# Letters to the Editor

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## Angular Dependence of Crystalline Nuclear Resonance Absorption

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IN a series of brief notes, Pound[1] has reported his observations on the absorption spectrum of $Al^{27}$ and $Li^7$ nuclei in crystals. His experiments prompted us to investigate the radiofrequency absorption for nuclei of spin $I$ and of electric quadrupole moment $Q$, as a function of the orientation of the steady magnetic field with respect to the crystalline electric field. In particular, we have investigated the consequences of assuming that the nuclei are subject to the interactions of the nuclear magnetic moment with the external magnetic field and of the quadrupole moment with the static crystalline electric field of tetragonal symmetry.

Let $\theta$ represent the angle which the steady magnetic field makes with the tetragonal axis ($z$ axis), and for convenience choose the $x$ axis so that it lies in the plane determined by the tetragonal axis and the external field vector, $H_0$. It follows then from group-theoretical considerations, that the wave functions which diagonalize the magnetic energy, are given by[2]

$$\Psi_m=(a\xi+b\eta)^{I+m}(-b\xi+a\eta)^{I-m}/((I+m)!(I-m)!)^{\frac{1}{2}}$$

where $a=\cos\theta/2$, $b=\sin\theta/2$, and $\xi$ and $\eta$ determine the nuclear spin functions $\chi_m$, given by

$$\chi_m=\xi^{I+m}\eta^{I-m}/((I+m)!(I-m)!)^{\frac{1}{2}}$$

whose axis of quantization coincides with the tetragonal axis. The advantage of the above representation of the spin functions lies in the fact that the matrix elements are easily evaluated.

The quadrupole energy operator is known to be[3] $DQ(3I_Z{}^2-I^2)$, where $D=(\partial^2V/\partial Z^2)[e/4I(2I-1)]$. Therefore, to carry out the perturbation calculation, the matrix elements

$$(m|I_Z{}^2|m)=\tfrac{1}{2}m^2(3\cos^2\theta-1)+\tfrac{1}{2}I(I+1)\sin^2\theta,$$

$$(m\pm1|I_Z{}^2|m)=\mp\tfrac{1}{2}\sin\theta\cos\theta(\pm2m+1)((I\mp m)(I\pm m+1))^{\frac{1}{2}}$$

and

$$(m\pm2|I_Z{}^2|m)=\tfrac{1}{4}\sin^2\theta((I\mp m)(I\pm m-1)(I\pm m+1)(I\pm m+2))^{\frac{1}{2}}$$

are needed, The first-order perturbation calculation then gives

$$E_m=-m\mu_Ng_NH_0+DQ[\tfrac{3}{2}m^2(3\cos^2\theta-1)+\tfrac{3}{2}I(I+1)\sin^2\theta-I(I+1)].$$

Since the allowed transitions in a perpendicular radiofrequency field are those for which $\Delta m=\pm1$, the absorption spectrum consists of the components

$$h\nu(m\rightarrow m\pm1)=\pm g_N\mu_NH_0\mp3mDQ(3\cos^2\theta-1)-\tfrac{3}{2}DQ(3\cos^2\theta-1)$$

with the intensities proportional to $(I(I+1)-m(m\pm1))$. The upper sign is to be taken for positive $g_N$ and the lower sign for negative $g_N$. It is readily seen that the absorption spectrum consists of $2I$ components all equally spaced, having separations between adjacent components given by $3DQ(3\cos^2\theta-1)$. Furthermore, the component separations $\Delta\nu_{\parallel}$ and $\Delta\nu_{\perp}$ for the steady field parallel and perpendicular respectively to the tetragonal axis are in the ratio of 2 to 1, and the spectrum coalesces into a single line when the external magnetic field is inclined at $\theta=\cos^{-1}(3)^{-\frac{1}{2}}$ with respect to the tetragonal axis. The term values to the second approximation can also be readily calculated by means of the off-diagonal matrix elements given above. According to Pound, it appears that second or even higher approximations are necessary in calculating the $Al^{27}$ spectrum.

We are also investigating the angular dependence of the hyperfine structure of the paramagnetic absorption spectrum reported by Penrose *et al.*[4]

[1] R. V. Pound, Phys. Rev. **73**, 1247 (1948); **76**, 1410 (1949); Proc. Phys. Soc. London **61**, 576 (1948).
[2] E. Wigner, *Gruppentheorie* (Edwards Brothers, Inc., Ann Arbor, Michigan), p. 175.
[3] C. Kikuchi, Phys. Rev. **77**, 746(A) (1950).
[4] Penrose, Abragam, and Pryce, Nature **163**, 992 (1949); for further references, see C. Kikuchi and R. D. Spence, Am. J. Phys. **18**, 167 (1950).

## The Schmidt Model and Odd-Even Staggering of the Isotopic Shift

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CENTROIDS of the hyperfine structure patterns of isotopes with odd $A$ and even $Z$ usually fall between the lines of isotopes with even $N$ and closer to those of the lighter even isotope. This phenomenon is referred to as the odd-even staggering. An explanation of it has been proposed[1] in terms of nuclear polarization. This explanation involves assumptions about nuclear wave functions and leaves room for the consideration of other effects, with which it might combine.

Schmidt's model[2] appears to have a bearing on the phenomenon. Schmidt accounts for features of nuclear magnetic moments on the single nucleon+core picture. The nucleon is supposed to have the same spin magnetic moment as in its free condition. The orbital magnetic moment of the proton is used as though the proton were free.

If the success[3] of Schmidt's model is not accidental then the unpaired nucleon must be mostly outside the core. Nuclear reactions suggest strong and intimate binding of nucleons with nuclear matter and the formation of compound states. While the nucleon is in the core it would be expected to affect the state of the nuclear matter as a whole and the effective value of the nucleon's charge and magnetic moment can be expected to be affected by correlated motions of the nuclear matter.

When two neutrons are added to a nucleus with even $Z$ the spin and magnetic moment are often practically unchanged. On Schmidt's model this situation is easily explained provided the neutron pair is supposed to form part of the core. An odd neutron, however, has to be pictured as mostly outside. The addition of a neutron pair to the interior will result in an expansion of the positive charge distribution. The odd neutron, remaining mostly outside the positive charge, would cause only secondary expansion effects accounting for odd-even staggering. Such a view is not at variance with general features of shell structure of nuclei.[4]

Considerations related to the virial theorem appear at first sight to indicate an abnormally large nuclear volume for the odd isotopes. But the current hypothesis[5] of strong spin-orbit coupling makes such reasoning inapplicable. The weaker binding of odd neutron need not be connected with an anomalous expansion of the nucleus, therefore.

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[1] Breit, Arfken, and Clendenin, Phys. Rev. **77**, 569 (1950). A longer paper on the same subject by the same authors is in press.
[2] T. Schmidt, Zeits. f. Physik **106**, 358 (1937); H. Schueler, Zeits. f. Physik **107**, 12 (1937); F. Hund, Physik. Zeits. **38**, 929 (1937).
[3] Aage Bohr and V. F. Weisskopf, Phys. Rev. **77**, 94 (1950).
[4] W. Elsasser, J. de phys. et rad. **5**, 625 (1934); A. Berthelot, J. de phys. et rad. **3**, 17, 52 (1942); Maria Goeppert Mayer, Phys. Rev. **74**, 235 (1948); **75**, 1969 (1949); Eugene Feenberg and Kenyon C. Hammack, Phys. Rev. **75**, 1877, 1968 (1949); L. W. Nordheim, Phys. Rev. **75**, 1894 (1949).
[5] Papers by M. G. Mayer in preceding footnote.