements on mesons was suggested by Powell and Rosenblum;[2] an investigation of the possibilities of this type of experiment and development of the necessary techniques had also been independently undertaken in this laboratory[3] and preliminary results have been published.[4]

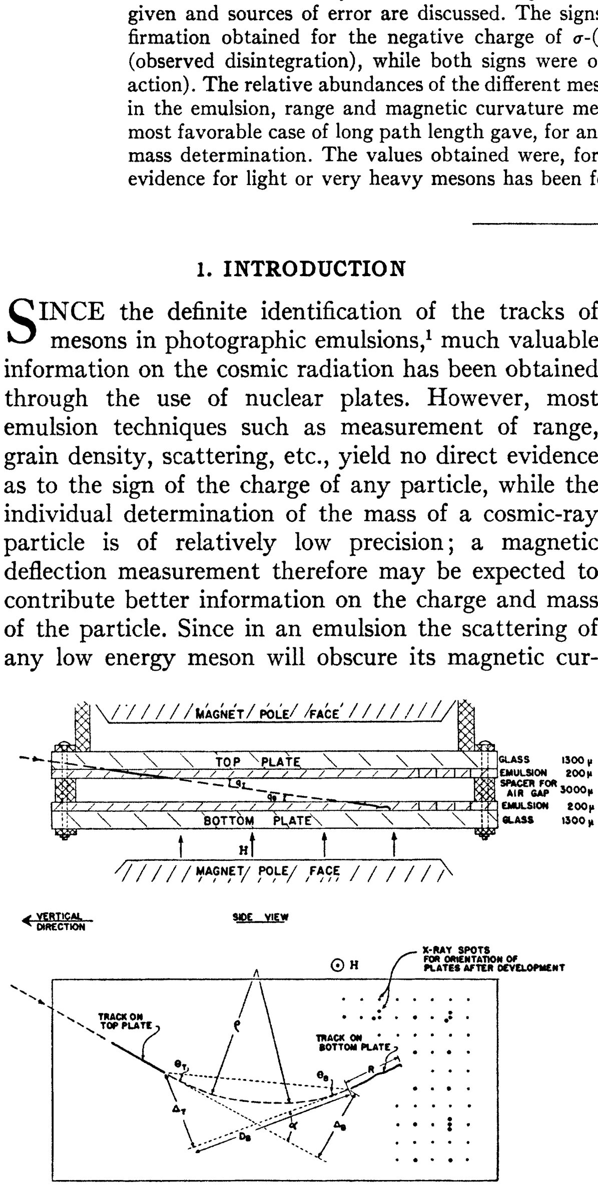

FIG. 1. Top and side view of plates as exposed in the field of the magnet. A typical meson is schematically represented, crossing the top emulsion and stopping in the bottom emulsion.

Because of the difficulty of reconstructing the orientation of the plates after development, Powell suggested[2] placing the developed emulsions in contact with each other; since this necessitated viewing the emulsion through the glass backing of standard thickness about 1 mm, the usual refracting microscope with short working distance could not be used. He therefore proposed the use of a specially constructed reflecting microscope[5] without objective lenses, having a large working distance. Prior to this time we were using emulsions on thin glass slides[3] so that an ordinary microscope could be used to view through the thin glass backing, but later the desired accuracy was achieved by the method[4] described in detail below, in which a pantograph[6] was used to map separately the events found on each plate on to large sheets of tracing paper, which were then superimposed for measuring deflection angles.

Several factors favored an experiment at stratospheric rather than mountain altitudes: (a) The frequency of events involving mesons is very considerably greater under a given quantity of matter at the top of

* Assisted by the Joint Program of the ONR and the AEC. The author is also indebted to the du Pont Company for a fellowship, and to the Research Corporation for a Cottrell Grant, during certain periods of this work.
† Now at Kalamazoo College, Kalamazoo, Michigan.
[1] Lattes, Occhialini, and Powell, Nature **160**, 453 (1947).
[2] C. Powell and S. Rosenblum, Nature **161**, 473 (1948); see also C. Franzinetti in *Cosmic Radiation* (Butterworth's, Ltd., London, 1949), p. 159. *Note added in proof.*—a more complete account has appeared after the present paper was submitted for publication: C. Franzinetti, Phil. Mag. **41**, 86 (1950).
[3] I. Barbour, Phys. Rev. **74**, 507 (1948).
[4] I. Barbour, Phys. Rev. **76**, 170 (1949); **76**, 320 (1949).
[5] C. R. Burch, Proc. Phys. Soc. London **59**, 41 (1947).
[6] I. Barbour, Rev. Sci. Inst. **20**, 530 (1949).

the atmosphere[7] and hence the number of mesons with long trajectories obtainable per plate per hour is correspondingly greater; also there is greater ease and certainty in matching the track segments due to one meson as it passes through the two emulsions, as the ratio of mesons stopping to protons stopping is very much increased under matter at balloon altitudes; (b) at mountain elevations the scattering of low energy mesons in the air of the gap between the plates is a serious problem, whereas close to the top of the atmosphere the scattering in the air of very low pressure introduces negligible errors; (c) it was desirable to investigate the presence of mesons of various masses and charges in the upper atmosphere since the creation of mesons takes place largely at the higher altitudes. Preliminary investigations with a 10-lb. magnetron magnet in a B-29 airplane showed the possibility of obtaining magnetic deflections. A larger magnet, described below, was accordingly constructed and four successful balloon flights were carried out through the cooperation of the ONR, using General Mills balloons.

2. EXPERIMENTAL PROCEDURE

Two nuclear plates were mounted as a "sandwich" in an aluminum plate-holder which held them rigidly face-to-face with a 3-mm air gap separating their parallel emulsion surfaces, as in Fig. 1. Ilford C2 plates were used throughout; on the first flight, 100-micron emulsions were employed, but in subsequent flights 200-micron emulsions were used because, as will be discussed later, better statistics could be obtained with thicker emulsions, without introducing significantly greater distortion errors. Special Ilford plates with an added quantity of plasticizer were tried on a test run, to see if this agent would increase the rigidity and thus lessen the emulsion distortion. Since there was no evidence for an appreciable effect of this sort they were not used in the magnetic deflection experiments. On the first flights, 2 in.×4 in. plates were used. For the last two flights specially manufactured 4 in.×4 in. plates were obtained so that a margin of 1 in. of emulsion surrounded the central 2 in.×2 in. emulsion area which was in the region of strong magnetic field; hence the large errors in track angles which would have been caused by emulsion distortion near the plate edges was avoided. The mounted plates, wrapped in aluminum foil, were then exposed to an x-ray beam to produce on the emulsion the grid of reference dots[2,3] by which the relative orientation of the plates could later be reconstructed, after development. For this purpose a $\frac{1}{32}$-in. Pb plate, drilled with holes 100 microns in diameter on a $\frac{1}{2}$-cm square grid (with a coded 3-hole pattern at intervals for ease in identification) was placed between the plates and a distant x-ray source. The plate-holder was then secured to the magnet for the balloon flight.

The magnet weighs 65 lb., produces a field of 13,300 gauss in an air gap $\frac{1}{4}$ in. wide, with pole pieces 2 in.×2 in. in area (25.9 cm²). The side blocks are of Alnico V; the cross-yoke and pole-pieces are of soft iron. The magnet was mounted with the gap and plates vertically downward so that there was thus a considerable quantity of iron above the plates to act as a meson generator, since at the top of the atmosphere there are few mesons of low energy found in the free air. The magnet was rigidly mounted in a Dow-metal frame which was then suspended by 16 shock-absorbing springs from a larger aluminum frame, enclosed with celluloid sheets and reflecting surfaces to provide temperature control. The equipment was flown from Camp Ripley, Minnesota, and the magnet and parachute were released in the upper atmosphere by a clock mechanism at a predetermined time. The first flight was sent to about 90,000 ft. for a trial period of 1 hour; the second was flown at this altitude for 3 hours, a third at 70,000 ft. for 6 hours, and the fourth flight for 6 hours at 90,000 ft.

A number of different development procedures were tried on 200-micron test plates, to determine the method most suitable for processing the emulsions from the deflection experiments. The "cold method,"[8] in which development of the plates during the time of diffusion of the developer is inhibited by lowering the temperature, gave excellent evenness of development, but in general appeared to produce greater distortion of the emulsion than other methods, as measured by the *apparent* deflection of fast proton tracks between "sandwich" pairs of plates exposed with no magnetic field. Several variations of the "two-bath" method[9] were tried, in which developing action during diffusion in the first solution is almost absent because the alkali agent is in the second bath only. These methods produced even development and little distortion; however we were not able to prevent the appearance of a higher density of background and surface grains, which increase the difficulty in scanning the emulsions for tracks and measuring their angles of emergence from the emulsion. (The usual operation of wiping the emulsion surface to remove these surface grains would be undesirable in this experiment.) The most satisfactory method appeared to be a one-bath, one-temperature method, but with a somewhat weaker developer (D19 4:1) and lower temperature (18°C) than usually employed, so that a longer development time (around 55 minutes for 200-micron emulsions) is required; the time of diffusion is thus a smaller fraction of the total developing time, and a more even development results than in the standard method. However this procedure does produce tracks slightly denser near the surface than near the glass, and hence can not be recommended if accurate grain counts are desired *throughout* the emulsion. In this experiment, where grain counts were of secondary importance and were carried out only near the surface of the emulsion to identify the 2-track

[7] J. Lord and Marcel Schein, Phys. Rev. 75, 1956 (1949).

[8] Dilworth, Occhialini, and Payne, Nature 162, 102 (1948).

[9] M. Blau and C. DeFelice, Phys. Rev. 74, 1198 (1948).

segments due to one particle passing through the two emulsions, and where freedom from emulsion distortion is of primary importance, it was found to be the most satisfactory method. Test runs were made to see if the use of a separate hardening solution between development and fixation would reduce distortion; since the improvement found was not pronounced, a standard acid fixer with hardener was used in these experiments.[9a]

3. ANALYSIS OF PLATES

After development the plates were scanned and the location of all particles stopping in the emulsion, and of heavy nuclei, stars, meson events, etc., were plotted relative to the grid of x-ray dots. This was greatly facilitated by the construction of a pantograph[6] attached to the microscope stage, by which the motion of the stage is mechanically amplified twenty times. When the scanner sees an event of interest, he has only to close a switch which activates a magnetically operated pen at the free end of the pantograph, thus recording on a large sheet of paper a plot of the position of the event. The pantograph has precision bearings, so that by using a cross-hair in the microscope eye-piece, considerable accuracy in plotting a point on the emulsion can be obtained. The azimuth direction of the emerging track, projected in the plane of the emulsion surface, is then read off with a goniometer protractor[6] on the microscope eye-piece; these angles are later reconstructed on the sheet of plotting paper. The approximate dip angle between the track and the plane of the emulsion can be determined by measuring under oil immersion the difference in depth of two track points of known horizontal separation. From these data, knowing the factor by which the emulsion shrinks on development (approximately 2.4 for these plates with 200-micron emulsions), one can calculate the approximate point at which the particle would have emerged from the opposite plate of the "sandwich" if undeflected. If in addition the range of the particle has been measured and the magnetic field strength is known, it is possible to lay off an area in which one should expect to find the matching segment of track in the second plate. The correct matching of the two track segments in the two emulsions, due to a single meson, can be ascertained from the following characteristics of the tracks:

(a) The positions and locations of the two tracks must be such that the deflection angle (θ_T in Fig. 1) between the tangent to the track as the particle leaves the top plate and the line connecting the points of leaving and entering the air gap, must equal the angle (θ_B) between the tangent to the second segment of track as it enters the opposite plate, and this same connecting line. This is, of course, a consequence of the fact that the trajectory due to the magnetic field in the gap is an arc of a circle; the two measured θ-angles should not differ by much more than twice the total probable error in each angle (see "sources of error").

(b) The dip angles between each of the two track segments and their respective emulsion surfaces should agree closely since the two emulsions had the same shrinkage factor at development. The dip angle can be determined by measuring the depth below the emulsion surface of a number of points on the track; the Leitz microscope used in these experiments has a ball-bearing fine adjustment with almost no back-lash, and depths can be determined to $\frac{1}{4}$ micron by taking several readings. For more accurate measurements of the dip angle, a tilting stage[6] has been constructed for the microscope. With an objective and eyepiece combination having slight curvature of field, the angle through which the stage must be tilted to bring the portion of the track near the surface into simultaneous focus can be accurately measured.

(c) Grain counts on portions of the two tracks near the surface should agree closely.

(d) An estimate of the multiple scattering of the particle in the emulsion is also valuable. The tracks with small dip angle which, as pointed out later, are particularly important because of the smaller percentage error in their deflection measurements, will have long path lengths in the second emulsion and hence afford a good opportunity to estimate the scattering, which is often

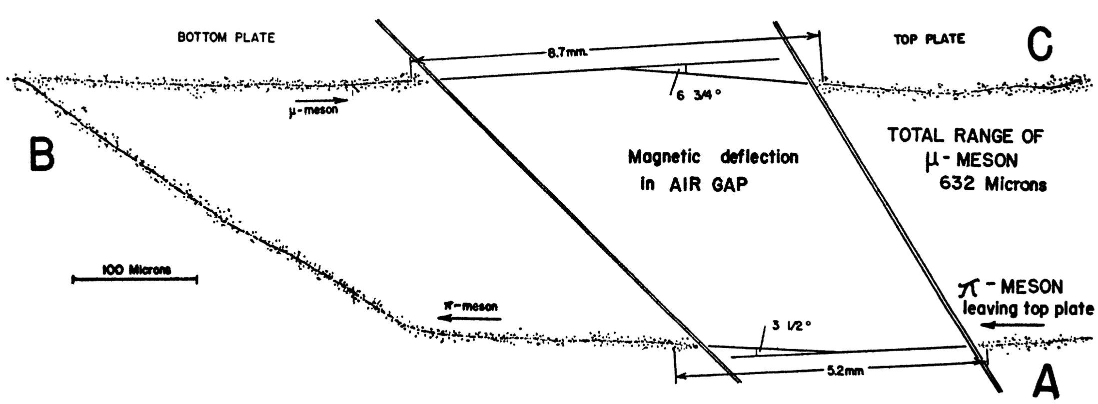

FIG. 2. Micro-projection of a $\pi-\mu$-decay occurring in 2 plates, arranged schematically to show magnetic deflection of both mesons. A π-meson traveled ~410 microns through the top emulsion to the point A and crossed the air gap, decaying in the bottom plate at B into a μ-meson which in turn re-crossed the air-gap to stop in the top emulsion at C. This case gave a mass determination of low accuracy, but was typical of cases showing the total range of a μ-meson.

[9a] It would be desirable in any extension of this type of experiment to investigate the use of a fine-grain developer with which a smaller grain size results, so that the azimuth angle at which a particle enters the emulsion could be more accurately determined from the first few grains of the track. Photo-glycine, for instance, has been recommended as giving a smaller size of grain, but in the developed emulsion the lighter tracks of low ionization appear harder to locate.

helpful in deciding which tracks passing through the emulsion might be due to mesons.

(e) The distance D_B, along the projection of one track, to the point of emergence from the opposite plate, must be consistent with the measured dip angles.

We have seldom found that there is more than *one* possible "matching" track in the second plate for which more than two or three of the characteristics above are in agreement. Matching is especially certain in the cases in which a π–μ-decay occurs in one plate and the μ-meson crosses the air-gap to stop in the second plate, for the total range in both plates of the μ-meson is known[1] to be about 600 microns. Figure 2 shows a typical case in which both mesons of a π–μ-decay traversed the air-gap and the total range of the μ-meson in both plates can be determined.

Once the tracks have been matched, the radius of magnetic curvature, ρ, in the plane of the emulsion is computed from the distance, D_B, and from measurement of the total angular change in direction, α, experienced by the particle. This angle should be equal to the sum of the angles, θ_B and θ_T, measured to the points of emergence of the tracks (see Fig. 1). However, unlike the measurements of the angle θ, the total angular deflection is not influenced by any errors in plotting positions, laying-off angles on the tracing paper, etc.; α depends only on the difference in the measured azimuth directions of the two emergent track segments. By purely geometrical considerations, one can show that $\rho=\frac{1}{2}D_B \sec\frac{1}{2}\alpha \csc\frac{1}{2}\alpha$. The exact value of the actual dip angle, q, is calculated for each case from the distance D_B and the known 3-mm separation of the plates. As shown below, the mass calculations then depend only on three quantities measured experimentally on the plates for each meson: (1) the range, R; (2) the deflection angle, α; and (3) the distance D_B; for from α and D_B one can compute ρ and q.

The calculation of the mass of the particles from these data is dependent on two assumptions: (1) that their charge is numerically equal to the electronic charge, and (2) that the energy loss is a function only of the velocity, v, as is known to be the case for ionization losses. It follows that for any particle of given mass m and range R in the non-relativistic region the relation holds: $R=mF(v)$, or $v=F'(R/m)$. Hence if H is the magnetic field strength, and v_p is the component of the velocity perpendicular to the magnetic field, we find that

$$R \cos q/H\rho = KR \cos q/mv_p = KR/mv = f(R/m), \quad (1)$$

where K is a constant. Values of the function f, were calculated and plotted using the range-energy data[10] for protons in an Ilford emulsion. Hence, using for any particle the experimentally determined values of R, ρ, q, and H, and knowing f, the unknown value of the mass m can be determined from the extreme left and right members of Eq. (1).

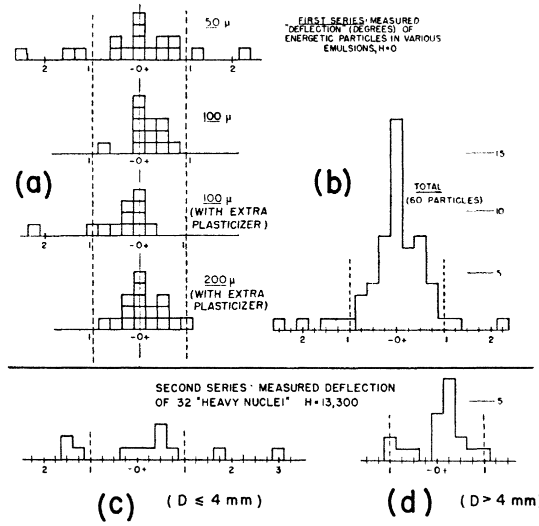

FIG. 3. Distortion test: measured deflection angles for (a) protons in various emulsions as indicated, no field; (b) total of 60 fast particles; (c) steep tracks of heavy nuclei; and (d) heavy nuclei at glancing angles.

4. SOURCES OF ERROR

The experimental value of ρ depends on the measured total angular deflection. The following sources of error in angular measurements may be listed:

(a) *Scattering.*—The greatest source of error in measuring the azimuth direction of a particle starting or finishing its trajectory in the air gap is scattering as it enters or leaves the emulsion. In practice, the azimuth is read off, relative to a "zero" direction established from the x-ray grid, by setting the intersection of the eye-piece cross-hairs, under high magnification, on the last grain of the track as it passes into the air, and then rotating the eye-piece until the cross-hair coincides as closely as possible with the center of each grain of the adjacent track segment which usually appears "straight." Except for cases where a visible scattering occurs very near the surface, it was found that for meson tracks of the average energy encountered, the direction of emergence could best be determined from the grains of about the last 20 microns of track before emergence; if too short a segment is used the limit of resolution of the optical system and the finite size of a grain and uncertainty in the position of its "center" introduce greater uncertainty in the direction of the azimuth line, whereas if too long a segment is taken, scattering may introduce greater changes in the direction of motion of the particle. Such a length is in fair agreement with the theoretical optimum segment of track from which to measure its direction of motion at a given point, as derived by Scott.[11] For particles of lower energy and smaller residual range, shorter track segments were used. To estimate the error in azimuth due to scattering, one can start by assuming that in setting the cross-hair on, say, the last 20 microns of a track which appears "straight" over that distance, one is in reality determining on the average the direction of motion of the particle at about the mid-point of this segment; then the average error, due to scattering, in determining the actual azimuth direction of the particle at emergence will be the average angle of multiple scattering of such a particle in the last 10 microns of emulsion. Using the

[10] Lattes, Fowler, and Cuer, Proc. Phys. Soc. London **59**, 883 (1947).

[11] W. T. Scott, Phys. Rev. **76**, 214 (1949), Eq. (11b).

Plate No.	Emulsion thickness (microns)	No. sq. inches observed	Total flight hours	Ceiling alt. (feet)	Hrs. within 5000' of ceiling	ρ a	ρ g	π-μ a	π-μ g	σ a	σ g	Ejected σ	Total
1	100	8	4.5	99,000	.74	13	10	4	4	2	1	1	35
2	200	8	5	95,000	3	47	62	19	25	10	14	2	179
3	200	7	8.5	69,000	6	38	42	13	31	14	17	0	155
4	200	7	8	94,000	5	49	53	23	27	12	17	1	182
					TOTALS-	147	167	59	87	38	49	4	551
TOTAL NO. OF MESONS OF EACH TYPE -						314		146		87		4	

FIG. 4. Number of mesons observed. Breakdown of total of 551 mesons according to plate, type of meson, etc. Mesons in columns "a" entered the emulsion from the air gap; those in columns "g" entered the emulsion from the glass backing.

results of the theory[11–13] and adapting these calculations to the C2 emulsion,[14] we can obtain for each meson the theoretical scattering angle (in the case above, for a path length of 10 microns), provided its range and an approximate value for its mass are known. For tracks of moderate range, this scattering angle, which represents an uncertainty inherent in this method, is of the order of 1 or 2 degrees.

(b) *Observational.*—There is also in practice a smaller uncertainty in reading each azimuth, depending on (1) the steepness of the track, (2) the particular pattern of the last few grains, and (3) the presence of any unusually large scattering which is directly visible in the last 20 microns, etc. An "observational" error, assigned to each measurement of the azimuth, is obtained by direct inspection of the track, and by the standard deviation or spread of 20 individual angle readings on each track (two observers each made a series of five successive readings, which were repeated with another microscope later). This source of error, while not independent of the scattering error above, is usually much smaller, and is included in the estimate of the total probable error to make allowance for the uncertainty occurring occasionally when a fairly straight track happens to have a visible sudden scatter very near the surface.

(c) *Emulsion distortion.*—During its life, development and drying, an emulsion inevitably experiences some distortion. It was found that contraction on drying causes severe distortion near the corners and lateral edges of a plate; this was the reason for obtaining plates large enough that measurements could be confined to areas of the plate at least 1 in. from the nearest plate edge. The total translatory motion of any point in the emulsion (distortion in position) does not introduce serious errors in this experiment. However, the derivative of this translation with respect to depth in the emulsion (i.e., the relative slippage of points at different depths) was found to be great enough to cause a rotary distortion in the angle of a track, particularly a steep one. In order to test the extent of this type of angle distortion, the apparent deflection was measured for a series of fast protons, passing through a sandwich in *no* magnetic field. These tests were made on several emulsions, developed in the method indicated above, as shown in Fig. 3, a. There is little variation among the emulsions of different thicknesses. At (b) the probable error of the distribution in angle is about $\frac{1}{2}°$, as seen in the figure. Below, similar measurements are shown of the deflection angle of 32 heavy nuclei[15] with $H = 13{,}300$ gauss. The distribution (d) refers to tracks of small or intermediate dip angles, i.e., path length D in the air-gap of at least 4 mm, which was about the minimum path length of any mesons used for deflection measurements; the distribution at (c) for very steep tracks ($D<4$) shows the greater errors in angle for steep tracks. These errors in the azimuth angles must include also any instrumental errors in the eye-piece protractor, or other parts of the microscope. These sources of error appear to be unavoidable and would represent the lowest limit ($\sim\frac{1}{2}°$) for angular errors in deflection measurements on very energetic mesons where scattering errors are small.

(d) *Scattering in air-gap.*—The scattering of particles in the air between the plates is proportional to the square root of the air pressure[12] and at these altitudes corresponding to about 1 cm Hg, turns out to be completely negligible compared to errors listed above.

In addition to errors in angle, there are four other factors which in this experiment cause smaller errors in the value of the calculated mass: (1) The *field* strength, H, was measured before and after each flight, with several different search coils calibrated with an accurate mutual inductance standard and ammeter. The field was mapped and found to be homogeneous to 2 percent to within $\frac{1}{4}$ in. of the edge of each pole-piece; and the absolute value of the field measurements are believed reliable to 2 percent. (2) The *range* of each particle in three dimensions is measured by making, for the cases of greater accuracy, a profile of depth *versus* horizontal component of path length. Since particles with reasonably long ranges were usually used, the error in range measurement produces an uncertainty in the mass value which is negligible compared with that due to the errors in measuring the deflection angle. (3) The value of the *dip angle q* in Eq. (1) is obtained from the path length D_B and the known 3-mm separation of the plates; these quantities are large enough to be measured with sufficient accuracy that the error in $\cos q$ (which is close to 1 for the tracks used) is negligible. (4) The *range-energy curve*[10] for protons, and its extrapola-

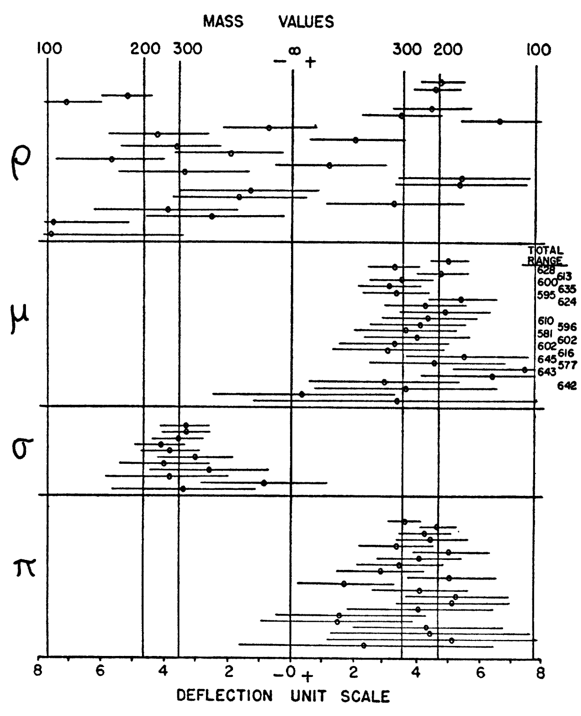

FIG. 5. Mass determinations on individual mesons, grouped vertically according to the type of meson and arranged in order of increasing precision. Values and probable errors are plotted on a deflection unit scale (see text); some corresponding mass values in units of m_e are given at the top. Positive charge is indicated if point lies to right of center-line, negative to left. For $\pi-\mu$-decays, in which the μ-meson was created in one plate and stopped in the other plate, the total range of the μ-meson is given at the extreme right of the diagram.

[12] H. Bethe, Phys. Rev. **70**, 821 (1946).
[13] E. J. Williams, Phys. Rev. **58**, 292 (1940).
[14] Goldschmidt-Clermont, King, Muirhead, and Ritson, Proc. Phys. Soc. London **61**, 183 (1948).
[15] Freier, Lofgren, Ney, Oppenheimer, Bradt, and Peters, Phys. Rev. **74**, 213 (1948).

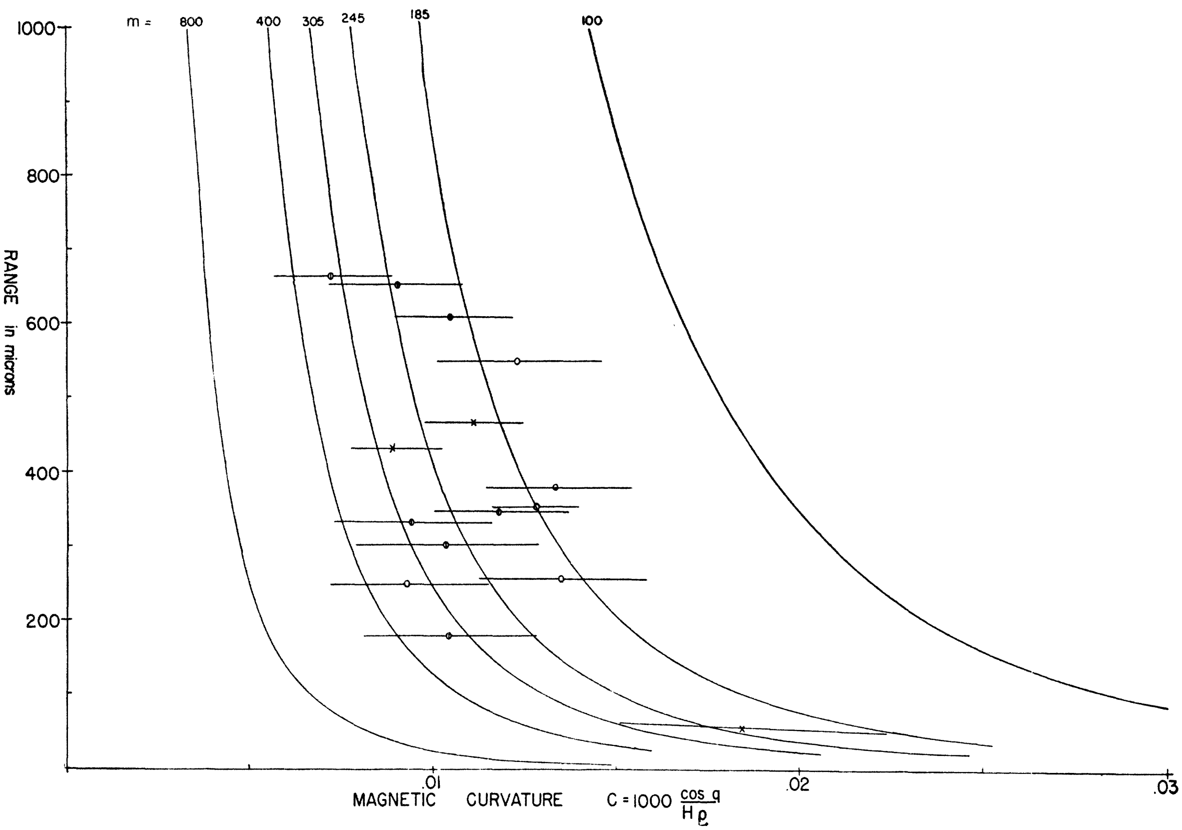

FIG. 6. Range-curvature plot. The solid curves represent the theoretical relationship between magnetic cuvature (C) and range in emulsion for a series of singly charged particles of different assumed mass values. A few experimental points and probable errors are given.

tion to higher energies, used for obtaining the function f in Eq. (1), may be assumed accurate to within a few percent.

The errors which would influence the mass determinations systematically (such as uncertainties in field or range) are in general small; the larger errors influencing the deflection angle α are Gaussian and hence tend to average out. Thus scattering errors are as likely to be positive as negative; similarly for observational errors, since the average measured angle for fast protons and heavy tracks was zero; even the occasional "mis-matching" of an unrelated track, which is possible though improbable, is as likely to occur on one side of the true corresponding track as on the other. The square of the *total* probable error in the angular deflection measurement for each meson is taken as the sum of the squares of the three sources of error in angle, a, b, and c above. Measurements of the apparent deflection of a few mesons in sandwiches with *no* magnetic field are consistent with this assignment of errors. (An exact experimental evaluation of angle errors in this manner would necessitate studying a very large number of such tracks, as there are these three independent sources of error in each case, and the scattering will give a Gaussian distribution of errors for each value of the range.) The plot of the final magnetic-deflection mass determinations, given in Fig. 5 below, indicates that the total probable errors have, if anything, been *over*estimated since for all types of mesons the *average* mass value for that group lies *within* the limits of probable error in the individual mass determination for considerably more than 50 percent of the cases; in fact, there are few cases in which the average value does not lie within the limits of probable error given. Of all the sources of error scattering is the most important. The average scattering angle is proportional to $1/E$, and decreases more rapidly than the curvature $(1/\rho)$ as one considers particles of greater and greater residual range; hence the percentage of error, due to scattering, in the deflection measurement is less for particles of longer range. It should also be emphasized that for a given curvature the angle of magnetic deflection is in first approximation proportional to the path length D in the air-gap. This has the important consequence that particles with small dip angle will have larger deflection angles; for such particles a given error in angle will represent a smaller percentage error in angle and hence greater accuracy in mass determination. These two factors, the range and the dip, account for the rather wide variations in the precision of individual mass determinations. A summary of all cases analyzed is given below.

5. RESULTS

Figure 4 indicates the number of events of different types found on the plates. The mesons have been grouped according to descriptive classifications,[1] where a σ is star-producing, π and μ are associated with an observed π–μ-decay, and a ρ is a particle which appears to have the scattering and density characteristic of a meson and stops with no visible interaction. For each individual stopped meson which had a trajectory in the air gap of at least 7 mm length, the mass was calculated from the observed experimental data as indicated in Eq. (1) (the plates of the 4th flight are still under analysis). The total probable error in angle, discussed in the preceding section, was then successively added to and then subtracted from the measured angle of deflection; using these two extreme angles new values of the mass were calculated which were assumed to correspond to the limits of probable error for that particular mass determination. The results are shown in Fig. 5 where, for each type of meson, the cases are arranged in order of increasing precision. In the figure there has been included a small sampling of the inaccurate cases. Since in averaging the mass determinations the values are weighted as the inverse squares of the probable errors, such cases contribute negligibly to such an average value; they are of interest because they indicate the sign of the charge (positive plotted on the right, negative on the left). The mass values have been plotted on a "deflection unit" scale in which equal increments of the deflection angle are measured by equal lengths in all parts of the scale. Since the errors in deflection angle must be Gaussian, probable errors in mass will be represented here by horizontal lines of equal length to the right and left of each point, which would not be the case on a linear mass abscissa. (The conversion from mass units to deflection units is actually made from a calculated curve relating the mass to the theoretical deflection angle measured in degrees for a particular range and dip angle which were arbitrarily chosen; hence, for mesons with other values of R and q, the reduced value expressed in these deflection units will not be equal to the actual deflection angle in *degrees*.)

The best statistical treatment of the experimental data to obtain an "average" mass value for each type of meson is perfoımed by taking a least-squares fit in a representation where abscissa and ordinate are variables with Gaussian error distributions; in our case, this will be a range-curvature plot. Figure 6 shows such a representation of a few selected experimental points, together with a few typical theoretical range-curvature curves for a series of assumed mass values. (All the experimental points are not shown here because with four types of mesons and with different probable errors for each point this is not a convenient representation for visualizing all the data on one graph.) In contrast to cloud-chamber techniques the uncertainties in the range using photographic emulsions are very much smaller than those in the curvature, so that they can safely be neglected. This is particularly true since the constant-mass curves can be seen to have such a steep slope that a very large error in range would be required to produce an error in mass comparable to that produced by errors in curvature; the curves have a less steep slope for small values of the range, but at such low energies scattering in any case gives a larger uncertainty in the curvature. The best value of the mass of a given type of meson is then determined by the least-squares method as the parameter of that curve of constant mass which for the experimental points on the range-curvature plot minimizes the sum $\sum(V_i/\epsilon_i)^2$, where V_i is the residual and ϵ_i the probable error of the curvature of an individual meson.[16,17] Such a least-squares solution yielded for the mass of the μ-meson 229 ± 21; for π-mesons 250 ± 25; for the σ-mesons, 302 ± 39; for the ρ^+-mesons 220 ± 30, and for the ρ^--mesons 226 ± 35.

A much simpler method for obtaining the approximate value for the mass of a given type of meson is to find the arithmetic average of the individual masses as expressed on the deflection unit scale (Fig. 5), weighting them inversely as the squares of the probable errors on that scale. The validity of such an approximation can be shown to depend on the following relationship: A given experimental point (mass-value) and the limits of probable error are obtained from curvature measurements which, for any mass parameter under consideration, have a certain residual V_i and error ϵ_i in the curvature ordinate (Fig. 6). If this point is represented, not on the actual curvature ordinate, but on the arbitrary deflection unit scale, as in Fig. 5, the new residual V_i' and error ϵ_i' on this scale will, of course, not in general be the same as V_i and ϵ_i. However one finds that over a considerable range of values, $V_i/\epsilon_i \cong V_i'/\epsilon_i'$. Because of this fact that the ratio of the residuals to the errors is almost unchanged by the transformation, one can, in place of minimizing the sum, $\sum(V_i/\epsilon_i)^2$, instead minimize $\sum(V_i'/\epsilon_i')^2$, on the deflection unit scale. The calculations are greatly simplified since with only one variable the best least-squares fit is equal to the arithmetic average. Values obtained in this manner were: $\pi\sim251\pm22$; $\sigma\sim304\pm40$; $\mu\sim231\pm21$; $\rho^-\sim225\pm32$; $\rho^+\sim220\pm25$. Combining the π- and σ-values gives 270 ± 19; while the μ- and ρ-values give 228 ± 15. The approximation method is seen to agree with the least-squares method to within 1 percent.

6. DISCUSSION

From the results in Fig. 4 concerning the number of mesons of different types observed on the plates several

[16] W. E. Deming, "Least squares," Graduate School of Department of Agriculture, Washington, D. C. (1938).

[17] R. B. Brode, Phys. Rev. **75**, 904 (1949). The residuals should be weighted as the inverse squares of the probable errors (ΔR and ΔC) rather than as the inverse squares of the *percent* probable errors as given. The author of the article acknowledges the mistake here (private communication).

comments can be made. The total number of mesons per hour per cc of emulsion at about 95,000 ft. found in the plates of flights 1, 2, and 4 respectively, are in good agreement. The number found on flight 3, whose ceiling was 69,000 ft., is significantly lower; comparing the rates at the two altitudes and assuming exponential absorption, a mean free path of $\sim$85 g/cm^2 is indicated for the absorption of the radiation which produced these low energy mesons. It is also interesting to compare the number of mesons of different types. Thus the ratio of the total number of ρ-mesons observed to the number of π- and σ-mesons is 1.4, which is lower than the ratio reported in plates at mountain altitudes (around 4). The ratio of π- to σ-cases is 1.7. Since, if anything, one would be more likely to miss a π–μ-decay in scanning than a σ-meson, and since the identification of both types of mesons is rather certain, a positive excess is indicated for these low energy mesons, even allowing for the fact that some negative mesons may produce no visible star. It will also be noted from the figure that slightly more mesons enter the emulsion from the glass backing than from the air gap.

The mass measurements on π-, μ-, and σ-mesons are consistent with the assumption of a unique value for the mass of each type of particle; and, while the averaged value of the σ-meson mass measurements was greater than that of the π-mesons, the limits of probable error of the two groups overlap, so the data are not inconsistent with the assumption that π- and σ-mesons have the same mass of 270$\pm$20 m_e, but opposite charge sign. Similarly, the μ-, ρ^+-, and ρ^--groups have mass averages which agree within the probable errors, giving for the whole group 226$\pm$15. These values may be compared with the Berkeley measurements[18] on artificial mesons, where the value found for π- and σ-mesons was 285 m_e; measurements on μ^+-cases yielded 216 m_e, but no measurements on negative light mesons (ρ^-) were obtainable. From cloud-chamber measurements, which are restricted to the light μ-mesons, Retallack and Brode[19] report a value of 215$\pm$4 m_e. It will be noted in Fig. 5 that in general the ρ-mesons were not as well-grouped about single values as were the other types of mesons. Since in this group were classified particles of "no visible interactions but appearance (scattering, density, etc.) characteristic of mesons," there may be included a certain percentage of heavier negative mesons captured without the emission of visible charged particles, such as have been reported from the Berkeley cyclotron experiments. There is also a slight possibility that one or two protons might have been included in this group.

Because of the fact that these experiments were carried out at very high altitudes, where one would expect mesons of all types to be produced in abundance in the 65 lb. of iron in the magnet, it is interesting to see what evidence is given in this data concerning the presence in this experiment of other types of mesons different from the usual π- and μ-types (outside of the range 200–300 m_e, like the "τ"- and "λ"-mesons reported). In the first place, the presence in an appreciable abundance of any type of meson producing nuclear disintegrations could not have escaped detection; all measured cases in which mesons produced visible stars in the emulsion had negative charges and had masses consistent with 270 m_e. Secondly, the percentage of low energy mesons present in these plates with mass less than 150 m_e must be comparatively small ($<$5 percent). (Very light mesons, of mass smaller than about 15 m_e, however, would have been difficult to detect in our C2 emulsions.) Thirdly, there is no evidence for the presence in appreciable abundance of *negative* mesons of greater than 400 m_e, stopping without visible interaction; such particles would be readily distinguishable from protons because of their charge sign. A *positive* heavy meson of, say, 1000 m_e would be more difficult to differentiate from a proton if it stopped without the emission of a visible particle. A series of mass determinations, carried out on a group of particles selected at random on these plates but presumed to be protons, gave values consistent with the accepted proton mass; however an extended series of measurements to obtain a more exact mass spectrum would be necessary before any definite statement could be made concerning such very heavy positive mesons.

The accuracy of these experiments was limited by the field strength and gap width of the magnet. A balloon experiment will be carried out using a stronger magnetic field and wider air-gap, which should give considerably greater accuracy to the measurements.

The author wishes to express his gratitude to Dr. Marcel Schein for his encouragement and cooperation throughout this project. Indebtedness is also acknowledged to the du Pont Company for a fellowship and to the Research Corporation for a research grant during different periods of the work. The ONR and the General Mills Company arranged for the balloon flights. W. Ferris assisted with the scanning of the emulsions and T. Kinyon, H. Martinek, and D. Orr helped with the computations.

[18] A. S. Bishop, Phys. Rev. **75**, 1468 (1949).
[19] J. G. Retallack and R. B. Brode, Phys. Rev. **75**, 1716 (1949).