Decay Constants from Coincidence Experiments

T. D. Newton
Chalk River Laboratories, National Research Council of Canada, Chalk River, Ontario, Canada
April 3, 1950

A HIGH speed coincidence circuit[1] has been used in this laboratory to study half-lives of the order of 10^{-9} sec.[2] This prompted a consideration of the mathematical form of delayed coincidence resolution curves. Simple methods have given results comparable to those recently published by Bay.[3]

In a typical experiment, one counter of a coincidence circuit is excited by a parent radiation from a radioactive source, the other counter by a daughter radiation whose lifetime is to be measured. Artificial delays can be inserted in either side of the circuit. Experiment relates the number of observed coincidences to the (negative or positive) artificial delay, x.

If there is no natural delay; i.e., if the daughter has a lifetime much too short to be measured by the coincidence circuit, one obtains the "prompt coincidence resolution curve," $P(x)$. If there is a measurable lifetime, the daughter emission has a probability $f(t)dt$ of occurring t sec. after the parent emission. Experiment then provides a "delayed coincidence resolution curve," $F(x)$.

If $F(x)$, $P(x)$, and $f(t)$ are all normalized as differential probability distributions, i.e., are all normalized to unit area, then

$$F(x)=\int_{-\infty}^{\infty} f(t)P(x-t)dt. \tag{1}$$

Here $P(x)$ represents the response of the circuit to simultaneous emission of particles of the same kind and energy as are used to give $F(x)$.

If only a single decay is involved, whose mean life is $\tau=1/\lambda$ then $f(t)=0$ for $t<0$, and $f(t)=\lambda e^{-\lambda t}$ for $t\geqslant 0$. With $y=x-t$, (1) then becomes

$$F(x)=\lambda e^{-\lambda x}\cdot\int_{-\infty}^{x} e^{\lambda y}P(y)dy. \tag{2}$$

Differentiation of (2) gives

$$dF(x)/dx=\lambda\{P(x)-F(x)\} \tag{3}$$

and

$$d/dx[\ln F(x)]=-\lambda\{1-P(x)[F(x)]^{-1}\}. \tag{4}$$

Equation (3) shows that the maximum of $F(x)$ occurs at its intersection with $P(x)$, and Eq. (4) shows when it is safe to fix λ by the slope of the "tail" of $F(x)$.

For short lifetimes it is preferable to use most of the experimental points. By integration of (3)

$$\lambda=\{F(A)-F(B)\}\left[\int_A^B (F(x)-P(x))dx^{-1}\right], \tag{5}$$

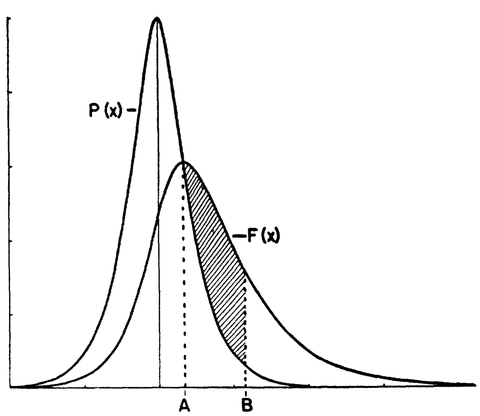

FIG. 1. By Eq. (5) λ is equal to the difference $\{F(A)-F(B)\}$ divided by the shaded area. The delayed curve $F(x)$ is calculated by relation (2) from the assumed prompt curve $P(x)$.

so that λ can be found from that section $A\leqslant x\leqslant B$ of the delay curve which is best determined (Fig. 1). The statistical errors of the tail occur only in the determination of total areas for normalization.

The parameter λ characterizes the asymmetry of $F(x)$. It could be determined from $F(x)$ alone if the true zero of the delay were known and if $P(x)$ were symmetrical.

Let $G(x)=F(-x)$ and $Q(x)=P(-x)$. Then from (5) one has

$$\lambda=\{F(A)+G(A)-F(B)-G(B)\}\left[\int_A^B (F(x)-G(x))dx-C\right]^{-1} \tag{6}$$

in which

$$C=\int_A^B [P(x)-Q(x)]dx$$

measures of the asymmetry of $P(x)$.

If the circuit itself introduces no asymmetry, $G(x)$ may be found experimentally by reversing the roles of the two sides of the circuit. This fixes the zero of delay. The correction C may still not vanish, due to asymmetries introduced by the excitation, but C will be small in a good experiment.

This discussion applies to simple decays, and experimental arrangements in which the daughter radiation is able to excite only one side of the circuit. More complicated cases can be similarly treated. The general relation between moments given by Bay[3] is an immediate consequence of (1).

The integrals in (5) and (6) should be computed numerically from experimental data. This allows a definite statement of statistical error. The arithmetic error so introduced becomes negligible if experimental settings are arranged in triples to which Simpson's rule can be applied.

1 R. E. Bell and H. E. Petch, Phys. Rev. **76**, 1409 (1949).
2 R. E. Bell and R. L. Graham, Phys. Rev. **78**, 490 (1950).
3 Z. Bay, Phys. Rev. **77**, 419 (1950).

Measurement of a Second Half-Life in Yb^{170}

R. E. Bell and R. L. Graham
Atomic Energy Project, National Research Council of Canada, Chalk River, Ontario, Canada
April 3, 1950

THE half-life of the 83-kev transition in Yb^{170} has been measured as $(1.60\pm0.2)\times10^{-9}$ sec. by delayed coincidence techniques, using the coincidence apparatus of Bell and Petch.[1] The experimental data were analyzed by the methods of Newton[2] and Bay.[3]

The beta-spectrum of Tm^{170} is complex,[4] a fraction of the beta-disintegrations leading to an excited state of Yb^{170}. This state is de-excited by the emission of 83-kev gamma-rays and K, L, and M conversion electrons. A source of Tm^{170} on a backing of 1.5-mg/cm² Al foil was mounted in a thin-lens beta-ray spectrometer. Coincidences were observed between nuclear beta-rays incident on an anthracene-1P21 counter placed immediately behind the source, and 73-kev L-conversion electrons focused by the spectrometer on a second similar counter. The delayed-coincidence resolution curve was recorded by an automatic device which inserted lengths of cable to delay the pulses and recorded the corresponding coincidence counting rates. Counting was carried out for equal times at the different delays. The resulting curve is shown in Fig. 1.

The resolution curve for prompt coincidences was obtained by replacing the source of Tm^{170} by one of Au^{198} without otherwise changing the apparatus. The coincidences then observed were due to conversion electrons of the 411-kev excited state of Hg^{198} in the source counter, and 73-kev nuclear beta-rays of Au^{198} in the spectrometer counter. This method of comparing Tm^{170} with Au^{198} ensures that any effects on the resolution curves due to electron transit time in the spectrometer or small pulse size from the 73-kev electron counter will cancel. In order to establish the promptness of the Au^{198} coincidences, the Au^{198} beta-conversion line coin-

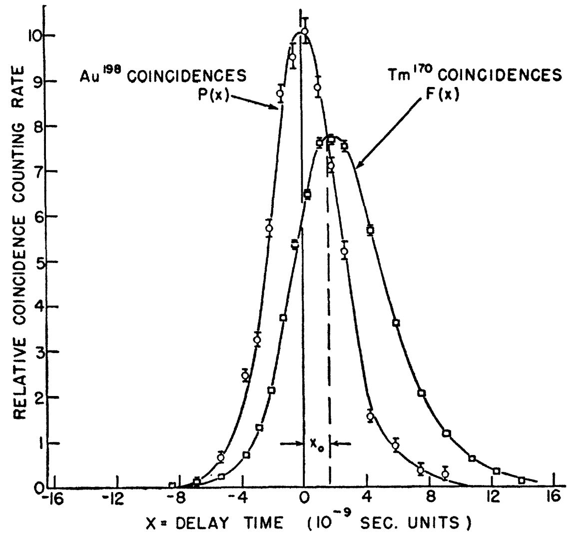

FIG. 1. Coincidence resolution curves. $P(x)$ is the prompt resolution curve obtained with an Au^{198} source and $F(x)$ is the delayed resolution curve observed with a Tm^{170} source, normalized to equal included areas. The curves intersect at $x=x_0$ where $F(x)$ is a maximum. Positive values of the delay time x correspond to delaying the pulses from the Tm^{170} beta-ray counter behind the source. The standard deviations of the points are indicated by vertical bars.

cidence resolution curve was compared in the spectrometer with the resolution curve obtained when hard beta-rays were passed through the anthracene crystals of both counters. The results showed that the half-life of the 411-kev transition in Hg^{198} does not exceed 2×10^{-10} sec.[5] The Au^{198} coincidences are thus prompt for purposes of this experiment.

Figure 1 shows the prompt resolution curve $P(x)$ observed for Au^{198} and the delayed resolution curve $F(x)$ observed for Tm^{170}, plotted to the same included area. Positive values of the delay, x, correspond to delaying the pulses from the Tm^{170} beta-ray counter behind the source. The areas were determined by numerical integration and the statistical error in equalizing the areas is less than two percent. The two curves obey the criterion[2] of intersecting at the maximum of $F(x)$.

The mean life $\tau=1/\lambda$ for the Yb^{170} transition has been evaluated from the curves of Fig. 1 in three ways. Newton's[2] Eq. (5) applied with $A=-\infty$, $B=x_0$ gives $\tau=(2.34\pm 0.12)\times 10^{-9}$ sec., and with $A=x_0$ and $B=\infty$ gives $\tau=(2.31\pm 0.09)\times 10^{-9}$ sec., the average being $\tau=(2.32\pm 0.07)\times 10^{-9}$ sec. Bay's[3] Eq. (4) gives $\tau=(2.28\pm 0.08)\times 10^{-9}$ sec. The value of τ obtained in the usual way from the slope of $\log F(x)$ at large positive delay is $(2.3\pm 0.4)\times 10^{-9}$ sec.

The new analyses give much improved statistical accuracy for a case such as this one, where $P(x)$ and $F(x)$ overlap to a large extent. The standard deviations quoted were obtained by carrying the standard deviations of the measured points through the numerical integrations. Averaging the above determinations of τ and making allowance for the fact that the Au^{198} coincidences are only known to be prompt within 2×10^{-10} sec., we get $\tau=(2.3\pm 0.3)\times 10^{-9}$ sec., or $T_{\frac{1}{2}}=(1.6\pm 0.2)\times 10^{-9}$ sec. for the 83-kev transition in Yb^{170}.

The new methods of analysis enable the present coincidence apparatus to set upper limits on half-lives of the order of 2×10^{-10} sec., and to measure half-lives of the order of 5×10^{-10} sec. with fair accuracy.

[1] R. E. Bell and H. E. Petch, Phys. Rev. **76**, 1409 (1949).
[2] T. D. Newton, Phys. Rev. **78**, 490 (1950).
[3] Z. Bay, Phys. Rev. **77**, 419 (1950).
[4] J. S. Fraser, Phys. Rev. **76**, 1540 (1949).
[5] Bell and Petch (reference 1) showed that this half-life was less than 2.1×10^{-9} sec. The new reduced limit was obtained with the same apparatus but using improved methods of analysis of coincidence resolution curves.

Saturation Backscattering of Positive Electrons

H. H. SELIGER
National Bureau of Standards, Washington, D. C.
April 3, 1950

THE percentage backscattering of positive electrons from a source backing thick enough to ensure complete absorption of the beta-particles (saturation thickness) has been studied under the conditions of geometry shown in Fig. 1. It was found that the backscattering of the positrons of Na^{22} under the same geometrical conditions is consistently 30 to 40 percent lower than the electron backscattering of I^{131}, RaE, and P^{32}.

As a check on the accuracy of the experimental technique, the backscattering of the negative electrons of I^{131}, Bi^{210}, and P^{32} was also investigated. In one set of experiments the weightless deposit approximately 12 mm in diameter was mounted on a thin aluminum leaf (0.22 mg/cm²) and net counting rates were measured with saturation thicknesses of Be, C, Al, Fe, Cu, Ag, and Pb backings placed directly beneath the weightless source. The net counting rate with no backing material beneath the source was assumed to be due only to forward radiation. The ratio of the net counting rate measured with a backing of atomic number Z to the net counting rate due to forward radiation will then indicate the percentage backscattering. In a different set of experiments, aliquots identical in activity and geometric size within one percent were deposited directly on polished faces of disks of saturation thickness again ranging from Be to Pb. The net counting rate in each case was plotted against the atomic number of the backing material and this curve was extrapolated to $Z=0$. The net counting rate extrapolated for $Z=0$ was assumed to be due to forward radiation only. The percentage backscattering was then calculated in the same manner as before.

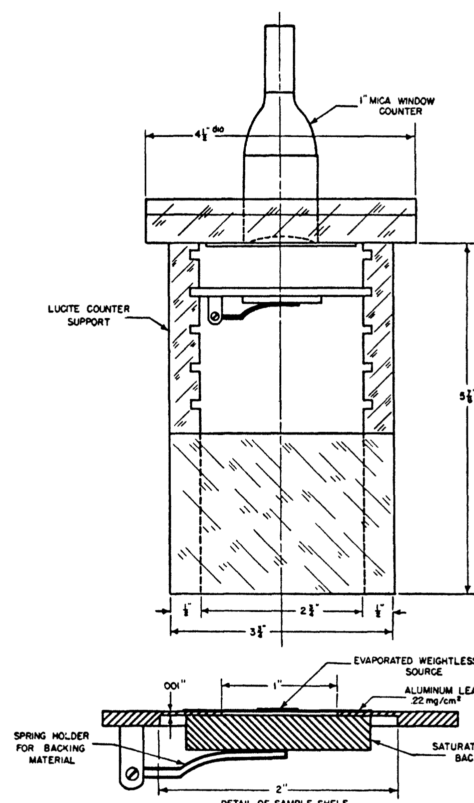

FIG. 1. Diagram showing dimensions of counter and source used in the backscattering experiments.