scattering due to thermal motions. However, as Holstein[8] has pointed out, the use of such kinetic theory concepts as a mean free path and a diffusion coefficient in this calculation of σ is not completely satisfactory in describing accurately the diffusion of imprisoned resonance radiation, where the atomic absorption coefficient varies considerably with frequency. Holstein has circumvented these difficulties by analyzing the diffusion process in terms of the probability of a resonance quantum going a given distance between successive absorptions. The averaging of this probability over the resonance line takes account of incoherent scattering due to thermal motions, and this averaged probability is used in an equation of radiative equilibrium which is solved approximately by variational methods for the decay constant. However, Zemansky's method for calculating σ, which gives the right order of magnitude, is used in this paper.

The optical thicknesses obtained from (23) for the values of N and l involved in the laboratory measurements of the decay constant are large, ranging from ten to one hundred. In this range $\log\beta$ as calculated by the first approximation is less than one percent larger than $\log\beta$ as calculated by the second approximation. In Fig. 1, values of $\log\beta$ are plotted for various values of $\log N$ and for $l = 1.95$ cm, as calculated for the second approximation, as calculated by Zemansky[6] using Milne's theory and as calculated by Holstein.[8] The measurements of Zemansky[3] for the same value of l are also plotted. Better agreement between measured and calculated decay constants have been obtained with the improved measuring techniques of Alpert *et al.*[4] The discrepancy between measured and calculated values of $\log\beta$ for large values of $\log N$ can be explained in part by the effects of collisional broadening which have been neglected in the computations.

It is a pleasure to express my appreciation to Dr. S. Chandrasekhar for suggesting this problem and for helpful discussions concerning it.

[8] T. Holstein, Phys. Rev. 72, 1212 (1947).

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The Growth of Circularly Polarized Waves in the Sun's Atmosphere and Their Escape into Space

V. A. Bailey
School of Physics, University of Sydney, Sydney, Australia
(Received May 24, 1949)

The theory, previously published, of plane waves in an ionized medium pervaded by static electric and magnetic fields is shown to predict wave amplification, and consequent electromagnetic noise, in certain frequency bands. It is then developed in detail for the case in which the static fields are both parallel to the direction of wave propagation and the perturbations are transverse to this direction.

It is shown that for any given frequency and electron drift velocity there are two trios of such waves, E_1 and E_2 waves, all circularly polarized; the E_1 and E_2 waves are oppositely polarized. It is found that any transverse perturbation temporally prescribed at a given plane can be split up into two such trios which can then be considered independently.

Necessary and sufficient conditions are then found under which a growing flux of energy carried by E_1 or E_2 waves can pass normally through the boundary between two different ionized media.

The theory is applied to show that under simple hypotheses about the drift of electrons in the atmosphere above a large sunspot strong circular waves can arise by growth of random transverse perturbations and can then escape from the sun. The consequences of two such hypotheses are compared with known observations of solar noise and used to interpret them.

It is concluded that the general hypothesis that electrons in a sunspot have a drift motion leads to results which are in good agreement with many facts about strong solar noise and which do not disagree with any others.

The ultimate intensity which a growing perturbation can attain is also discussed.

1. INTRODUCTION

In two previous publications[1,2] the general equations which specify the dispersion and polarization of plane waves in an ionized medium, pervaded by static electric and magnetic fields, have been derived. This theory may be conveniently referred to as the electro-magneto-ionic theory of wave propagation and more briefly as the E.M.I. theory. As a limiting case it includes the well-known magneto-ionic theory (M.I. theory) and its application is in general subject to the same conditions of validity as the latter.

We shall here apply the E.M.I. theory to the important solar phenomenon of emission of strong circularly polarized radio noise by a large sunspot.

As it appears that the Australian publications referred to above are not yet readily available in the United States and elsewhere, a summary of the E.M.I. theory is given here in the approximation which neglects the motions of the positive ions.

In Appendix 1 we also give the relativistic form of the

[1] V. A. Bailey, J. Roy. Soc. N.S.W. 82, 107 (1948).
[2] V. A. Bailey, Australian J. Sci. Res. A, 1, 351 (1948).

equation of dispersion which is needed for certain purposes.

The physical problem is to determine the wave properties of an ionized medium which, in the steady state, consists of N_0 electrons per cc, N_{i0} positive ions per cc and gas molecules, and is pervaded by a static electric field $\mathbf{E}_0$ and a static magnetic field $\mathbf{H}_0$. Consequently in the steady state the electrons and ions have the mean drift velocities $\mathbf{U}_0$ and $\mathbf{U}_{i0}$, respectively, relative to a frame of reference in which the gas is at rest. In general also the electrons and ions will have random motions specified by the temperatures θ and θ_i, respectively, and frequencies of collision (with gas molecules) ν and ν_i, respectively. It has been found more convenient to specify the random motions by the quantities τ and τ_i which are equal to one-third of the mean square velocities of agitation of the electrons and ions, respectively. According to circumstances we may have $\tau \lesssim U_0^2$ and $\tau_i \lesssim U_{i0}^2$ (as for example in traveling-wave tubes and the ionosphere, respectively).

In the theory it is assumed as a first approximation that ν, ν_i, τ, τ_i are constants and that the distribution of random velocities is constant. Of course ν and ν_i are related to τ and τ_i and to U_0 and U_{i0} through the mean free paths, but the actual relations are best considered when the general theory is applied to a particular problem, for then known experimental data may be available or can be obtained. When the gas pressure is very low, as in electron-beam tubes or in the sun's atmosphere above 2000 km from its surface, the quantities ν, ν_i may in general be neglected when waves of several Mc/sec. frequency are being considered. The motions of the positive ions may also be neglected when the ionic density-frequency and gyro-frequency are both much less than the wave frequency.

The E.M.I. theory is developed from the following laws of physics:

I. Maxwell's laws of the electromagnetic field.
II. The conservation of electrons and positive ions.
III. Maxwell's laws of the transfer of momentum in mixtures of different kinds of particles which are also subject to fields of force.

As is well known, III represents only a first approximation to the exact laws. But the exact laws are in general much more complicated than Maxwell's and cannot be used without an exact knowledge of the laws of force which operate at collisions and which will differ according to the nature of the molecules and positive ions present. In any event it is known that in the few cases which have been studied rigorously the results show that Maxwell's laws of transfer are very good approximations. Also these laws become exact when the effects of collisions are negligible.

The laws of transfer of energy of agitation are here neglected in the interests of simplicity. The effect is unimportant when ν, ν_i, τ, and τ_i are negligible, but even when they are not, the effect may be important only in special circumstances. The analogous problem of the propagation of sound waves in air shows that the corresponding neglect of energy transfer leads only to the correction which Laplace made in Newton's estimate of the velocity of sound, namely about ten percent. In the E.M.I. theory we therefore neglect possible perturbations of ν, ν_i, τ, and τ_i and treat them as constants.

It has also been found convenient to introduce the vector and scalar potentials $\mathbf{A}$ and V.

The physical problem now consists of the study of perturbations of the quantities N_0, N_{i0}, $\mathbf{E}_0$, $\mathbf{H}_0$, $\mathbf{U}_0$, $\mathbf{U}_{i0}$, $\mathbf{A}_0$ and V_0. These perturbations are denoted by the corresponding lower case symbols n, n_i, $\mathbf{e}$, $\mathbf{h}$, $\mathbf{u}$, $\mathbf{u}_i$, $\mathbf{a}$ and v.

The resulting equations for the perturbations are non-linear. In a first approximation to their solutions we retain only the linear terms and so can obtain plane wave (or cylindrical wave) solutions of the form

$$Ae^{i(\omega t - Lx)}.$$

This first approximation therefore holds true only when the perturbations are small. But when the angular frequency ω lies within certain frequency bands,* the perturbations may grow sufficiently to make it necessary to retain the non-linear terms, and then the solutions become much more complicated and difficult to apply except in special cases. With such growing perturbations their reaction on the medium may become large enough to consider. It is clear that one important result is that the wave energy grows at the expense of the kinetic energies $\frac{1}{2}mU^2$, $\frac{1}{2}m_iU_i^2$, of the electrons and ions, which in turn draw upon the energy associated with the static electric and magnetic fields.

In this paper we cannot make an exhaustive study of all these questions as this would require too much space. We will therefore limit ourselves to a fairly exact study of the origin and escape of circularly polarized waves from a large, isolated sun spot and to a discussion of the reaction of such waves on the medium. For the same reason we shall postpone to a future occasion the full discussion of any longitudinal (plasma) growing waves and their interaction with the circular waves through non-linear terms in the equations or otherwise.

It may also be stated here that when the relativistic E.M.I. theory (in Appendix 1) is used it is found that as the drift velocity U_0 approaches c, the velocity of light, the possibility of growth tends to disappear. We will therefore restrict our discussion to situations in which $U_0^2 \ll c^2$.

The principal symbols used here are set out as follows, mostly in alphabetical order. Considerable use has been made of auxiliary symbols in order to keep the formulas and some of the discussion from becoming unwieldy.

* In which, for example, two possible values of L are conjugate complex numbers.

$\mathbf{A}_0$, $\mathbf{a}$ the static and varying vector potentials respectively.
βi the imaginary part of M.
c the velocity of light.
$\nabla = \mathbf{i}(\partial/\partial x)+\mathbf{j}(\partial/\partial y)+\mathbf{k}(\partial/\partial z)$.
$D_t = \partial/\partial t$.
$\Box^2 = \nabla^2 - c^{-2}D_t^2$.
$D = D_t + (\mathbf{U}\cdot\nabla)$.
e the electronic charge, in e.s.u.
$\mathbf{E}_0$, $\mathbf{e}$ the static and varying electric field vectors respectively.
E_1, E_2 denote circularly polarized waves defined under (20) or (20.1) and in Appendix 2.
$f=\omega/2\pi$, the wave frequency in Mc/sec.
$f_0=p_0/2\pi$, the electron density frequency (or plasma frequency) in Mc/sec.
$f_H=\Omega_0/2\pi$, the electron gyro-frequency in Mc/sec.
$f_x, f_x' = \frac{1}{2}[(f_H^2+4f_0^2)^{\frac{1}{2}} \pm |f_H|]$ at x km above the sun's surface.
$\mathbf{H}_0$, $\mathbf{h}$ the static and varying magnetic field vectors respectively.
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ unit vectors parallel to the x, y, and z axes respectively.
$k_n = (-)^{n-1}$ with $n=1$ or 2.
L the angular wave number (complex).
M the complex refractive index.
μ the real part of M.
m the electronic mass.
N_0, n the static and varying number-density of electrons.
ν the frequency of collisions of an electron with other kinds of particles.
$p_0 = (4\pi N_0 e^2/m)^{\frac{1}{2}}$ the angular electron density frequency (or plasma frequency).
P the Poynting flux of energy.
$R = \omega - U_x L$.
$\rho = \nu/\omega$.
$\sigma_n = 1 - k_n\eta$.
τ one-third of the mean square velocity of electron agitation.
$\mathbf{U}_0$, $\mathbf{u}$ the mean steady drift velocity and varying velocity, respectively, of the electrons.
U_x, U_y, U_z the components of $\mathbf{U}_0$.
U an abbreviation for U_0.
V_0, v the static and varying scalar potentials.
$\mathbf{V}_0 = \mathbf{U}_0/c$.
$\mathbf{\Omega}_0 = (-e/mc)\mathbf{H}_0$ the angular electron gyro-frequency vector.
Ω_x, Ω_y, Ω_z the components of $\mathbf{\Omega}_0$.
Ω an abbreviation for Ω_0.
ω the angular wave frequency, always taken as positive.
$\omega_1, \omega_2 = \frac{1}{2}[(\Omega^2+4p_0^2)^{\frac{1}{2}} \pm |\Omega|]$.
ω_a, ω_a' values of ω_1, ω_2 respectively in a medium A.
$X = R^2 - \tau L^2 - p_0^2 - i\nu R$.
$\xi = p_0^2/\omega^2$.
$Y = R(Z+p_0^2) - i\nu Z$.
$\eta = \Omega/\omega$.
$Z = c^2L^2 - \omega^2$.
x, y, z space coordinates.
t time.

With the notation

$$\left.\begin{array}{lll} N=N_0+n, & \mathbf{E}=\mathbf{E}_0+\mathbf{e}, & \mathbf{H}=\mathbf{H}_0+\mathbf{h}, \\ \mathbf{U}=\mathbf{U}_0+\mathbf{u}, & \mathbf{A}=\mathbf{A}_0+\mathbf{a}, & V=V_0+v, \end{array}\right\} \quad \text{(A)}$$

and the motions of the positive ions neglected, the complete set (S) of fundamental equations which express the laws I, II and III are

$$\mathbf{E} = c^{-1}D_t\mathbf{A} - \nabla V, \tag{1}$$

$$\mathbf{H} = \nabla \times \mathbf{A}, \tag{2}$$

$$4\pi e(N - N_{i0}) = -\Box^2 V, \tag{3}$$

$$4\pi e N\mathbf{U} = -\Box^2\mathbf{A}, \tag{4}$$

$$D_t N = -\nabla(N\mathbf{U}), \tag{5}$$

$$D\mathbf{U} + \nu\mathbf{U} + N^{-1}\nabla\tau N = (e/m)(\mathbf{E} + c^{-1}\mathbf{U}\times\mathbf{H}). \tag{6}$$

The set (S_0) of equations for the steady state of the medium are obtained from this set (S) by adding the suffix 0 to all the variables and omitting all the terms in which D_t occurs.

As a check it will be found that the Eq. (6_0) obtained from (6) is consistent with the classical formulas of J. S. Townsend[3] for the steady motions of electrons under the action of uniform electric and magnetic fields. When N_0 is uniform in the steady state (6_0) becomes

$$(\mathbf{U}_0\cdot\nabla)\mathbf{U}_0 + \nu\mathbf{U}_0 - (e/mc)\mathbf{U}_0\times\mathbf{H}_0 = (e/m)\mathbf{E}_0; \tag{7}$$

this equation determines the drift velocity $\mathbf{U}_0$ in terms of the static electric and magnetic fields and the collision frequency or, alternatively, determines the electric field required to set up a given drift velocity.

To obtain the set of equations (S_p) for the perturbations we substitute in the set (S) given above the expressions for N, $\mathbf{E}$, $\mathbf{H}$, etc., given under (A), and subtract the corresponding equations in the set (S_0). As some of (S_p) are non-linear, we may in a first approximation, which is valid for sufficiently small pertubations, retain only the linear terms.

When the medium and the static fields are uniform in the steady state the quantities with suffix 0 are all constants and $N_{i0}=N_0$. To study any (small) plane perturbation of this state it is sufficient, by Fourier's integral theorem, to take all the lower case variables in the linear equations (S_p) to be proportional to

$$e^{i(\omega t - Lx)}. \tag{8}$$

Then (S_p) reduces to the following set:

$$\mathbf{e} = -ic^{-1}\omega\mathbf{a} + iLv\mathbf{i}, \tag{1.1}$$

$$\mathbf{h} = -iL\mathbf{i}\times\mathbf{a}, \tag{2.1}$$

$$n = (Z/4\pi ec^2)v, \tag{3.1}$$

$$N_0\mathbf{u} + \mathbf{U}_0 n = (Z/4\pi ec)\mathbf{a}, \tag{4.1}$$

$$Rn = N_0 L u_x, \tag{5.1}$$

$$\delta\mathbf{u} - \mathbf{\Omega}_0\times\mathbf{u} - i\tau N_0^{-1}Ln\mathbf{i} = (e/m)(\mathbf{e} + c^{-1}\mathbf{U}_0\times\mathbf{h}), \tag{6.1}$$

where Z, R, δ and $\mathbf{\Omega}_0$ are defined below under (11).

Without loss of generality we shall take the xy plane so that it contains the direction of the drift velocity, i.e., we take $U_z=0$.

On eliminating $\mathbf{e}$, $\mathbf{h}$, n and $\mathbf{u}$ from (6.1) by means of the Eqs. (1.1) to (5.1) we obtain the following equations

[3] J. S. Townsend, *Electricity in Gases* (Clarendon Press, Oxford, 1915), Sections 91 and 92.

in the components a_x, a_y, a_z of **a**:

$$\left.\begin{aligned}\alpha_x a_x+\alpha_y a_y+\alpha_z a_z=0,\\ \beta_x a_x+\beta_y a_y+\beta_z a_z=0,\\ \gamma_x a_x+\gamma_y a_y+\gamma_z a_z=0,\end{aligned}\right\}\quad(9)$$

where

$$\left.\begin{aligned}&\alpha_x=X+iL\Omega_z U_y,\quad \alpha_y=-i\omega\Omega_z+p_0^2\omega L U_y Z^{-1},\\ &\qquad\qquad\qquad\qquad\alpha_z=i\omega\Omega_y,\\ &\beta_x=-(\Omega_z R+L\delta U_y)Z,\quad \beta_y=i\omega Y,\quad \beta_z=\omega\Omega_x Z,\\ &\gamma_x=(\Omega_y R+L\Omega_x U_y)Z,\quad \gamma_y=-\omega\Omega_x Z,\quad \gamma_z=i\omega Y,\end{aligned}\right\}\quad(10)$$

with

$$\left.\begin{aligned}X&=R^2-\tau L^2-p_0^2-i\nu R,\\ Y&=R(Z+p_0^2)-i\nu Z,\\ Z&=c^2L^2-\omega^2,\\ R&=\omega-U_xL,\\ \delta&=iR+\nu,\\ p_0^2&=4\pi N_0e^2/m,\\ \mathbf{\Omega}_0&=(-e/mc)\mathbf{H}_0=(\Omega_x,\Omega_y,\Omega_z).\end{aligned}\right\}\quad(11)$$

The necessary and sufficient condition for a non-zero solution of (9) is that the determinant Δ formed by the coefficients is zero; this yields the equation of dispersion.

When we take into account the fact that $U_y^2\ll c^2$ it is found that the terms in U_y^2 are always small compared with certain others. On neglecting the terms in U_y^2 the equation of dispersion reduces to the form

$$X(Y^2-\Omega_x^2Z^2)-(\Omega_y^2+\Omega_z^2)RYZ-2p_0^2LRZ\Omega_xU_y\Omega_y=0,\quad(12)$$

which is of the eighth degree in L and ω.

This form differs from the Eq. (3A) in Appendix 1, which is derived by means of a relativistic theory, only by quantities which are of the second or higher orders of smallness in U_x, U_y and τ.

When (12) is satisfied then, by (9), a_x, a_y, a_z are proportional to the co-factors of any row of the determinant.

Then **e** and **h** are given by

$$\left.\begin{aligned}&e_x=-\phi\omega^{-2}Za_x,\quad e_y=\phi a_y,\quad e_z=\phi a_z,\\ &h_x=0,\quad h_y=-\phi Ma_z,\quad h_z=\phi Ma_y,\end{aligned}\right\}\quad(13)$$

where $\phi=-i\omega/c$, $M=cL/\omega$.

When the drift velocity $\mathbf{U}_0$ and the electron temperature (proportional to τ) are negligible (12) reduces to the (bi-quadratic) equation of dispersion in the M.I. theory. For small enough collision frequencies (e.g., $\nu\ll\omega$) the latter equation reduces to

$$(\omega^2-p_0^2)[\omega^2(Z+p_0^2)^2-\Omega_x^2Z^2]-\Omega_T^2\omega^2Z(Z+p_0^2)=0,\quad(12.0)$$

where

$$Z=c^2L^2-\omega^2=\omega^2(M^2-1),\quad \Omega_T^2=\Omega_y^2+\Omega_z^2.$$

The (ω, L) curve corresponding to (12.0) is symmetrical about both axes. It cuts the ω-axis at the points ± 0, $\pm p_0$, $\pm\omega_1$, $\pm\omega_2$, where

$$\omega_1,\omega_2=\tfrac{1}{2}[(\Omega_0^2+4p_0^2)^{\frac{1}{2}}\pm|\Omega_0|].\quad(14)$$

It has the oblique asymptotes $cL=\pm\omega$ and the asymptotes $\omega=\pm\omega_3$, $\omega=\pm\omega_4$ parallel to the L axis where

$$\omega_3,\omega_4=+[\tfrac{1}{2}(\Omega_0^2+p_0^2)\pm(\tfrac{1}{4}(\Omega_0^2+p_0^2)^2-\Omega_x^2p_0^2)^{\frac{1}{2}}]^{\frac{1}{2}}.\quad(15)$$

For example when the plasma frequency is 100 Mc/sec., the gyro-frequency is 600 Mc/sec. and the angle θ between the directions of the magnetic field and of propagation is more than 45°, then

$$\omega_1/2\pi\doteqdot 616,\quad \omega_2/2\pi\doteqdot 16.5,$$
$$\omega_3/2\pi\doteqdot 605,\quad \omega_4/2\pi\doteqdot 100\cos\theta\ \text{Mc/sec.}$$

The frequencies 0, p_0, ω_1, ω_2, ω_3 and ω_4 are the edges of frequency bands within which the wave number L (and the refractive index M) is purely imaginary. In these bands waves cannot propagate through the given medium.

It can be shown that in general

$$\omega_1>(\Omega_0^2+p_0^2)^{\frac{1}{2}}>\omega_3>(\tfrac{1}{2}(\Omega_0^2+p_0^2))^{\frac{1}{2}}>\omega_4$$

and

$$\omega_2<p_0^2/(\Omega_0^2+p_0^2)^{\frac{1}{2}}<\omega_1,$$

and that when $\Omega_0^2>p_0^2$ than $\omega_1>\omega_3>\omega_2$ and ω_4.

One of the bands then lies between ω_1 and ω_3 and one of the other bands lies between ω_2 and 0.

When an electron drift velocity $\mathbf{U}_0$ exists the corresponding (ω, L) curve has some branches which are similar to the ones just considered but distorted in a skew manner so that they become *unsymmetrical* about the ω-axis. The principal consequence is that in general we now obtain bands in which L (and M) is a *complex* number $a\pm ib$.

These bands approximate to the bands considered above in the M.I. theory. We thus see that the effect of electron drift is to create wave amplification and consequent electromagnetic noise in frequency bands in which otherwise waves cannot propagate.† If $L=a\pm ib$ the phase velocity is ω/a and one of the two corresponding waves or wave groups grows by the factor $\exp|bx|$ in the distance x.

No wave amplification is possible when the wave is propagated transversely to the drift velocity and at the same time the magnetic field is perpendicular to either the direction of propagation or the drift velocity. This follows from the fact that when $U_x=0$ and $\Omega_xU_y\Omega_y=0$ then (12) reduces to the Eq. (12.0) corresponding to the M.I. theory.

In particular when the drift and magnetic field have a common direction (as in the case (C 11) considered below) wave growth is in general increasingly favored by orientation of the direction of propagation towards this common direction.

In general a root of (12) can be determined only by some method of successive approximation, like Newton's, from some first approximation. In one method of

† This prediction of noise occurring in separate bands has now been confirmed by means of experiments carried out in collaboration with Dr. K. Landecker. These experiments will be described elsewhere.

TABLE I.

Height x in km.	ϵ_1 <	ϵ_2 <	ϵ_3 <	ϵ_4 <	ϵ_5 <
$>2\times10^3$	2×10^{-4}	10^{-4}	2×10^{-3}	50	10^{-2}
$>10^4$	10^{-8}	10^{-9}	2×10^{-8}	50	5×10^{-7}

getting the first approximation we take $\nu=0$ and in another $\mathbf{U}_0=0$. Both have their special uses. The second method has the special virtue that it develops four of the roots from those of the M.I. theory; moreover the latter are expressible in a closed form.

We shall now consider in detail the important case in which the drift and magnetic field are both in the direction of propagation, i.e., when

$$U_x=U_0,\ U_y=U_z=0 \text{ and } \Omega_x=\Omega_0,\ \Omega_y=\Omega_z=0. \quad \text{(C 11)}$$

From now on we may conveniently drop the subscript 0 from the symbols U_0 and Ω_0.

Then (10) and (9) combined with (13), yield

$$\left.\begin{aligned} Xe_x&=0,\\ iYe_y+\Omega_x Ze_z&=0,\\ -\Omega_x Ze_y+iYe_z&=0, \end{aligned}\right\} \quad (16)$$

The first equation yields two waves with the dispersion equation $X=0$, i.e.,

$$(\omega-UL)^2-\tau L^2-p_0^2-i\nu(\omega-UL)=0, \quad (17)$$

which have their electric vectors in the direction of propagation and no magnetic vectors. These waves will grow when

$$\tau>U^2>0 \quad\text{and}\quad \nu^2\ll\omega^2<p_0^2(1-U^2/\tau).$$

Since they have no corresponding Poynting flux they are not of primary importance in considering observed circular solar noise. In a more exhaustive study they may require to be considered when (and only when) account is taken of the neglected non-linear terms. Otherwise, as will be seen, the origin and escape of circular solar noise can be adequately accounted for quite independently of these waves. For these reasons their detailed discussion here is omitted.‡

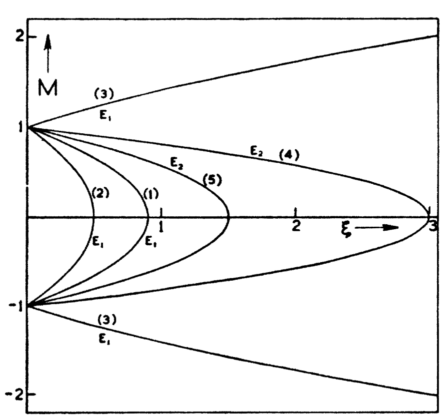

FIG. 1. Curves of (ξ, M) for E_1 and E_2 waves with $V=0$ and the following values of $k_n\eta$: (1) 0.1; (2) 0.5; (3) 2; (4) -2; (5) -0.5.

The second and third equation under (16), combined with (13) yield the following results:

There are two types of waves with dispersion equations given by

$$Y-k_n\Omega Z=0, \quad (n=1, 2), \quad (18)$$

where

$$k_1=1, \quad k_2=-1,$$

and polarizations given by

$$h_z/h_y=-k_n i. \quad (19)$$

These two types may be conveniently labeled

$$\left.\begin{aligned} &E_1 \text{ waves when } k_n\Omega>0;\\ &E_2 \text{ waves when } k_n\Omega<0. \end{aligned}\right\} \quad (20)$$

In conformity with the M.I. theory (as shown in Appendix 2) the E_1 and E_2 waves may be termed extraordinary and ordinary waves, respectively.

By (19) they are both circularly polarized, but in opposite senses. Also by (18) and (19),

$$\left.\begin{aligned} &E_1 \text{ waves are } RH \text{ or } LH \text{ and } E_2 \text{ waves are}\\ &LH \text{ or } RH \text{ according as } \Omega\gtrless 0, \text{ i.e., } H\gtrless 0, \end{aligned}\right\} \quad (20.1)$$

where RH and LH mean, respectively, right-handed and left-handed direction of rotation of the magnetic vector when viewed in the direction of $0x$.

From (16), (1.1), (3.1), and (5.1) we see that for these waves e_x, v, n and u_x are zero, i.e., the electric and magnetic vectors and the electron vibrations are purely

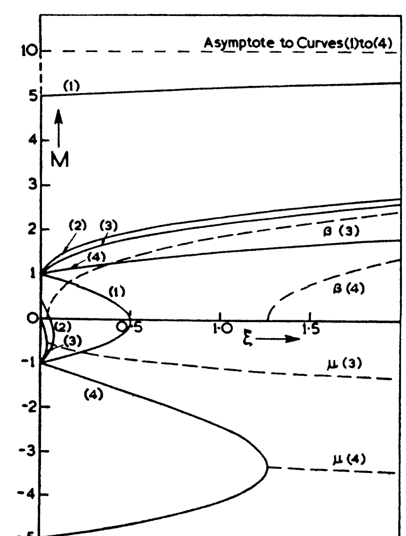

FIG. 2. Curves of (ξ, M) for E_1 waves with $V=0.1$ and the following values of $k_n\eta$: (1) 0.5; (2) 0.95; (3) 1; (4) 1.5.

‡ Some numerical examples of such longitudinal waves are given in reference 2 and in a joint paper with J. A. Roberts in the Australian J. Sci. Res. A, **2**, 307 (1949). Also it now appears that such waves have been previously discussed by W. O. Schumann in Zeits. f. Physik **121**, 7 (1943).

transverse, and there are no perturbations of the electron density.

Also from (4.1) and (13) we have

$$\mathbf{u}=(1/4\pi N_0 e)i\omega(M^2-1)\mathbf{e}. \quad (21)$$

These circular waves and the longitudinal waves (corresponding to (17)) thus form a complete contrast. Also they are entirely without mutual interactions (i.e., coupling) unless and until one of them can grow so large that the non-linear terms of the theory become important. Therefore in considering the growth of the circular waves it is initially unnecessary to consider the possibility of having growing longitudinal waves.

2. THE CIRCULARLY POLARIZED WAVES

The dispersion Eqs. (18) of circular waves, when expanded by means of (11), are the two cubics

$$(VM-1)\{\omega^2(M^2-1)+p_0^2\} + (k_n\Omega+i\nu)\omega(M^2-1)=0, \quad (22)$$

where M is the complex refractive index; $V=U/c$ and $V^2 \ll 1$.

These show that the circular waves depend on the random motions of the electrons only through the terms in ν.

In order to discuss the propagation of waves in a medium which is gradually varying in space we shall adopt the following convenient notation¶

$$\xi=p_0^2/\omega^2, \quad \eta=\Omega/\omega, \quad \sigma_n=1-k_n\eta, \quad \rho=\nu/\omega. \quad (23)$$

Then the cubics (22) assume the form

$$f(M)\equiv VM^3-(\sigma_n-i\rho)M^2 + V(\xi-1)M+\sigma_n-\xi-i\rho=0. \quad (24)$$

In general the roots of each cubic are unequal.

In our applications of the theory of the sun's atmosphere (in Section 5) the adopted numerical values of p_0, Ω and ν are those given by Smerd,[4] the magnetic field H being that above the center of a large sunspot. His values of $f_0=p_0/2\pi$ and $f_H=|\Omega|/2\pi$ are also indicated in our Fig. 4 by means of curves.

From such data we find that in regions at different heights x above the sun's surface and for waves of frequencies ω such that they can grow in these regions, the following relations exist.

$$\nu=\epsilon_1 p_0, \quad \nu=\epsilon_2|\Omega|, \quad \nu=\epsilon_3\omega, \quad \omega=\epsilon_4 p_0, \quad \rho=\epsilon_5\xi,$$

where ϵ_1, ϵ_2, ϵ_3, ϵ_4 and ϵ_5 $(=\epsilon_1\epsilon_4)$ have values less than the numbers given in the corresponding columns of Table I.

Thus in our application we have:

Above the heights 2×10^3 and 10^4 km, respectively, $\rho<2\times10^{-3}$ and 2×10^{-8}, $\rho<10^{-4}|k_n\eta|$ and $10^{-9}|k_n\eta|$, $\rho<10^{-2}\xi$ and $5\times10^{-7}\xi$. (25)

¶ This is in part similar to one often used in discussing propagation in the ionosphere.

[4] S. F. Smerd, Australian Council for Scientific and Industrial Research, R. P. L. 14 (January, 1948).

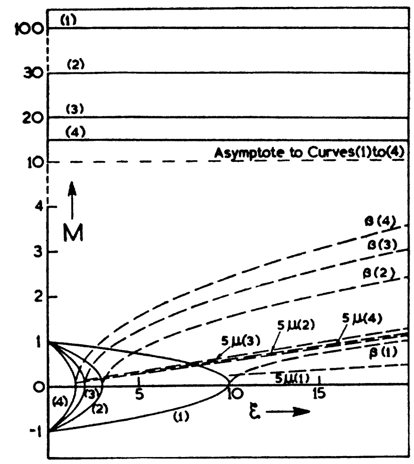

FIG. 3. Curves of (ξ, M) for E_2 waves with $V=0.1$ and the following values of $k_n\eta$: (1) -9; (2) -2; (3) -1; (4) -0.5.

Therefore in (24) the terms in ρ may be neglected except perhaps when $|\xi-\sigma_n|$ or $|\sigma_n|$ is comparable with, or less than, ρ.

We shall first consider the general circumstances in which ρ is negligible, i.e., when (24) approximates to

$$f(M)\equiv VM^3-\sigma_n M^2+V(\xi-1)M+\sigma_n-\xi=0. \quad (24.0)$$

In general neither of these cubics has more than two roots equal.

The six roots of these equations will be denoted by M_1, M_2, M_3 for the E_1 waves (when $\sigma_n<1$), and by M_{-1}, M_{-2}, M_{-3} for the E_2 waves (when $\sigma_n>1$).

The dependence of real values of M on the electron density N_0 is conveniently shown by drawing curves with ξ and M as coordinates for given values of η and V and for $\xi>0$. Such curves are illustrated by Figs. 1 to 3.

Points on the curve defined by (24.0) are most easily determined by calculating ξ for real values of M.

The intercept on the ξ-axis is σ_n and the intercepts on the M axis are 1, -1, $V^{-1}\sigma_n$. The only asymptote to the curve is

$$M=V^{-1}$$

and is approached from above or below according as $\sigma_n \gtrless 1$.

The important ranges of values of ξ in which growing waves can occur are indicated by means of the vertical tangents. These tangents occur at points P_1, P_2, P_3 determined by (24.0) and the equation

$$M=V^{-1}\theta,$$

where θ is a root of

$$\theta^3-\tfrac{1}{2}(\sigma_n+3)\theta^2+\sigma_n\theta-\tfrac{1}{2}V^2(\sigma_n-1)=0.$$

Since $V^2\ll1$ and in general $\sigma_n\neq0$ the two larger roots θ_1, θ_2 and the third root are given approximately by

$$\theta_1,\ \theta_2 \doteq \tfrac{1}{4}\{(\sigma_n+3)\pm(\sigma_n^2-10\sigma_n+9)^{\frac{1}{2}}\}, \quad \theta_3 \doteq \tfrac{1}{2}V^2(1-\sigma_n^{-1}).$$

The values of ξ at the vertical tangents are given by

$$\xi=(1-V^{-2}\theta^2)(\sigma_n-\theta)/(1-\theta).$$

The two tangents corresponding to θ_1 and θ_2 exist only when $\sigma_n\leqslant 1$ or $\geqslant 9$ i.e., always for E_1 waves and when $k_n\eta<-8$ for E_2 waves. The third vertical tangent, occurs near (but not at) the point $\xi=\sigma_n$ where the curve cuts the ξ-axis. When $\sigma_n<0$ this tangent does not exist for values of $\xi>0$.

Figure 1 shows examples when $V=0$. This corresponds to the well-known case of longitudinal propagation in the magneto-ionic theory. The curves are parabolas and correspond to the usual straight lines which depict M^2 in terms of ξ. The waves are E_1 or E_2 according as $k_n\eta\gtreqless 0$. The values of M are either real or pure imaginary.

Figures 2 and 3, respectively, give examples of E_1 and E_2 waves when $V>0$ i.e., when the electrons have a motion of drift in the direction of $0x$. These show that the values of M are in general either real or complex with non-zero real parts. Thus for a given frequency and type E_n there are values of the electron density for which all the waves remain constant in amplitude, apart from attenuation due to electron collisions. These may be called waves of the First Species. With other values of the electron density one of the waves grows and another attenuates (apart from the results of collisions) as they travel. These may be called waves of the Second Species.

For *all* values of the electron density at least two progressive waves of each type can exist. The only values of electron density at which just two such waves can exist are those corresponding to the points $\xi=\sigma_n$, where $M=0$.

In those parts of Figs. 2 and 3 where only one real root M_3 occurs the complex roots M_1, M_2 can be calculated as follows by means of the values of M_3:

$$M_1=\mu+i\beta,\quad M_2=\mu-i\beta, \tag{26}$$

where

$$\left.\begin{aligned}\mu&=\tfrac{1}{2}(V^{-1}\sigma_n-M_3),\\ \beta&=[V^{-1}M_3^{-1}(\xi-\sigma_n)-\tfrac{1}{4}(V^{-1}\sigma_n-M_3)^2]^{\frac{1}{2}}.\end{aligned}\right\}\tag{27}$$

From (27) and (24.0) we have

$$\frac{\xi}{2\mu}=\frac{M_3^2-1}{M_3-V^{-1}},$$

and since $\xi>0$ it follows that $\mu>0$ when $M_3<V^{-1}\sigma_n$ and either (a) $V>0$ with $|M_3|<1$ or $M_3>V^{-1}$, or (b) $V<0$ with $|M_3|>1$ and $M_3>V^{-1}$.

From this we deduce that

$$\left.\begin{aligned}&\mu>0 \text{ only when}\\ &V>0 \text{ and } k_n\eta<1+V,\\ \text{or}\quad &V<0 \text{ and } k_n\eta>0.\end{aligned}\right\}\tag{28}$$

Similar remarks may be made about M_{-3}, M_{-1}, M_{-2}.

The dashed curves in Figs. 2 and 3 show μ and β in terms of ξ. They relate to E_1 or E_2 waves according as $k_n\Omega\gtreqless 0$.

Physically μ is the real refractive index and β is the index of growth or attenuation.

When $|V|\ll 1$, direct approximations to the roots M_1, M_2 can be made as follows.

As first approximations to them we take the roots M_{10}, M_{20} of the equation derived from (24.0) when V is made zero, namely

$$M_{10},\, M_{20}=\pm ia=\pm i(\xi\sigma_n^{-1}-1)^{\frac{1}{2}}.$$

Then by Newton's method of iteration applied to (24.0) we obtain the second approximations

$$M_1,\, M_2=\pm ia\pm\frac{Va\xi(1-\sigma_n^{-1})}{\pm 2\sigma_n a+iV(2+\xi-3\xi\sigma_n^{-1})}.$$

Hence, if

$$\left.\begin{aligned}a&=(\xi\sigma_n^{-1}-1)^{\frac{1}{2}},\\ b&=\{1+\tfrac{1}{2}\xi(1-3\sigma_n^{-1})\}a^{-1},\\ \psi&=k_n\eta\xi/2(\sigma_n^2+V^2b^2),\end{aligned}\right\}\tag{29}$$

then,

$$M_1,\, M_2=\mu\pm i\beta, \tag{30}$$

where

$$\mu\doteqdot -V\psi,\quad \beta\doteqdot a+V^2\psi\sigma_n^{-1}b. \tag{31}$$

Clearly in this approximation M_1, M_2 are complex when $\xi>\sigma_n>0$, i.e., when $0<\omega<\delta\omega_2$ and $|\Omega|<\omega<\omega_1$, where ω_1, ω_2 are defined under (14).

For E_1 waves in the critical case $\sigma_n=0$ we have to proceed otherwise. Thus, when in (24.0)

$$\sigma_n=0 \quad\text{and}\quad |V|\ll|\xi(\xi-1)^{-\frac{3}{2}}|$$

we may take $M_3\doteqdot(V^{-1}\xi)^{\frac{1}{3}}$ and then (27) yields the following result:

$$\left.\begin{aligned}&\text{When } \omega=k_n\Omega \text{ and } |V|\ll|\xi(\xi-1)^{-\frac{3}{2}}|,\\ &\text{then}\\ &\qquad \mu\doteqdot-\tfrac{1}{2}(V^{-1}\xi)^{\frac{1}{3}},\quad \beta\doteqdot\tfrac{1}{2}\sqrt{3}(V^{-1}\xi)^{\frac{1}{3}}.\end{aligned}\right\}\tag{31.1}$$

The corresponding waves may be termed E_1 gyro waves of the Second Species.

Values of M, μ, and β calculated by means of (31) and (31.1) are found to agree well with those given by the curves in Figs. 2 and 3.

The formulas (31), (20), and (31.1) yield the following conditions for μ to be positive:

$$\left.\begin{aligned}&\text{When } |V|\ll 1 \text{ and } \sigma_n\geqslant 0 \text{ then } \mu>0 \text{ only}\\ &\quad\text{with } E_1 \text{ waves and } V<0, \text{ or}\\ &\qquad E_2 \text{ waves and } V>0.\end{aligned}\right\}\tag{28.1}$$

These conclusions are substantially independent of collisions when the terms in ρ of (24) are negligible. As mentioned earlier this is so when $|\xi-\sigma_n|$ and $|\sigma_n|$ are large compared with ρ, i.e., when the wave frequency does not lie near one of the frequencies ω_1, ω_2 or $|\Omega|$.

When $|\xi-\sigma_n|$ or $|\sigma_n|$ is either (a) comparable with ρ, or (b) less than ρ, a special investigation is necessary.

Under (a) the calculations are very lengthy whereas under (b) they are both relatively simple and more sensitive to the effects of collisions. Accordingly we shall here consider only the following special cases:

I. $|\xi-\sigma_n|\ll\rho\ll\sigma_n$ and ξ;
II. $|\sigma_n|\ll\rho\ll\xi$.

The condition $\rho\ll\xi$ is added on account of (25). In the case I we obtain from (24)

$$M_1 \doteqdot (-k_nV\eta/\sigma_n)(1+i\rho/\sigma_n), \quad M_2 \doteqdot -i\rho/k_nV\eta.$$

This shows that for $\mu=\mu_1$ (28.1) also holds true in I.

In the case II we obtain the same approximate results as under (31.1) and (28.1) when $|V|\gg\rho$.

We may therefore conclude, with the help of (25), that in our application to the sun's atmosphere, as exemplified by Fig. 4, the effect of collisions on growth may be neglected in regions above 2000 km when the drift velocity $|U|\gg 6\times 10^7$ cm/sec. and in regions above 10^4 km when $|U|\gg 600$ cm/sec.

3. THE WAVES WHICH CORRESPOND TO ANY TRANSVERSE PERTURBATION TEMPORALLY PRESCRIBED AT THE PLANE x=0

In this section it will be shown that (a) any transverse perturbation at a given plane gives rise to two trios of circular waves which are oppositely polarized and can be treated independently of each other; (b) within certain frequency bands each trio will in general grow; (c) the mean Poynting flux of a wave with refractive index $M_m=\mu_m+i\beta_m$ is in the direction of propagation $0x$ and grows in that direction when μ_m and β_m are both positive.

Also the type of growing waves (E_1 or E_2) is found which carries field energy in the direction of $0x$ under different conditions ((48) and (48.1)).

The arguments are set out in some detail partly for the sake of clarity and partly because some of the relevant formulas ((32), (33), (42.1), and (42.2)) are required in the next section.

The vital question of escape of field energy from the sun's atmosphere makes the discussion of the Poynting flux of primary importance. It has not been found necessary here to consider the behavior of wave groups, i.e., of integral expressions like those in (32) but with the limits of integration replaced by $\omega_0+\delta\omega$ and $\omega_0-\delta\omega$, respectively. This is fortunate since the mathematical definition of a wave group is apt to become physically vague (and perhaps misleading) when the rate of growth is very large.‖

We shall represent the given transverse perturbation by the electric vector $\mathbf{e}$, the magnetic vector $\mathbf{h}$ and the electron velocity $\mathbf{u}$. Their dependence on time will be indicated, when necessary, by the notation $\mathbf{e}(t)$, $\mathbf{h}(t)$, $\mathbf{u}(t)$.

We shall now determine the values of the corresponding vectors at the plane x, namely

$$\mathbf{e}(t,x), \quad \mathbf{h}(t,x), \quad \mathbf{u}(t,x).$$

These can be expressed in the following general forms which all satisfy the fundamental Eqs. (1.1) to (6.1).

$$\left.\begin{aligned} \mathbf{e}(t,x)&=\int_0^\infty \sum_m \mathbf{A}_m e^{i\omega(t-M_mx)}d\omega,\\ \mathbf{h}(t,x)&=\int_0^\infty \sum_m \mathbf{B}_m e^{i\omega(t-M_mx)}d\omega,\\ \mathbf{u}(t,x)&=\int_0^\infty \sum_m \mathbf{C}_m e^{i\omega(t-M_mx)}d\omega, \end{aligned}\right\} \tag{32}$$

where $m=1, 2, 3, -1, -2, -3$, and for convenience the velocity of light c is taken as the unit of velocity.

M_m is a known function of ω derivable from (24.0), and $\mathbf{A}_m$, $\mathbf{B}_m$, $\mathbf{C}_m$ are functions of ω which remain to be determined.

On setting $x=0$ and inverting the resulting Fourier integrals we obtain

$$\left.\begin{aligned} \sum_m \mathbf{A}_m&=\mathbf{I}_e\equiv 1/\pi\int_0^\infty \mathbf{e}(\alpha)e^{-i\omega\alpha}d\alpha,\\ \sum_m \mathbf{B}_m&=\mathbf{I}_h\equiv 1/\pi\int_0^\infty \mathbf{h}(\alpha)e^{-i\omega\alpha}d\alpha,\\ \sum_m \mathbf{C}_m&=\mathbf{I}_u\equiv 1/\pi\int_0^\infty \mathbf{u}(\alpha)e^{-i\omega\alpha}d\alpha. \end{aligned}\right\} \tag{33}$$

The quantities $\mathbf{I}_e$, $\mathbf{I}_h$, $\mathbf{I}_u$ are thus known in terms of the given perturbation.

Since the component waves are circularly polarized we may by (13) and (19) write

$$\mathbf{A}_m=(\mathbf{j}-ik_n\mathbf{k})A_m, \tag{34}$$

where, according as $\Omega \gtrless 0$,

$k_n=\pm 1$ for E_1 waves and $k_n=\mp 1$ for E_2 waves.

Hence from (33) we obtain

$$A_1+A_2+A_3=J_e^1, \tag{35.1}$$

$$A_{-1}+A_{-2}+A_{-3}=J_e^2, \tag{35.2}$$

where

$$J_e^1=\tfrac{1}{2}(I_{ey}\pm iI_{ez}), \quad J_e^2=\tfrac{1}{2}(I_{ey}\mp iI_{ez}), \tag{36}$$

and we take the upper or lower signs according as $\Omega\gtrless 0$. Similarly we have

$$B_1+B_2+B_3=J_h^1, \tag{37.1}$$

$$C_1+C_2+C_3=J_u^1, \tag{38.1}$$

and analogous relations (37.2), (38.2).

But from (13) and (19) we obtain

$$\mathbf{B}_m=ik_nM_m\mathbf{A}_m, \tag{39}$$

‖ This difficulty is absent in the M.I. theory and so the notion of group-velocity may there be profitably used.

and from (21) combining with (13),

$$\mathbf{C}_m = iK^{-1}\omega(M_m^2 - 1)\mathbf{A}_m, \tag{40}$$

where

$$K = 4\pi N_0 e. \tag{41}$$

Thus (35.1), (37.1), and (38.1) yield the following system of three simultaneous equations:

$$\left.\begin{aligned} A_1 + A_2 + A_3 &= J_e^1, \\ M_1A_1 + M_2A_2 + M_3A_3 &= -k_n J_h^1, \\ M_1^2A_1 + M_2^2A_2 + M_3^2A_3 &= -K\omega^{-1}J_u^1 + J_e^1. \end{aligned}\right\} \tag{42.1}$$

A similar set (42.2) relates A_{-1}, A_{-2}, A_{-3}.

These results show that the original perturbation can be split up into two kinds of circular waves which are oppositely polarized.

Thus (32) may be replaced by two similar sets of Eqs. (32.1) and (32.2). In (32.1), $\mathbf{e}(t, x)$, etc., are replaced by $\mathbf{e}_1(t, x)$ etc., and m takes the values 1, 2, 3, while in (32.2) $\mathbf{e}(t, x)$ etc., are replaced by $\mathbf{e}_2(t, x)$ etc., and m takes the values -1, -2, -3.

Accordingly each trio of waves may be treated in every respect independently of the other.

When M_1, M_2, M_3 are all different (42.1) always yields finite values of A_1, A_2, A_3.

When $M_1 = M_2 \neq M_3$ the terms A_2, M_2A_2 and $M_2^2A_2$ in (42.1) are replaced respectively by 0, $i\omega^{-1}A_2'$ and $2i\omega^{-1}M_1A_2'$, but finite values of A_1, A_2' and A_3 are always obtained.

As (24.0) cannot in general have three equal roots this case need not be considered.

Similarly we always obtain finite values of A_{-1}, A_{-2}, A_{-3}.

When

$$M_m = \mu_m + i\beta_m,$$

with μ_m, β_m both real functions of ω, the corresponding term in (32.1) is

$$\int_0^\infty \mathbf{A}_m e^{\omega\beta_m x} \cdot e^{i\omega(t-\mu_m x)} d\omega. \tag{43}$$

From (24.0) we see that when ω exceeds a certain finite value ω_0 all the values of M_m are real, i.e., $\beta_m = 0$. It then follows that all the integrals (43) are finite.

Also when $\omega < \omega_0$, β_m has two equal non-zero values of opposite signs and therefore the E_j perturbations represented by (32.1) will in general ultimately grow with increasing $|x|$.

Similar results hold for (32.2).

The mean Poynting flux for the wave with index M_m is given by

$$\bar{P}_m = c/4\pi |A_m|^2 \mu_m e^{2\omega\beta_m x}, \tag{44}$$

where

$$M_m = \mu_m + i\beta_m. \tag{45}$$

Thus

$$\bar{P}_m > 0 \text{ when } \mu_m > 0 \tag{46}$$

and

$$\bar{P}_m \text{ increases with } x \text{ when } \beta_m > 0. \tag{47}$$

From (46), (28), and (28.1) we derive these results:

The only growing waves which carry field energy in the direction of $0x$ are as follows:
With $V > 0$ they are growing E_1 or E_2 waves according as $0 < k_n\eta < 1 + V$ or $k_n\eta < 0$.
With $V < 0$ they are always E_1 waves. (48)

When $|V| \ll 1$, the only growing waves which carry field energy in the direction of $0x$ are E_2 or E_1 waves according as $V \gtrless 0$. (48.1)

When $V > 0$ these waves may in general be either RH or LH but when $1 \gg V > 0$ they are LH or RH according as $\mathbf{H}$ and $0x$ have the same or opposite directions. Also when $V < 0$ they are RH or LH according as $\mathbf{H}$ and $0x$ have the same or opposite directions.

By (31), (31.1) and (44) when $|V| \ll 1$ then for a growing wave $\bar{P}_m$ is nearly proportional to V, except when the wave is an E_1 gyro wave.

For the waves of constant amplitude $\bar{P}_m$ is positive when $M_m > 0$.

4. PASSAGE OF A GROWING CIRCULARLY POLARIZED PERTURBATION NORMALLY THROUGH THE BOUNDARY BETWEEN TWO DIFFERENT MEDIA

In this section it is proposed first to determine the transverse perturbations in a medium A which lead to a given transverse perturbation, in a second medium B, for which the mean energy flux $\bar{P}$ travels away from A, with $V \neq 0$ in one or both media.

This is done in the following way. First the wave components on one side of the common boundary of A and B are related to those on the other side and then expressed separately in terms of the latter. Next it is shown that these expressions always yield finite values for the components sought. Then it is shown that even with $V = 0$ in either A or B, waves of sufficiently high frequency which exist in one medium must lead to waves in the other. Lastly, necessary and sufficient conditions are found under which growing energy fluxes in A can pass into B.

It is also proposed to determine G_{11} the factor by which the mean flux of a growing wave increases when passing from a plane in the medium A to a distant plane in B.

The boundary is here taken normal to the direction of propagation, i.e., to the x axis.

At the boundary the necessary and sufficient conditions which must be satisfied are that the three vectors $\mathbf{e}$, $\mathbf{h}$, and $\mathbf{u}$ vary continuously across it. The first two need not be justified here, while the third is imposed by the fact that the electrons are drifting across the boundary and that under finite forces they cannot suddenly change their transverse velocity $\mathbf{u}$.

From these boundary conditions we deduce that the Poynting flux $\mathbf{P}$ also varies continuously across the boundary.

If a new origin of coordinates be taken in the boundary plane, we may use the formulas of Section 3 for the present discussion. Thus we will take the vectors $\mathbf{e}(t)$, $\mathbf{h}(t)$, $\mathbf{u}(t)$ to represent the perturbations at the boundary $x=0$ and regard the expressions (32) as representing the perturbations in the medium A, when $x<0$ and M_m, $\mathbf{A}_m$, etc., are replaced by M_{am}, $\mathbf{A}_{am}$, etc., and those in the medium B when $x>0$ and M_m, $\mathbf{A}_m$, etc., are replaced by M_{bm}, $\mathbf{A}_{bm}$, etc.

The quantities $\mathbf{I}_e$, $\mathbf{I}_h$, $\mathbf{I}_u$ defined in (33) now represent the perturbations of frequency ω at, and near, the boundary. The same is therefore true of the quantities on the right of (42.1) and (42.2) which determine the two constituent trios of waves.

From (42.1) we obtain the following relations between the amplitudes A_m of the trio of E_1 waves in the two media.

$$\left.\begin{aligned}\sum_m A_{am}&=\sum_m A_{bm}\\ \sum_m M_{am}A_{am}&=\sum_m M_{bm}A_{bm}\\ \sum_m M_{am}^2A_{am}&=\sum_m M_{bm}^2A_{bm}\end{aligned}\right\}(m=1,2,3). \quad (49.1)$$

As was proved in Section 3, for (42.1), when M_{a1}, M_{a2}, M_{a3} are all different the system of Eqs. (49.1) always yields finite values of A_{a1}, A_{a2}, A_{a3}, namely

$$A_{a1}=(M_{a1}-M_{a2})^{-1}(M_{a1}-M_{a3})^{-1}\times\sum_m(M_{bm}-M_{a2})(M_{bm}-M_{a3})A_{bm}, \quad (50.1)$$

and formulas for A_{a2}, A_{a3} obtained from this by cyclic permutation of subscripts.

Similar expressions are obtained when, exceptionally, two of these indices are equal.

On calculating $\bar{P}$, the mean value in time of the Poynting flux $\mathbf{P}$ of the E_1 perturbation, we find that it consists of terms which are periodic in x and three terms $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_3$ which are not. The average value of $\bar{P}$ in a given region of space is therefore given approximately by the formula

$$\bar{P}=\bar{P}_1+\bar{P}_2+\bar{P}_3. \quad (51.1)$$

For the trio of E_2 waves we have similar relations (49.2), (50.2), and (51.2) and can draw similar conclusions.

From (50.1) and (50.2) it follows that when progressive waves can exist in both media then such waves can in general pass from A to B or B to A.

Waves cannot exist in a medium when $V=0$ and $\xi>\sigma_n$, i.e., $\omega^2-k_n\Omega\omega-p^2<0$.

Hence, if

$$\left.\begin{aligned}\omega_a,\ \omega_a'&=\tfrac{1}{2}[(\Omega_a^2+4p_a^2)^{\frac{1}{2}}\pm|\Omega_a|],\\ \omega_b,\ \omega_b'&=\tfrac{1}{2}[(\Omega_b^2+4p_b^2)^{\frac{1}{2}}\pm|\Omega_b|],\end{aligned}\right\} \quad (52)$$

it follows that

When $V_a=0$ there can be an energy flux in medium A with E_1 waves only when $\omega>\omega_a$ and with E_2 waves only when $\omega>\omega_a'$. } (52A)

A similar conclusion** (52B) holds true for medium B when $V_b=0$.

We shall now establish the following result when $V=0$ in one of the media alone.

When $V=0$ in either A or B alone and transverse waves, of frequencies $\omega>\omega_a'$ or ω_b', respectively, exist in one of these media, then such waves always exist in the other. } (53)

For to suppose the contrary is equivalent to having a non-zero solution of equations like (49.1) when their right-hand sides or left-hand sides are all zero; this is impossible because in general the three numbers M_1, M_2, M_3 for either case are all different. Even when $M_1=M_2\neq M_3$ the result (53) is still derived, for the terms in A_2 of (49.1) are then modified as indicated in Section 3.

We shall now determine the conditions under which a mean outward flux in medium B can exist, i.e.,

$$\bar{P}_b>0. \quad (54)$$

Three possible cases, covering growing waves, which can arise will now be considered in detail sufficient for use in later sections.

Case I: $V_a\neq 0$, $V_b\neq 0$

From (50.1) and (50.2) we see that both E_1 and E_2 waves can pass from A to B.

An E_1 wave which grows in A will or will not lead to a growing wave in B with a positive flux, according as

(a) $A_{b1}\neq 0$ and $\mu_b>0$ or (b) $A_{b1}=0$.

With (a) $\bar{P}_{b1}>0$ and at a sufficiently large distance $x_b>0$ the flux $\bar{P}_{b1}$ is much larger than $\bar{P}_{b2}$ and $\bar{P}_{b3}$. We

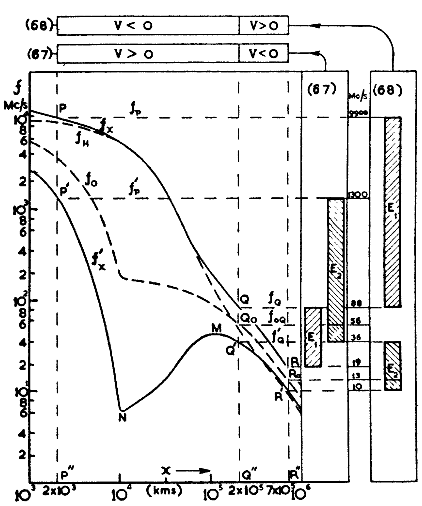

FIG. 4. Illustrating the determination of the frequency bands of growing E_1 and E_2 waves which can escape from the sun's atmosphere over a large sunspot, under hypotheses (67) and (68) about the drift velocity.

** These results are well-known in the magneto-ionic theory of propagation.

TABLE II. Summary of results from hypotheses (67) and (68). $x_P=2\times10^3$, $x_Q=2\times10^5$, $x_R=7\times10^5$ km.

Hypothesis	Drift V_{PQ}	Drift V_{QR}	Frequency bands of escaping waves (Mc/sec.) E_2	E_1
67	>0	<0	1300 to 56 or 36* growing in PQ below 8×10^3 km. 44* to 36* growing partly above 10^5 km.	88 to 19 growing in QR, within broad regions.
68	<0	>0	36 to 13 or 10* growing in QR.	9900 to 88 growing in PQ. 6000 to 600 growing in narrow regions below 5×10^4 km.

* If the region near Q is penetrated.

can therefore simplify the discussion by taking A_{b2} and A_{b3} to be zero since other values will give approximately the same value for $\bar{P}_b$.

On doing so we obtain from (50.1)

$$A_{a1}=\rho_{11}^{-1}A_{b1}, \tag{55}$$

where

$$\rho_{11}=\frac{(M_{a1}-M_{a2})(M_{a1}-M_{a3})}{(M_{b1}-M_{a2})(M_{b1}-M_{a3})}. \tag{56}$$

Also the growing wave amplitude $A_{a1}(x_a)$, at the plane $x=x_a<0$ inside the medium A is given by

$$A_{a1}(x_a)=A_{a1}e^{\omega\beta_a x_a}. \tag{57}$$

From (56), (57), (51.1), and (44) (and on reversion to the unit of velocity 1 cm/sec.) we obtain

$$G_{11}\equiv\frac{P_b(x_b)}{P_{a1}(x_a)}\doteqdot|\rho_{11}|^2\frac{\mu_b}{\mu_a}\exp[2\omega(\beta_b x_b-\beta_a x_a)/c], \tag{58}$$

with $\mu_b>0$.

When $\mu_a>0$ the formula (58) gives the factor of increase G_{11} of a growing flux in passing from the plane $x_a<0$ in A to a distant plane $x_b>0$ in B.

G_{11} can never be zero since μ_a is always finite and since, in (56), M_{a1} cannot be equal to either M_{a2} (for $\beta_a\neq0$) or to M_{a3} (which is real).

With (b) we can similarly for a sufficiently large distance set $A_{b2}=0$ and so obtain a similar, simple approximate expression for the ratio G_{11}. Also a necessary and sufficient condition for a perturbation in A to give rise to a positive flux $\bar{P}_b$ in B is, by (44) and the boundary conditions, that

$$\{|A_{a1}|^2+|A_{a2}|^2\}\mu_a+|A_{a3}|^2M_{a3}=4\pi c^{-1}\bar{P}_b. \tag{59}$$

Similar results hold true for an E_2 wave which grows in A.

Among the random perturbations which can occur at the plane $x=x_a$ in the medium A there can arise some which lead to the situation (a) and others which lead to (b). We may therefore conclude as follows:

When $V_a\neq0$ and $V_b\neq0$, certain transverse perturbations in medium A can lead to a positive mean flux of energy in medium B which grows in the direction A to B; also certain other transverse perturbations in A can lead ultimately to a positive, constant mean flux in B. (60)

By (48) the first kind of perturbations are E_1 or E_2 waves when $V_b>0$ but only E_1 waves when $V_b<0$. Also when $1\gg V_b>0$ they are only E_2 waves.

Case II: $V_a\neq0$, $V_b=0$

We have here the following necessary condition (which is similar to (52A)):

When $V_b=0$ there can be an energy flux in B, with E_1 waves only when $\omega>\omega_b$, and with E_2 waves only when $\omega>\omega_b'$. (61)

Also by (53) it follows that in general when transverse waves of different frequencies $\omega>\omega_b'$ exist in A there is an energy flux in B. Hence,

An energy flux in B can arise from a growing wave in A when, and only when, Eq. (24.0) has complex roots with

$\omega>\omega_b$, $\xi=p_a^2/\omega^2$, $\sigma_n=1-|\Omega_a|/\omega$ for E_1 waves

and with

$\omega>\omega_b'$, $\xi=p_a^2/\omega^2$, $\sigma_n=1+|\Omega_a|/\omega$ for E_2 waves. (62)

The occurrence of such complex roots can be examined by means of curves like those in Figs. 2 and 3, by means of the discriminant of a cubic or, approximately, by means of the formulas (29), (31), and (31.1).

Thus, for example,

Complex roots of (24.0) occur when $|V|\ll1$, $\sigma_n>0$ and approximately $\xi>\sigma_n$, i.e., for E_1 waves when $|\Omega_a|<\omega<\omega_a$ and for E_2 waves when $\omega<\omega_a'$, (63)

where ω_a, ω_a' are defined in (52).

When $|V_a|\ll1$ we deduce from (62) and (63) the following approximate necessary and sufficient conditions under which growing energy fluxes in A can pass into B:

With $|V_a|\ll1$ and $V_b=0$, E_1 growing waves can pass when $\omega_a>\omega>\omega_b$ and $|\Omega_a|$ and E_2 growing waves can pass when $\omega_a'>\omega>\omega_b'$. (64)

Case III: $V_a=0$, $V_b\neq0$

Here we deduce, from (53), that:

With $V_a=0$ and $V_b\neq0$ a part of the waves in A (with frequencies $\omega>\omega_a'$) can pass into B. (65)

5. ESCAPE FROM THE SUN OF CIRCULAR WAVES GROWING IN THE ATMOSPHERE ABOVE A LARGE ISOLATED SUNSPOT

We will now apply some of our theoretical conclusions to the waves which can arise by growth of random transverse perturbations in the sun's atmosphere above the center of a large sunpot on the equator.

For this purpose four curves are drawn, as in Fig. 4, with x the distance above the sun's surface as abscissa and f_0, f_H, f_x, f_x' as ordinates, where f_0, f_H are respectively the corresponding electron density frequency and the electron gyro frequency (in cycles/sec.) and

$$f_x, f_x' = \tfrac{1}{2}[(f_H^2+4f_0^2)^{\frac{1}{2}} \pm |f_H|]. \tag{66}$$

The values of f_0 and f_H are those given by Smerd,[4] H being the magnetic field above the center of a sunspot with a maximum field at the spot of 3600 gauss.

With this spot the curve for f_x' has a valley N in the region between 10^4 and 10^5 km and a peak M at $x=1.1\times10^5$ km with the corresponding frequency $f_M'=44$ Mc/sec.

In the language of the magneto-ionic theory of propagation f_x is the frequency of the extraordinary wave and f_x' that of the ordinary wave for which $\mu=0$ in a medium with no electron drift.†† As the literature on solar noise often uses these terms it is necessary here to relate them to the E_1 and E_2 waves considered in the present theory with the electrons drifting. In Appendix 2 it is shown that we may term the E_1 waves extraordinary and the E_2 waves ordinary.

Following a well-known method of approximation we may consider the sun's atmosphere as composed of a succession of uniform strata each differing slightly from the preceding one.

We shall now suppose that throughout the sun's atmosphere above the spot the electrons drift relatively to the ions with a vertical velocity V which is a continuous function of the distance x.

Since very little is known concerning V, we are at liberty to adopt various hypotheses about it and then compare the theoretical consequences with known observations.

As an example we shall adopt the following hypothesis concerning V in the regions bounded by the lines PP', QQ' and RR', in Fig. 4, which are at the respective distances

$$x_P=2\times10^3, \quad x_Q=2\times10^5, \quad x_R=7\times10^5 \text{ km}.$$

These lines will be named the lines P, Q, and R, respectively.

$$\left.\begin{array}{l} |V|\ll1 \text{ everywhere;} \\ V>0 \text{ in the region } P \text{ to } Q; \\ V<0 \text{ in the region } Q \text{ to } R; \\ V \text{ negligible near } Q \text{ and beyond } R. \end{array}\right\} \tag{67}$$

†† The term ordinary is sometimes used exclusively for that wave for which, when propagated transversely to the magnetic field, $f=f_0$ when $\mu=0$. The current terminology is unfortunately somewhat confused.

Let f_P, f_P'; f_Q, f_Q'; f_R, f_R' denote the pairs of frequencies given by (66) which correspond to P, Q, R, respectively.

On applying (60) and (48.1) we see that in the region P to Q there can be growing waves which proceed outward and that these are E_2 waves. By (63) each such wave grows only in the regions for which its frequency lies below the curve for f_x'.

All these E_2 waves with frequencies $f>f_0$ will, by the usual considerations of ray propagation in the magneto-ionic theory, be able to pass into the region near Q where V is negligible. They lie in the band of frequencies f_P' to f_{0Q} where f_{0Q} is the value of f_0 corresponding to the distance x_Q.

By (53) or (65) a part of the waves in this band can pass into the region between Q and R and then into the region beyond R, thus escaping from the sun. All these escaping E_2 waves have grown solely in regions below $x=10^4$ km. Their frequencies lie in the band 1300 to 56 Mc/sec.

Similarly by (60) and (48) there can be growing waves in the region Q to R and these are necessarily E_1 waves. By (63) they grow in the regions bounded by the curves for f_H and f_x. Then, by (64), these waves can pass into the region beyond R and so escape from the sun. Each of these escaping E_1 waves has grown in some part of the region above $x=2\times10^5$ km. Their frequencies lie in the band f_Q to f_R, i.e., 88 to 19 Mc/sec.

The lower frequency limit f_{0Q} for E_2 waves which was adopted above is based on a ray treatment of propagation (with no electron drift) which assumes that when $f_0=f$ then $\mu=0$ and as a result E_2 waves cannot penetrate the region concerned. The universal correctness of this treatment and its consequence has been called into question by Saha and his co-workers[5] and it appears also to be at variance with certain observations of Toshniwal,[6] Harang,[7] and Newstead.[8] It is therefore possible that the lower limit of the E_2 band is set by (64), i.e., by f_Q'. This entails replacing the frequency 56 Mc/sec. given above by 36 Mc/sec.

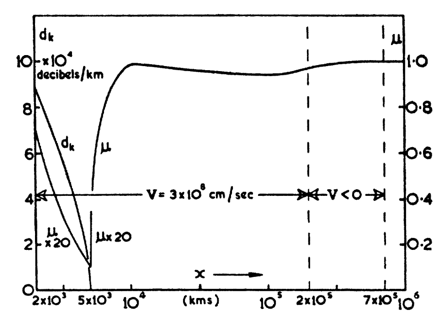

FIG. 5. Curves giving the refractive index μ and rate of growth of intensity d_k for an escaping E_2 wave of frequency 200 Mc/sec. under hypothesis (67) with $V=3\times10^8$ cm/sec.

[5] M. N. Saha *et al.*, Nature **158**, 549 (1946); Ind. J. Phys. **21**, 181, 199 (1947).
[6] G. R. Toshniwal, Nature **135**, 471 (1935).
[7] L. Harang, Terr. Mag. **41**, 143 (1936).
[8] G. Newstead, Nature **161**, 312 (1948).

As another example we shall take the hypothesis complementary to (67), namely:

$$\left.\begin{array}{l} |V| \ll 1 \text{ everywhere;} \\ V<0 \text{ in the region } P \text{ to } Q; \\ V>0 \text{ in the region } Q \text{ to } R; \\ V \text{ negligible near } Q \text{ and beyond } R. \end{array}\right\} \quad (68)$$

By reasoning as before we obtain the following results: E_2 waves in the band 36 to 13 (or 10) Mc/sec. grow in the region Q to R and escape from the sun. E_1 waves in the band 9900 to 88 Mc/sec. grow in those parts of the region P to Q which lie between the curves for f_H and f_x and also escape. Of these the E_1 waves with frequencies between 6000 and 600 Mc/sec. grow in the relatively small regions (below 5×10^4 km) where the f_H curve lies very close to the f_x curve. A simple calculation shows that the lengths Δx of these regions are approximately

$$f_x'/\tan\theta,$$

where $\tan\theta$ is the slope of the f_H curve; therefore Δx is of the order of 20 km for f about 4000 Mc/sec.

This distance is about 0.004 times the distance over which the E_2 waves of frequency 200 Mc/sec. (for example) in the complementary situation (67) can grow. It may therefore result that the latter waves escape with much larger intensities than the E_1 waves of frequencies between 6000 and 600 Mc/sec. in the situation (68). To decide this question definitely it would be necessary to use the exact statement (62) in the place of the approximate ones (63) and (64) and make an estimate of the fraction of the energy flux which escapes. This will not be attempted here.

These results are summarized in Table II.

An example of an E_2 wave which can grow and escape in the situation (67) is illustrated in Fig. 5 by the curves representing the refractive index μ and the rate of growth of intensity d_k, in decibels/km, for a frequency of 200 Mc/sec. and a drift velocity of 3×10^8 cm/sec. These show that the wave grows in the region between $x=2\times10^3$ and $x=5\times10^3$ km at different rates between 9×10^4 decibels/km‡‡ and zero and has wave-lengths between 43 and 300 meters.

It will be seen from Table II that in each region of growth the escaping E_1 waves lie in a higher frequency band.

Also by (48.1) when E_2 waves are observed to escape we may conclude that $V>0$ in the regions where they grew. Similarly when E_1 waves are observed we may conclude that if $|V|\ll1$ then $V<0$ in the regions where they grew. In the language of the magneto-ionic theory these conclusions may be expressed as follows:

When ordinary circular waves escape then the electrons in the regions of growth must drift outward. } (69)

When extraordinary circular waves escape the corresponding electrons, if not too fast, must drift inward. } (70)

Also from (20.1) we deduce the following:

According as the sunspot is a positive or negative pole, the escaping ordinary waves are respectively LH or RH and the extraordinary waves are RH or LH. } (71)

These theoretical conclusions for a large sunspot may now be compared with observation.

During a period of sunspot activity in February, 1946, Appleton and Hey[9] found that the observed strong solar noise was most intense when the sunspot concerned was near the central meridian. During the period of activity in the following July they[10] observed that the solar noise was circularly polarized.

The work of Pawsey, Payne-Scott, and McCready[11] and of Allen[12] shows that peaks of power received on 200 Mc/sec. coincide with the passage of large sunspot groups across the meridian.

Similar observations have been reported by Ryle and Vonberg[13] who, on 175 Mc/sec., also found the responsible solar waves to be largely circular. These observations show that in general the strongest solar noise is emitted normally to the sunspot group from which it originates.

The observation by Martyn,[14] that on 200 Mc/sec. the dominant polarization of the waves from a large northern group of sunspots changed from RH to LH as it crossed the meridian, does not contradict this view.

Dr. Pawsey has very kindly supplied the following as yet unpublished information. During the eclipse of November 1, 1948, Christiansen, Yabsley, and Mills¶¶ observed the following facts on a wave-length of 50 cm:

(1) Strong noise came from small areas located near visible sunspots or near places at which a visible sunspot existed one solar rotation earlier.

(2) The noise radiation from these areas was notably circularly polarized.

(3) As the edge of the Moon's disk crossed these areas the RH waves and the LH waves alternated in becoming the stronger.

These observations are consistent with the view that strong circular waves were traveling outward from *each* spot (visible or invisible). They are also consistent with the theoretical conclusion, given in Section 1, that wave growth is increasingly favored by orientation of the direction of propagation toward the common direction of the drift and the magnetic field.

‡‡ I.e., 10^{90} in about one-quarter of a wave-length.

9 E. V. Appleton and J. S. Hey, Phil. Mag. **37**, 73 (1946).

10 E. V. Appleton and J. S. Hey, Nature **158**, 339 (1946).

11 Pawsey, Payne-Scott, and McCready, Nature **157**, 158 (1946).

12 C. W. Allen, M. N. R. A. S. **107**, 396 (1947).

13 M. Ryle and D. D. Vonberg, Nature **158**, 339 (1946).

14 D. F. Martyn, Nature **158**, 308 (1946).

¶¶ All of the Radiophysics Division of the Australian Council for Scientific and Industrial Research.

Ryle and Vonberg[15] also found that out of sixteen observations of polarization at 175 Mc/sec. nine corresponded to ordinary waves, three corresponded to extraordinary waves, and four were inconclusive. According to (69) and (70) the corresponding electron drift would be outward for nine of these observations and inward for three. Also under our hypotheses (67) and (68) the nine observations would relate to ordinary waves growing in the chromosphere or lower and the three observations would relate to extraordinary waves growing in the corona near or below 10^5 km.

In Martyn's observations mentioned above the (northern) negative sunspots would have preceded the positive spots in their passage across the meridian. It therefore follows from his observations and (71) that the dominant waves concerned were ordinary. Also under our hypothesis (67) they would have grown in the chromosphere or lower. His own theoretical view was contrary to this conclusion.

Bolton[16] found on 100 Mc/sec. that the dominant waves from a southern group of sunspots changed from LH to RH as they crossed the central meridian. By (71) and (67) these also would relate to ordinary waves growing in the chromosphere or lower.

We may therefore conclude that under the hypothesis (67) most of the strong circular waves, the polarization of which has been determined, were ordinary waves and originated in the chromosphere, or lower, with the electrons drifting out of the sunspots concerned.

The consequences of our two hypotheses may also be compared with the observations of Appleton and Hey[9] made in February, 1946. They found that the noise intensity increased rapidly with the wave-length between 2 and 5 meters and then decreased between 5 and 12 meters. Table II shows that under hypothesis (68) no waves between 88 and 36 Mc/sec., i.e., 3.4 and 8.3 meters, escape; so this hypothesis is here excluded. But under hypothesis (67) the observed waves between 2 and 3.4 meters would have been ordinary ones which have grown in the chromosphere, those between 3.4 and 5.3 (or 8.3) meters would have been a mixture of ordinary and extraordinary waves and those between 5.3 (or 8.3) and 12 meters would have been extraordinary waves which have grown in the corona.

The scarcity of observed strong, circular solar waves of frequencies below about 20 Mc/sec. is explained under hypothesis (67) by the fact that according to Table II the lowest possible frequency for an escaping wave is then 19 Mc/sec.

When the region Q, where V changes sign, occurs near the top of the chromosphere then under hypothesis (67) the frequency band of the ordinary waves would lie entirely within the band of the extraordinary waves, but under the hypothesis (68) they would separate widely, with the ordinary band below the extraordinary band.

[15] M. Ryle and D. D. Vonberg, Proc. Roy. Soc. **193**, 98 (1948).
[16] See J. L. Pawsey, J. Inst. Elec. Eng. (to be published).

These results and the corresponding ones which can be similarly derived for smaller sunspots may be used to guide future observations of solar noise.

For a more exact study of the rate of growth and attenuation of waves in the solar atmosphere it is necessary to consider in greater detail the effects due to the collisions of electrons with other particles. This should be done in general for $x<2\times10^3$ km and in particular for E_1 waves of frequencies near the local gyro-frequency f_H in higher regions. One such effect is that in regions where V is negligible the E_1 waves of the lower frequencies would tend to be strongly absorbed.

From all the foregoing theoretical and observational results we may conclude that observed strong circular solar noise of wave-lengths less than 15 meters can originate from random disturbances in different regions of the solar atmosphere over sunspots whenever the electrons in those regions have a drift motion relative to the positive ions, e.g., when a constant (or slowly varying) electric field exists with a component parallel to the spot's magnetic field. That such an electric field can occur has been shown by Giovanelli.[17]

This conclusion is a particular example of the theory previously published[18] that in general abnormal solar noise may be attributed to growing waves which can arise when static electric and magnetic fields are present in the sun's atmosphere.

It also appears possible that the observed waves with frequencies above 60 Mc/sec. have been mainly ordinary waves which grew in the chromosphere by interaction with electrons drifting outward, and that those with frequencies below 60 Mc/sec. have been mainly extraordinary waves which grew in the corona by interaction with electrons drifting inwards.

This suggests that other possible causes of the drift are: (a) Electron emission from low regions over the sunspot, and (b) greater absorption of outgoing radiation by electrons than by ions. either when free or when being freed from one another.

These suggestions of course need closer examination.

It is interesting to note that our conclusion that ordinary waves can escape from the lower regions of the solar atmosphere is in agreement with Saha's[5] view; but, unlike the latter, it is substantially independent of the question whether an ordinary wave in the magneto-ionic theory can penetrate the region in which $f_0=f$.

Summing up, we can see how the single hypothesis that the electrons in a sunspot have a drift motion leads us, through Maxwell's equations and generally accepted data about the sun's atmosphere, to conclusions which are in good agreement with many of the well-established facts about strong solar noise and which do not appear to disagree with the remainder.

An important feature of this theory is that it does not require us to invoke large temperatures of an order exceeding 10^4 °K.

[17] R. G. Giovanelli, M. N. R. A. S. **107**, 338 (1947).
[18] V. A. Bailey, Nature **161**, 559 (1948).

This is in contrast to the theory published recently by Ryle[19] in which, among others, the following two hypotheses are adopted: (1) An electric field exists; (2) this electric field causes the electrons near sunspots to have mean temperatures of from 10^6 to 10^8 °K and sometimes to 10^{10} °K.

The first hypothesis alone is sufficient, on our present theory, to account for strong, circular solar noise.

The second hypothesis may or may not be true, but it does not appear to be necessary.

Also recently a theory has been proposed by Haeff[20] which resembles the present one in that it too makes use of growing plane waves. In that theory the electrons are supposed to be divided into two or more streams with different but parallel drift velocities, and magnetic fields are not considered. The resulting growing waves are longitudinal with no Poynting flux or polarization[21] and the theory gives no information on the process by which these waves give rise to the observed radiation which is necessarily accompanied by a Poynting flux. Also in order "to interpret the observed data on the intensity of solar radiation and its spectral distribution" in terms of his theory Haeff makes a large number of special assumptions. Of these the assumptions that the current density at "the surface of the Sun" is about 10^{-5} amp./cm² and that the mean electron velocity is about 2×10^8 cm/sec. lead to an electron density near the sun's surface which is about 10^{-6} times the correct value according to Smerd.

6. ULTIMATE LIMIT OF A GROWING FLUX OF RADIATION

When the transverse perturbations become larger it is necessary to take account of the non-linear terms, depending on time, which occur in the fundamental Eq. (6).

On taking mean values in time of all terms, with the perturbations periodic in time and propagated along the x axis, this equation yields:

$$U_x(\partial \mathbf{U}_0/\partial x)+\langle u_x(\partial \mathbf{u}/\partial x)\rangle_{\mathrm{Av}}+\nu \mathbf{U}_0 = (e/m)(\mathbf{E}_0+c^{-1}\mathbf{U}_0\times\mathbf{H}_0+\mathbf{F})-G\mathbf{i}, \quad (72)$$

where

$$\mathbf{F}=c^{-1}\langle\mathbf{u}\times\mathbf{h}\rangle_{\mathrm{Av}}, \quad G=\tau\frac{\partial}{\partial x}\langle \log N\rangle_{\mathrm{Av}}. \quad (73)$$

For the case (C 11) we obtain from (72) the following relation between the components along $0x$:

$$\tfrac{1}{2}\frac{\partial}{\partial x}U_x^2+\nu U_x=(e/m)(E_x+F_x)-L, \quad (74)$$

where

$$F_x=c^{-1}\langle\mathbf{u}_T\times\mathbf{h}_T\rangle_{\mathrm{Av}}, \quad (75)$$

$$L=\tfrac{1}{2}\frac{\partial}{\partial x}\langle u_x^2\rangle_{\mathrm{Av}}+G. \quad (76)$$

F_x arises from the transverse perturbations and L arises from the longitudinal ones.

On using (13) and (21) it is easily found that

$$F_x \doteqdot -\int_0^\infty \frac{\omega\beta}{8\pi N_0 ec}(|M|^2+1)|e_T|^2 d\omega, \quad (77)$$

where

$$M=\mu+i\beta, \quad |e_T|^2=|e_y|^2+|e_z|^2.$$

Since $e<0$ it follows that for the *growing* components of the transverse perturbations (which ultimately dominate the others) we have

$$F_x>0. \quad (78)$$

From (74) and (78) we see that as the transverse perturbations grow, they impose on the electrons a notable mean force eF_x which opposes the velocity of drift U_x and so tends to reduce it.

It is interesting to note that this opposing force eF_x can also be regarded as arising from the gradient of the mean Maxwellian pressure p_{xx} due to the growing transverse perturbations. For if

$$p_{xx}=1/8\pi\langle(|e_T|^2+|h_T|^2)\rangle_{\mathrm{Av}}, \quad (79)$$

then it can be deduced without much difficulty‖‖ that in the situation considered here

$$N_0 eF_x=-(\partial p_{xx}/\partial x). \quad (80)$$

(For small perturbations this can also be deduced from (77) and (79) combined with (13).)

With perturbations growing in the direction of $0x$ it is evident that p_{xx} increases with x and so, by (80), it follows that $F_x>0$, which confirms (78).

Thus, with large growing perturbations the radiation pressure gradient acts on the stream of electrons so as to reduce their mean velocity of drift U_x.

Similarly from the nature of the term L in (74) we see that U_x is reduced by the growth of longitudinal waves.

Hence it follows that ultimately a *kinetic steady state* in the medium is reached in which U_x is so small that, on account of the energy lost by collisions of electrons, the transverse perturbations cease to grow.

The ultimate value reached by F_x will depend on circumstances. Thus, if initially $\tau>U_x^2$ and a suitable random longitudinal perturbation did occur, this could lead to growing longitudinal perturbations and then F_x might not be able to attain a large value. On the other hand, if τ were small or if the initial perturbation were not longitudinal, then when ν is small, F_x would ultimately approximate to the reversed static electric field $-E_x$.

Thus the ultimate intensity of the growing waves could at the most be such that the effect of their radia-

[19] M. Ryle, Proc. Roy. Soc. **195**, 82 (1948).
[20] A. V. Haeff, Phys. Rev. **74**, 1532 (1948); **75**, 1546 (1949).
[21] V. A. Bailey, Phys. Rev. **75**, 1104 (1949).

‖‖ For example, from Eqs. (6.3) and (7.6) in R. Becker's *Theorie der Elektrizität* (B. G. Teubner, Leipzig, 1933), Vol. II, pp. 34–37.

tion pressure gradient on the electrons approximately balances that of the static electric field.

We may also consider the situation in which some or all of the electrons are ejected with an initial drift velocity U_1 and the static electric field is zero. Then we can readily deduce that, with ν small, the initial drift kinetic energy of the electrons is ultimately converted into oscillatory kinetic energy and radiation of transverse wave energy. This sets an upper limit to the intensity of the growing transverse waves.

ACKNOWLEDGMENTS

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I am also indebted to Dr. J. L. Pawsey for putting me in quick touch with much of the available observational work on solar noise and for freely giving me the benefit of his wide knowledge of this subject. Lastly I wish to thank Dr. E. J. Bowen, Chief of the Radio-Physics Division of the Council for Scientific and Industrial Research, for his permission to mention the unpublished work of Messrs. Christiansen, Yabsley, and Mills.

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APPENDIX 1

In a frame of reference which is at rest relative to the positive ions and with the axis $0x$ in the direction of the drift velocity $\mathbf{U}$, the dispersion equation in the relativistic E.M.I. theory, for a wave of the form $A\exp[i(\omega t-Llx-Lmy)]$ turns out to have either of the following forms:

$$XY^2-(\beta^2\Omega_x^2+\Omega_T^2)\beta RZY+(\beta^2S\Omega_x+Lm\Omega_y)^2Z(c^2p_0^2+\tau Z)=0; \quad (1A)$$

$$X[Y^2-(\beta^2\Omega_x^2+\Omega_T^2)Z^2]-[\beta^2(S\Omega_y-Lm\Omega_x)^2+c^{-2}(Z+\beta^2R^2)\Omega_z^2]Z(c^2p_0^2+\tau Z)=0, \quad (2A)$$

where

$X=\beta^2R^2-\tau c^{-2}(Z+\beta^2R^2)-p_0^2-i\beta^2\nu R$,
$Y=\beta R(Z+p_0^2)-i\beta\nu Z$,
$Z=c^2L^2-\omega^2$,
$\beta=(1-U^2/c^2)^{-\frac{1}{2}}$,
$R=\omega-ULl$,
$S=Ll-\omega U/c^2$,
$(l, m, 0)$ = direction cosines of the direction of propagation,
$\mathbf{\Omega}=(-e/\beta m_0c)\mathbf{H}_0$, $\Omega_T^2=\Omega_y^2+\Omega_z^2$,
$p_0^2=4\pi N_0e^2/m_0$,
m_0 = the rest-mass of an electron,
N_0 = the rest-density of the electrons.

When $|U|$ approximates to the velocity of light, (1A) approximates to

$$\{R^2(1-\tau c^{-2})-i\nu R\}\{R(Z+p_0^2)-i\nu Z\}^2=0,$$

which does not yield growing waves.

When $U^2\ll c^2$, $\tau\ll c^2$ and the frame of reference is changed by rotation about $0z$ until $0x$ is parallel to the direction of wave propagation, (1A) is transformed to the following approximation for the equation of dispersion of a wave of the form $A\exp[i(\omega t-Lx)]$:

$$XY^2-\Omega^2RZY+(L\Omega_x-c^{-2}\omega\mathbf{U}\cdot\mathbf{\Omega})^2Z(c^2p_0^2+\tau Z)=0, \quad (3A)$$

where

$X=R^2-\tau L^2-p_0^2-i\nu R$,
$Y=R(Z+p_0^2)-i\nu Z$,
$Z=c^2L^2-\omega^2$,
$R=\omega-U_xL$,
$\mathbf{\Omega}=(-e/m_0c)\mathbf{H}_0$,
$p_0^2=4\pi N_0e^2/m_0$.

Equation (3A) differs from Eq. (12), in Section 1, only by quantities which are of the second or higher orders of smallness in $\mathbf{U}$ and τ.

In the case (C11), when

$$\Omega_x=\Omega,\quad \Omega_y=0,\quad \Omega_z=0,\quad l=1,\quad m=0,$$

(2A) becomes

$$X(Y^2-\beta^2\Omega^2Z^2)=0.$$

This yields the approximate Eqs. (17) and (18) when $U^2\ll c^2$ and $\tau\ll c^2$.

The detailed derivation of the formulas (1A), (2A) and (3A) will be given in another publication.

APPENDIX 2

The use of the terms ordinary and extraordinary, which is current in the magneto-ionic theory of wave propagation, is not in *general* feasible in the electro-magneto-ionic theory.

As the literature of solar noise often contains these terms we shall here show how, nevertheless, they can be related to the E_1 and E_2 waves defined in our present special case (C11).

The ordinary wave as usually defined, with $V=0$, is that which is the less affected by the magnetic field; since

$$M^2=1-\xi/(1-k_n\eta)$$

the ordinary wave therefore corresponds to $k_n\eta<0$. Hence by (20) the ordinary and extraordinary waves correspond to our E_2 and E_1 waves respectively when $V=0$. These identifications may be retained when $V\neq 0$.