A New Criterion for the Occurrence of Slip in Thin Single Crystals

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It has been found that in thin crystals of aluminum slip does not occur in the plane or direction which corresponds to the maximum resolved shear. Instead, slip directions are favored which correspond to a short path of slip across the crystal. A formula is derived which permits one to predict the behavior of thin single crystals in tension.

IN the course of preparation of thin, single crystal specimens of aluminum for the study of internal friction as a function of elongation, it was found that the slip did not occur according to the usual rules governing the maximum resolved shear. This being a serious deviation from the generally accepted behavior, a systematic investigation of this phenomenon has been undertaken.

By using the method of carefully controlled strain-anneal, single crystals of aluminum 10 to 20 cm long and a few centimeters wide were obtained in a sheet 0.03 cm thick. Since the samples were to be used for the study of internal friction, they had to be cut out in such a manner as to avoid any straining. This was achieved by covering a single crystal with an asphalt lacquer and etching[1] along an outline from which the lacquer was removed prior to its complete hardening. The specimens were then electrolytically polished[2] to the desired thickness and shape. The final size was 2 cm length, 2 mm width and around 0.2 mm thickness, with somewhat larger butts at the ends for clamping in a tensile device. The stretching was performed under continuous microscopic observation which indicated a uniform appearance of slip lines along the length of the specimen.

TABLE I. Observed slip systems and calculated values of S and S' for four crystals. Only those having high values of S or S' and the active ones are shown.

Crystal	Slip plane	Slip direction	S	S'	Observed slip
#7	$\bar{1}11$	$0\bar{1}1$	0.245	0.179	2nd
		101	0.365	0.066	
	111	$\bar{1}10$	*0.469*	0.125	
		$0\bar{1}1$	0.301	0.192	
	$1\bar{1}1$	$\bar{1}01$	0.230	0.227	1st
#8	111	$\bar{1}01$	*0.442*	*0.348*	1st
		$0\bar{1}1$	0.418	0.214	
	$\bar{1}\bar{1}1$	011	0.373	0.313	2nd
#9	$\bar{1}\bar{1}1$	011	0.361	0.289	2nd
		101	*0.472*	0.136	
	111	$\bar{1}01$	0.372	*0.342*	1st
#10	$\bar{1}\bar{1}1$	101	0.383	0.152	
		011	0.338	*0.322* }	—simult.
	111	$\bar{1}01$	*0.438*	*0.326* }	
		$0\bar{1}1$	0.400	0.113	

Orientation of the crystallographic axes of the crystal with respect to the direction of tension was determined by means of x-rays or by optical methods. A face-centered cubic lattice slips on a (111)-type plane in a [110]-type direction. The actual index of the active slip plane is easily determined from the measurement of the inclination of the slip lines to the direction of tension as observed on the surface of the sample. Inasmuch as the samples were very small it was found advantageous to determine the direction of slip by comparing the Laue x-ray spots of planes perpendicular to the various possible directions of slip.[3] The spot corresponding to the active direction of slip was much broader than those of the other, inactive, directions in the same slip plane.

According to the usual criterion,[4] the first slip should occur on a slip system which corresponds to the maximum resolved shear. The latter is given by

$$S=(F/A)\cos\theta\sin\theta\cos\eta=(F/A)\cos\theta\cos\lambda, \quad (1)$$

where F/A is the tension per unit cross section of the sample, θ is the angle between the direction of tension and the normal to the slip plane, λ is the angle between the direction of tension and the direction of slip and η is the angle between the slip direction and the projection of the direction of tension on the slip plane. Assuming $F=A$, the relative resolved shear S was computed for all possible slip systems; the maximum value should correspond to the system which is first to become active. A comparison with experiments showed a complete lack of agreement.

It is difficult to imagine any "macroscopic" reason why formula (1) would not apply. Thus attention was directed to the actual mechanism of slip according to which for a slip to occur a dislocation has to form at or near the surface and cross the specimen. There it undergoes reflection and retraces its path producing additional slip and so on until it is stopped by an obstruction. If we compare a dislocation moving across the thickness (0.2 mm) of the sample with a dislocation moving across the width (2 mm) of the sample we see that in the first case the area on which a dislocation can

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[1] Edwards, Frary, and Jeffries, *The Aluminum Industry* (McGraw-Hill Book Company, Inc., New York, 1930), p. 469.
[2] R. B. Mason, U. S. Patent No. 2108603.
[3] Turner, Wu, and Smoluchowski, Rev. Sci. Inst. (to be published).
[4] W. Boas and E. Schmid, *Kristallplastizität* (Verlag. Julius Springer, Berlin, 1935), p. 111.

form, and thus the probability of its occurrence, is much bigger. Also, in that case the path is shorter and thus the probability, per unit length of a dislocation, that a dislocation will be stopped in its run by an obstacle (a mosaic boundary, etc.) is smaller. It appears, therefore, that the resolved shear as computed from formula (1) should be multiplied by some factor P which takes into account these additional geometrical considerations which favor a short path. We assume this factor to be inversely proportional to the length of the path of the dislocation, which is given by

$$L_1=\frac{t\sin\delta}{\sin\Phi\cos\theta} \quad \text{or} \quad L_2=\frac{w\sin\delta}{\cos\Phi-\cos\lambda\cos\delta}, \tag{2}$$

whichever is the smaller. Thus, depending on whether the path is limited by the thickness t or by the width w of the sample the factor P is given by c/L_1 or c/L_2 where c is a proportionality constant. The angle Φ is the angle between the direction of slip and the trace of the plane of slip on that surface of the sample which is perpendicular to the thickness t, while δ is the angle between that trace and the direction of tension. In our case the samples were quite thin ($w=10t$) and so in all cases the limiting factor was due to the thickness of the sample. Thus

$$S'=(PF/A)\cos\theta\cos\lambda=(cF/L_1A)\cos\theta\cos\lambda$$
$$=\frac{c}{t}\frac{F}{A}\frac{\cos^2\theta\cos\lambda\sin\Phi}{\sin\delta} \tag{3}$$

and the corresponding relative values were computed assuming $cF=tA$. The values obtained from formulae (1) and (3) are given in Table I, for several crystals together with the actually observed first and second slip systems. For these and all other samples, observations agree with the new formula (3) and not with formula (1) except in one case (crystal #8) where both formulas gave the same result. It is important to note that in general the two formulas indicate not only different directions of slip but also different planes of slip. If the average distance between the obstacles in the metal is much smaller than the dimensions of the sample and the sample itself is approximately square or round in cross section then the usual formula (1) is applicable. It should be noted that a similar preference for slip in the direction of shortest path was observed by Smekal[5] in his experiments on the tensile deformation of rocksalt crystals at room temperature.

Whenever the rotation of the crystal during the operation of the first slip system is small, the relative values of S' do not change much and then, as expected, the second slip occurs in a slip system which has the next highest S' in a new plane. An interesting case was Sample No. 10 for which two slip-systems had almost identical values of S' while the corresponding values of S differed by 30 percent: the first slip occurred on both systems at once (so-called duplex slip) in agreement with formula (3). It is interesting to note that this crystal was extremely ductile. A measurement of the actual shear stresses is planned.

Slip systems in crystals in relation to the orientation of the tensile direction are conveniently described by means of a diagram used by Schmid and Boas.[6] In our case, however, the slip system depends also on the dimensions of the sample and on the orientation of the various surfaces. For that reason no similar general diagram can be constructed. It should be noted, however, that the second slip system was always a conjugate[7] of the primary slip system.

[5] A. Smekal, Zeits. f. Physik **93**, 166 (1934).

[6] Reference 4, p. 93.

[7] "Conjugate" in the sense used by Professor C. H. Mathewson (see, for instance, Maddin, Mathewson, and Hibbard, Jr., J. Metals, **1**, 527 (1949)), to whom the authors wish to express their appreciation for stimulating discussion concerning the results here reported.