Decay Constants from Coincidence Experiments

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A HIGH speed coincidence circuit[1] has been used in this laboratory to study half-lives of the order of 10^{-9} sec.[2] This prompted a consideration of the mathematical form of delayed coincidence resolution curves. Simple methods have given results comparable to those recently published by Bay.[3]

In a typical experiment, one counter of a coincidence circuit is excited by a parent radiation from a radioactive source, the other counter by a daughter radiation whose lifetime is to be measured. Artificial delays can be inserted in either side of the circuit. Experiment relates the number of observed coincidences to the (negative or positive) artificial delay, x.

If there is no natural delay; i.e., if the daughter has a lifetime much too short to be measured by the coincidence circuit, one obtains the "prompt coincidence resolution curve," $P(x)$. If there is a measurable lifetime, the daughter emission has a probability $f(t)dt$ of occurring t sec. after the parent emission. Experiment then provides a "delayed coincidence resolution curve," $F(x)$.

If $F(x)$, $P(x)$, and $f(t)$ are all normalized as differential probability distributions, i.e., are all normalized to unit area, then

$$F(x)=\int_{-\infty}^{\infty}f(t)P(x-t)dt. \tag{1}$$

Here $P(x)$ represents the response of the circuit to simultaneous emission of particles of the same kind and energy as are used to give $F(x)$.

If only a single decay is involved, whose mean life is $\tau=1/\lambda$ then $f(t)=0$ for $t<0$, and $f(t)=\lambda e^{-\lambda t}$ for $t\geqslant 0$. With $y=x-t$, (1) then becomes

$$F(x)=\lambda e^{-\lambda x}\cdot\int_{-\infty}^{x}e^{\lambda y}P(y)dy. \tag{2}$$

Differentiation of (2) gives

$$dF(x)/dx=\lambda\{P(x)-F(x)\} \tag{3}$$

and

$$d/dx[\ln F(x)]=-\lambda\{1-P(x)[F(x)]^{-1}\}. \tag{4}$$

Equation (3) shows that the maximum of $F(x)$ occurs at its intersection with $P(x)$, and Eq. (4) shows when it is safe to fix λ by the slope of the "tail" of $F(x)$.

For short lifetimes it is preferable to use most of the experimental points. By integration of (3)

$$\lambda=\{F(A)-F(B)\}\left[\int_A^B(F(x)-P(x))dx^{-1}\right], \tag{5}$$

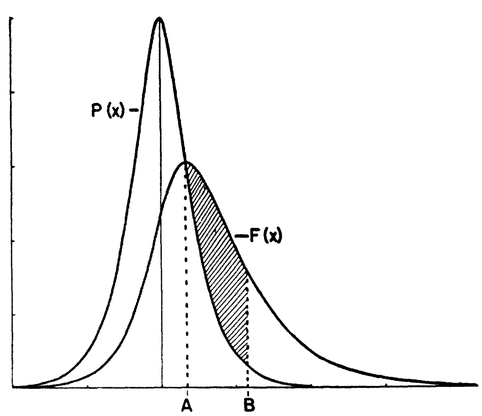

FIG. 1. By Eq. (5) λ is equal to the difference $\{F(A)-F(B)\}$ divided by the shaded area. The delayed curve $F(x)$ is calculated by relation (2) from the assumed prompt curve $P(x)$.

so that λ can be found from that section $A\leqslant x\leqslant B$ of the delay curve which is best determined (Fig. 1). The statistical errors of the tail occur only in the determination of total areas for normalization.

The parameter λ characterizes the asymmetry of $F(x)$. It could be determined from $F(x)$ alone if the true zero of the delay were known and if $P(x)$ were symmetrical.

Let $G(x)=F(-x)$ and $Q(x)=P(-x)$. Then from (5) one has

$$\lambda=\{F(A)+G(A)-F(B)-G(B)\}\left[\int_A^B(F(x)-G(x))dx-C\right]^{-1} \tag{6}$$

in which

$$C=\int_A^B[P(x)-Q(x)]dx$$

measures of the asymmetry of $P(x)$.

If the circuit itself introduces no asymmetry, $G(x)$ may be found experimentally by reversing the roles of the two sides of the circuit. This fixes the zero of delay. The correction C may still not vanish, due to asymmetries introduced by the excitation, but C will be small in a good experiment.

This discussion applies to simple decays, and experimental arrangements in which the daughter radiation is able to excite only one side of the circuit. More complicated cases can be similarly treated. The general relation between moments given by Bay[3] is an immediate consequence of (1).

The integrals in (5) and (6) should be computed numerically from experimental data. This allows a definite statement of statistical error. The arithmetic error so introduced becomes negligible if experimental settings are arranged in triples to which Simpson's rule can be applied.

[1] R. E. Bell and H. E. Petch, Phys. Rev. 76, 1409 (1949).
[2] R. E. Bell and R. L. Graham, Phys. Rev. 78, 490 (1950).
[3] Z. Bay, Phys. Rev. 77, 419 (1950).

Measurement of a Second Half-Life in Yb^{170}

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THE half-life of the 83-kev transition in Yb^{170} has been measured as $(1.60\pm0.2)\times10^{-9}$ sec. by delayed coincidence techniques, using the coincidence apparatus of Bell and Petch.[1] The experimental data were analyzed by the methods of Newton[2] and Bay.[3]

The beta-spectrum of Tm^{170} is complex,[4] a fraction of the beta-disintegrations leading to an excited state of Yb^{170}. This state is de-excited by the emission of 83-kev gamma-rays and K, L, and M conversion electrons. A source of Tm^{170} on a backing of 1.5-mg/cm^2 Al foil was mounted in a thin-lens beta-ray spectrometer. Coincidences were observed between nuclear beta-rays incident on an anthracene-1P21 counter placed immediately behind the source, and 73-kev L-conversion electrons focused by the spectrometer on a second similar counter. The delayed-coincidence resolution curve was recorded by an automatic device which inserted lengths of cable to delay the pulses and recorded the corresponding coincidence counting rates. Counting was carried out for equal times at the different delays. The resulting curve is shown in Fig. 1.

The resolution curve for prompt coincidences was obtained by replacing the source of Tm^{170} by one of Au^{198} without otherwise changing the apparatus. The coincidences then observed were due to conversion electrons of the 411-kev excited state of Hg^{198} in the source counter, and 73-kev nuclear beta-rays of Au^{198} in the spectrometer counter. This method of comparing Tm^{170} with Au^{198} ensures that any effects on the resolution curves due to electron transit time in the spectrometer or small pulse size from the 73-kev electron counter will cancel. In order to establish the promptness of the Au^{198} coincidences, the Au^{198} beta-conversion line coin-