On the Diffusion of Imprisoned Resonance Radiation

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The solution of the problem of diffusion of imprisoned resonance radiation obtained by Chandrasekhar is worked out in detail in the second approximation. Decay constants, characteristic roots and eigenfunction coefficients necessary for an explicit solution are tabulated for both large and small optical thicknesses. A comparison is made between computed and measured decay constants for the Hg 2537A resonance line.

1. INTRODUCTION

DIFFUSION of imprisoned resonance radiation has been the subject of both experimental and theoretical investigation and can be explained simply in terms of excitation of atoms. In escaping from a gas of atoms in their normal state, radiation at the frequency of a resonance line of the atoms will be absorbed and emitted many times. This transfer of excitation from atom to atom delays the escape of the resonance quanta from the gas because they are "imprisoned" within each atom for an average time equal to the mean life of the excited state. The escape of resonance quanta, therefore, can be considered statistically as a decay of atoms in the excited state, and it can be analyzed as a diffusion process, if the frequency dependence of the atomic absorption coefficient for the resonance line $\sigma(\nu)$ is approximated by assigning a constant value σ over a given frequency interval $\Delta\nu$ such that

$$\int \sigma(\nu)d\nu=\sigma\Delta\nu. \tag{1}$$

The equations of radiative transfer and radiative equilibrium for this diffusion process were first formulated and solved approximately by Milne.[1] Chandrasekhar[2] has analyzed this problem in a similar manner obtaining solutions in finite approximations. The solution in the first approximation, which is essentially equivalent to Milne's solution, is worked out and presented in detail.

In this paper Chandrasekhar's solution in the second approximation will be presented in a similar manner, including tabulation of decay constants of the exponential decay of excited atoms, characteristic roots and dominant eigenfunction coefficients necessary for an explicit solution. Also the decay constants for the Hg 2537A resonance line, as calculated in the first and second approximations and in other investigations, will be compared with laboratory measurements.

2. CHARACTERISTIC ROOTS AND DECAY CONSTANTS FOR THE SECOND APPROXIMATION

The characteristic equation (reference 2, p. 360) in the second approximation is

$$\frac{1}{\omega}=\frac{a_1}{1+(\mu_1 k)^2}+\frac{a_2}{1+(\mu_2 k)^2} \tag{2}$$

where the Gaussian weights a_1, a_2 and divisions μ_1, μ_2 have the values

$$a_1=0.652145,\quad \mu_1=0.339981,$$
$$a_2=0.347855,\quad \mu_2=0.861136,$$

and where ω takes one of its eigenvalues $\omega^{(l,m)}$ or $\omega^{(0,m)}$ with $m=1, 2, \cdots, \infty$. The characteristic roots of (2) corresponding to these eigenvalues consists of the real roots $k_1^{(l,m)}$ or $k_1^{(0,m)}$ and the imaginary roots $ik_2^{(l,m)}$ or $ik_2^{(0,m)}$. Using the definition of θ_{ij} (reference 2, p. 361), we have

$$\begin{aligned}
\theta_{11}^{(l,m)}&=\tan^{-1}\mu_1 k_1^{(l,m)}, & \theta_{11}^{(0,m)}&=\tan^{-1}\mu_1 k_1^{(0,m)},\\
\theta_{12}^{(l,m)}&=\tan^{-1}\mu_2 k_1^{(l,m)}, & \theta_{12}^{(0,m)}&=\tan^{-1}\mu_2 k_1^{(0,m)},\\
\theta_{21}^{(l,m)}&=\tanh^{-1}\mu_1 k_2^{(l,m)}, & \theta_{21}^{(0,m)}&=\tanh^{-1}\mu_1 k_2^{(0,m)}.
\end{aligned} \tag{3}$$

Similar definitions for $\theta_{22}^{(l,m)}$ and $\theta_{22}^{(0,m)}$ are not valid since

$$\mu_2 k_2^{(l,m)}>1;\quad \mu_2 k_2^{(0,m)}>1 \tag{4}$$

for all values of m. Therefore the equations which determine the eigenvalues of ω (reference 2, p. 361) take the modified forms,

$$\begin{vmatrix}
\cos\theta_{11}^{(l,m)}\cos(\theta_{11}^{(l,m)}+k_1^{(l,m)}\tau_1) & \cos\theta_{12}^{(l,m)}\cos(\theta_{12}^{(l,m)}+k_1^{(l,m)}\tau_1)\\
\cosh\theta_{21}^{(l,m)}\cosh(\theta_{21}^{(l,m)}+k_2^{(l,m)}\tau_1) & \dfrac{\cosh k_2^{(l,m)}\tau_1+\mu_2 k_2^{(l,m)}\sinh k_2^{(l,m)}\tau_1}{1-(\mu_2 k_2^{(l,m)})^2}
\end{vmatrix}=0, \tag{5}$$

which is satisfied by $\omega^{(l,m)}$, and

$$\begin{vmatrix}
\cos\theta_{11}^{(0,m)}\sin(\theta_{11}^{(0,m)}+k_1^{(0,m)}\tau_1) & \cos\theta_{12}^{(0,m)}\sin(\theta_{12}^{(0,m)}+k_1^{(0,m)}\tau_1)\\
\cosh\theta_{21}^{(0,m)}\sinh(\theta_{21}^{(0,m)}+k_2^{(0,m)}\tau_1) & \dfrac{\mu_2 k_2^{(0,m)}\cosh k_2^{(0,m)}\tau_1+\sinh k_2^{(0,m)}\tau_1}{1-(\mu_2 k_2^{(0,m)})^2}
\end{vmatrix}=0, \tag{6}$$

[1] E. A. Milne, J. Math. Soc. London 1, 40 (1926).
[2] S. Chandrasekhar, *Radiative Transfer* (Clarendon Press, Oxford, 1950), p. 354.

TABLE I. Decay constant, characteristic roots and dominant eigengunction coefficients appearing in the second approximation solution for large optical thicknesses.

Decay constant*		Characteristic roots		Dominant eigenfunction coefficients		Optical thickness τ_1 for	
$(\omega^{(l,m)}-1)/\omega^{(l,m)}$ or $(\omega^{(0,m)}-1)/\omega^{(0,m)}$	$-\log\left(\frac{\omega^{(l,m)}-1}{\omega^{(l,m)}}\right)$ or $-\log\left(\frac{\omega^{(0,m)}-1}{\omega^{(0,m)}}\right)$	$k_1^{(l,m)}$ or $k_1^{(0,m)}$	$k_2^{(l,m)}$ or $k_2^{(0,m)}$	$\frac{a^{(l,1)}A_1^{(1)}}{I^{(0)}}$	$-\log_e\left(\frac{-a^{(l,1)}A_2^{(1)}}{I^{(0)}}\right)$	$(l, 1)$	$(0, 2)$
1.0000×10^{-6}	6.00000	0.00173205	1.9720	0.6367	1795.20441	906.182	...
3.1623×10^{-6}	5.50000	0.00308008	1.9720	0.6366	1012.00442	509.289	1019.28
1.0000×10^{-5}	5.00000	0.00547723	1.9720	0.6366	571.35874	286.107	572.904
3.1623×10^{-5}	4.50000	0.00974064	1.9720	0.6366	313.18413	160.569	321.832
1.0000×10^{-4}	4.00000	0.0173222	1.9720	0.6366	183.41917	89.9873	180.669
3.1623×10^{-4}	3.50000	0.0308096	1.9718	0.6365	104.55968	50.2903	101.274
1.0000×10^{-3}	3.00000	0.0548215	1.9712	0.6361	59.92813	27.9597	56.6157
3.1623×10^{-3}	2.50000	0.0976786	1.9695	0.6349	34.65068	15.3897	31.4710
1.0000×10^{-2}	2.00000	0.174780	1.9641	0.6315	19.97762	8.3009	17.2882
3.1623×10^{-2}	1.50000	0.317058	1.9468	0.6218	11.46733	4.2842	9.2386
1.0000×10^{-1}	1.00000	0.60235	1.8902	0.5981	6.43080	1.9942	4.5979
3.1623×10^{-1}	0.50000	1.3605	1.7071	0.5480	3.71399	0.6919	1.850

* $(\omega^{(0,1)}-1)/\omega^{(0,1)}\equiv 0$.

which is satisfied by $\omega^{(0,m)}$. These equations are for a slab of gas of infinite extent and a total optical thickness of $2\tau_1$.

It is simpler to solve (5) or (6) together with (2) and (3) for τ_1 for assigned values of the decay constants $(\omega^{(l,m)}-1)/\omega^{(l,m)}$ or $(\omega^{(0,m)}-1)/\omega^{(0,m)}$, rather than solving for decay constants for assigned values of the optical thickness. The results, together with characteristic roots, are tabulated in Table I for large values of τ_1 and in Table II for small values of τ_1, using a sufficient number of values of m to give the final solution of the problem the same order of accuracy as the tables. This accuracy is better than ± 5 in the last digit. The decay constants are given in units in which the mean life of the excited state is unity.

3. NORMALIZED EIGENFUNCTIONS AND THE SECOND APPROXIMATION SOLUTION

The orthogonal, normalized eigenfunctions used in the final solution of the problem are in the second approximation (reference 2, p. 362)

$$\psi^{(l,m)}(\tau)=A_1^{(m)}[\cos k_1^{(l,m)}\tau+(A_2^{(m)}/A_1^{(m)})\cosh k_2^{(l,m)}\tau] \quad (7)$$

TABLE II. Decay constant, characteristic roots and dominant eigenfunction coefficients appearing in the second approximation solution for small optical thicknesses.

Decay constant*		Characteristic roots		Dominant eigenfunction coefficients		Optical thickness τ_1 for						
$(\omega^{(l,m)}-1)/\omega^{(l,m)}$ or $(\omega^{(0,m)}-1)/\omega^{(0,m)}$	$\log\omega^{(l,m)}$ or $\log\omega^{(0,m)}$	$k_1^{(l,m)}$ or $k_1^{(0,m)}$	$k_2^{(l,m)}$ or $k_2^{(0,m)}$	$\frac{a^{(l,1)}A_1^{(1)}}{I^{(0)}}$	$-\frac{a^{(l,1)}A_2^{(1)}}{I^{(0)}}$	$(l, 1)$	$(0, 2)$	$(l, 2)$	$(0, 3)$	$(l, 3)$	$(0, 4)$	$(l, 4)$
0.100	...	0.6024	1.8902	0.5981	0.001611	1.9942	...	...	...	...	...	...
0.150	...	0.7766	1.8476	0.5815	0.005919	1.4435	...	...	...	...	...	...
0.200	0.09691	0.9464	1.8046	0.5695	0.01113	1.1177	...	...	...	...	...	...
0.250	...	1.1192	1.7620	0.5601	0.01720	0.8975	...	...	...	...	...	...
0.300	...	1.2998	1.7203	0.5507	0.02283	0.7354	1.9473	...	...	...	...	...
0.350	...	1.4912	1.6803	0.5432	0.02704	0.6117	1.6694	...	...	...	...	...
0.400	...	1.6979	1.6425	0.5369	0.02976	0.5121	1.4426	...	...	...	...	...
0.450	...	1.9222	1.6074	0.5317	0.03084	0.4305	1.2537	2.0699	...	...	...	...
0.500	...	2.1686	1.5750	0.5277	0.03082	0.3613	1.0930	1.8162	...	...	...	...
0.550	...	2.4432	1.5456	0.5248	0.02901	0.3035	0.9535	1.5957	...	...	...	...
0.600	0.39794	2.7541	1.5189	0.5221	0.02662	0.2528	0.8303	1.3995	1.9701	...	...	...
0.650	...	3.1137	1.4949	0.5199	0.02368	0.2083	0.7194	1.2227	1.7275	...	...	...
0.700	...	3.5413	1.4733	0.5173	0.0202	0.1690	0.6185	1.0597	1.5046	1.9481	...	...
0.7500	...	4.0692	1.4539	0.5147	0.0168	0.1336	0.5254	0.9094	1.2973	1.6822	2.0681	...
0.7750	...	4.3881	1.4449	...	...	...	...	...	...	...	1.9120	...
0.8000	0.69897	4.7559	1.4364	0.512	0.013	0.1019	0.4375	0.7653	1.0967	1.4262	1.7567	2.0870
0.8250	...	5.198	1.4283	...	...	...	...	...	...	...	1.6023	1.9038
0.8500	...	5.7235	1.4206	0.509	0.010	0.0730	0.3502	0.6238	0.8987	1.1729	1.4475	1.7220
0.8750	...	6.3942	1.4133	...	...	...	...	...	...	...	1.2899	1.5371
0.9000	1.00000	7.2862	1.4064	...	...	...	0.2646	0.4789	0.6952	0.9104	1.1262	1.3417
0.9250	...	8.5698	1.3997	...	...	...	...	...	...	...	0.9526	1.1351
0.9500	1.30103	10.6848	1.3934	...	...	...	0.1707	0.3168	0.4644	0.6110	0.7583	0.9051
0.97500	1.60206	15.3743	1.3874	...	...	...	0.1137	0.2155	0.3180	0.4199	0.5223	0.6244
0.98750	1.90309	21.9273	1.3845	...	...	...	0.0774	0.1488	0.2206	0.2920	0.3639	0.4355
0.993750	2.20412	31.1398	1.3831	...	...	...	...	0.1037	0.1543	0.2047	0.2552	0.3056
0.9968750	2.50515	44.1302	1.3824	...	...	...	...	0.0725	0.1082	0.1438	0.1794	0.2150
0.99843750	2.80618	62.4742	1.3821	...	...	...	...	...	0.0761	0.1013	0.1265	0.1516
0.999218750	3.10721	88.3976	1.3819	...	...	...	...	...	...	0.0714	0.0892	0.1070

* $(\omega^{(0,1)}-1)/\omega^{(0,1)}\equiv 0$.

and

$$\psi^{(0,m)}(\tau)=B_1{}^{(m)}[\sin k_1{}^{(0,m)}\tau+(iB_2{}^{(m)}/B_1{}^{(m)})\sinh k_2{}^{(0,m)}\tau], \tag{8}$$

where by the boundary conditions (reference 2, p. 362)

$$A_2{}^{(m)}/A_1{}^{(m)}=-\cos\theta_{11}{}^{(l,m)}\cos(\theta_{11}{}^{(l,m)}+k_1{}^{(l,m)}\tau_1)/\cosh\theta_{21}{}^{(l,m)}\cosh(\theta_{21}{}^{(l,m)}+k_2{}^{(l,m)}\tau_1) \tag{9}$$

and

$$iB_2{}^{(m)}/B_1{}^{(m)}=-\cos\theta_{11}{}^{(0,m)}\sin(\theta_{11}{}^{(0,m)}+k_1{}^{(0,m)}\tau_1)/\cosh\theta_{21}{}^{(0,m)}\sinh(\theta_{21}{}^{(0,m)}+k_2{}^{(0,m)}\tau_1), \tag{10}$$

and by the normalizing conditions

$$\frac{1}{(A_1{}^{(m)})^2}=\tau_1+\frac{\sin 2k_1{}^{(l,m)}\tau_1}{2k_1{}^{(l,m)}}+\left[\frac{A_2{}^{(m)}}{A_1{}^{(m)}}\right]^2\left[\tau_1+\frac{\sinh 2k_2{}^{(l,m)}\tau_1}{2k_2{}^{(l,m)}}\right]$$
$$+4\left[\frac{A_2{}^{(m)}}{A_1{}^{(m)}}\right]\left[\frac{k_1{}^{(l,m)}\sin k_1{}^{(l,m)}\tau_1\cosh k_2{}^{(l,m)}\tau_1+k_2{}^{(l,m)}\cos k_1{}^{(l,m)}\tau_1\sinh k_2{}^{(l,m)}\tau_1}{(k_1{}^{(l,m)})^2+(k_2{}^{(l,m)})^2}\right] \tag{11}$$

and

$$\frac{1}{(B_1{}^{(m)})^2}=\tau_1-\frac{\sin 2k_1{}^{(0,m)}\tau_1}{2k_1{}^{(0,m)}}-\left[\frac{iB_2{}^{(m)}}{B_1{}^{(m)}}\right]^2\left[\tau_1-\frac{\sinh 2k_2{}^{(0,m)}\tau_1}{2k_2{}^{(0,m)}}\right]$$
$$+4\left[\frac{iB_2{}^{(m)}}{B_1{}^{(m)}}\right]\left[\frac{k_2{}^{(0,m)}\sin k_1{}^{(0,m)}\tau_1\cosh k_2{}^{(0,m)}\tau_1-k_1{}^{(0,m)}\cos k_1{}^{(0,m)}\tau_1\sinh k_2{}^{(0,m)}\tau_1}{(k_1{}^{(0,m)})^2+(k_2{}^{(0,m)})^2}\right]. \tag{12}$$

The expansions in terms of these eigenfunctions of the mean intensity $J(t,\tau)$ and the number of excited atoms per cm^3 $N(t,\tau)$ constitute the final solution of the problem. These expansions (reference 2, p. 362) in the second approximation are

$$J(t,\tau)=\sum_{m=1}^{\infty}a^{(l,m)}\psi^{(l,m)}(\tau)\times\exp\{-(\omega^{(l,m)}-1)t/\omega^{(l,m)}\}+\sum_{m=2}^{\infty}a^{(0,m)}\psi^{(0,m)}(\tau)\times\exp\{-(\omega^{(0,m)}-1)t/\omega^{(0,m)}\} \tag{13}$$

and

$$N(t,\tau)=\sum_{m=1}^{\infty}a^{(l,m)}\omega^{(l,m)}\psi^{(l,m)}(\tau)\times\exp\{-(\omega^{(l,m)}-1)t/\omega^{(l,m)}\}+\sum_{m=2}^{\infty}a^{(0,m)}\omega^{(0,m)}\psi^{(0,m)}(\tau)\times\exp\{-(\omega^{(0,m)}-1)t/\omega^{(0,m)}\}, \tag{14}$$

where the summation of $\psi^{(0,m)}(\tau)$ starts with $m=2$, since $\psi^{(0,1)}(\tau)$ is identically zero. These solutions are for the time interval $t>0$ where $t=0$ marks the cutting off of all radiation incident on the gas and hence the starting of the diffusion process. Examination of Tables I and II shows that for a given τ_1, the decay constant $(\omega^{(l,1)}-1)/\omega^{(l,1)}$ is the smallest, and so the term containing $\psi^{(l,1)}(\tau)$ in (13) and (14) will dominate for sufficiently large values of t, this domination being more pronounced for large values of τ_1. Hence $\psi^{(l,1)}(\tau)$ will be designated the dominant eigenfunction, and its associated decay constant, the dominant decay constant.

The coefficients of expansion $a^{(l,m)}$ and $a^{(0,m)}$ are determined by equating $J(t=0,\tau)$ to the mean intensity $J(\tau)$ obtained from the solution of the stationary Schuster problem for the time interval $t<0$. In the second approximation this solution for the steady state conditions when radiation is incident on the gas, gives (reference 2, p. 363)

$$J(\tau)=L_0\tau-2L_1\sinh k^{(0)}\tau+L_2, \tag{15}$$

where

$$k^{(0)}=1.972027$$

is a root of the characteristic equation (reference 2, p. 363). The coefficients in (15) as determined by the boundary conditions (reference 2, p. 363) are

$$L_0=\tfrac{1}{2}I^{(0)}[0.228020\exp\{-k^{(0)}\tau_1\}-4.466748\exp\{k^{(0)}\tau_1\}]/L^*, \tag{16}$$

$$L_1=\tfrac{1}{2}I^{(0)}[0.521155]/L^* \tag{17}$$

and

$$L_2=\tfrac{1}{2}I^{(0)}, \tag{18}$$

where

$$L^*=(0.228020\tau_1+0.389506)\exp\{-k^{(0)}\tau_1\}-(4.466748\tau_1+3.100033)\exp\{k^{(0)}\tau_1\}, \tag{19}$$

and $I^{(0)}$ is the intensity of the isotropic radiation incident at $\tau=-\tau_1$ Expressions for the coefficients of expansion can be obtained in the usual manner, employing the orthogonality of the eigenfunctions and substituting $J(\tau)$ for $J(t=0,\tau)$. These coefficients of expansion are

$$a^{(l,m)}=\int_{-\tau_1}^{\tau_1} J(\tau)\psi^{(l,m)}(\tau)d\tau=2L_2A_1^{(m)}\left[\frac{\sin k_1^{(l,m)}\tau_1}{k_1^{(l,m)}}+\left(\frac{A_2^{(m)}}{A_1^{(m)}}\right)\frac{\sinh k_2^{(l,m)}\tau_1}{k_2^{(l,m)}}\right] \tag{20}$$

and

$$a^{(0,m)}=\int_{-\tau_1}^{\tau_1} J(\tau)\psi^{(0,m)}(\tau)d\tau=2L_0B_1^{(m)}\left\{\frac{\sin k_1^{(0,m)}\tau_1}{(k_1^{(0,m)})^2}-\frac{\tau_1\cos k_1^{(0,m)}\tau_1}{k_1^{(0,m)}}\right.$$
$$\left.-\left[\frac{iB_2^{(m)}}{B_1^{(m)}}\right]\left[\frac{\sinh k_2^{(0,m)}\tau_1}{(k_2^{(0,m)})^2}-\frac{\tau_1\cosh k_2^{(0,m)}\tau_1}{k_2^{(0,m)}}\right]\right\}$$
$$+4L_1B_1^{(m)}\left\{\frac{k_1^{(0,m)}\cos k_1^{(0,m)}\tau_1\sinh k^{(0)}\tau_1-k^{(0)}\sin k_1^{(0,m)}\tau_1\cosh k^{(0)}\tau_1}{(k^{(0)})^2+(k_1^{(0,m)})^2}\right.$$
$$\left.+\left[\frac{iB_2^{(m)}}{B_1^{(m)}}\right]\left[\frac{k_2^{(0,m)}\cosh k_2^{(0,m)}\tau_1\sinh k^{(0)}\tau_1-k^{(0)}\sinh k_2^{(0,m)}\tau_1\cosh k^{(0)}\tau_1}{(k^{(0)})^2-(k_2^{(0,m)})^2}\right]\right\}. \tag{21}$$

The coefficients of the dominant eigenfunction in the expression for $J(t,\tau)$ have been obtained in terms of $I^{(0)}$, using (9), (11), (18) and (20), and the results are included in Tables I and II.

4. DECAY CONSTANT FOR THE MERCURY 2537A RESONANCE LINE

Zemansky[3] and Alpert *et al.*[4] have measured the dominant decay constant for the Hg 2537A resonance line in the normal time scale

$$\beta=A_{21}(\omega^{(l,1)}-1)/\omega^{(l,1)} \tag{22}$$

where A_{21} is the Einstein coefficient for spontaneous emission. These measurements were made for several values of N, the number of absorbing atoms per cm^3, which is related to the optical thickness by the expression

$$\tau_1=\tfrac{1}{2}N\sigma l, \tag{23}$$

where l is the geometric thickness of a mercury vapor cell, and σ is the mean absorption coefficient as defied by (1).

Determining the correct value of σ for diffusion of resonance radiation, however, is not a simple matter of determining a line breadth $\Delta\nu$ from the Doppler breadth and fine structure splitting, as such calculations give a value for σ several times too large. As Zanstra[5] among others has pointed out, Doppler shifts arising from thermal motions of the Hg atoms will produce noncoherent scattering of the resonance radiation, and this will speed up the diffusion process since the probability of escape for a resonance quantum at the center of the line is much less than the product of the probability of its being scattered to the wings and the probability of escape at the scattered frequency. This phenomenon will qualitatively explain the necessary reduction in σ, but a satisfactory quantitative theory is still needed. An approximate method for determining σ is provided by Zemansky[6] who has equated a diffusion coefficient derived by Kenty[7] to $A_{21}/4N^2\sigma^2$ which appears as a diffusion coefficient in Milne's[1] theory. In obtaining his diffusion coefficient as a function of N, l and atomic constants, Kenty has taken account of incoherent

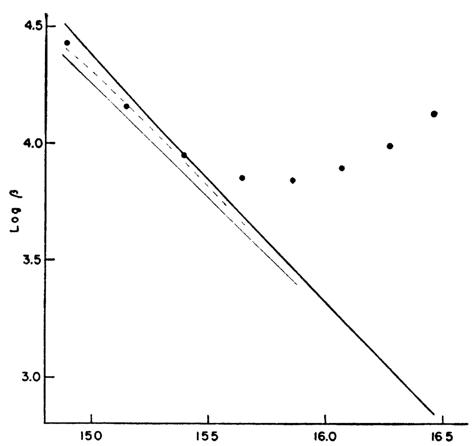

FIG. 1. Logarithm of β, the dominant decay constant for the Hg 2537A resonance line in reciprocal seconds, as a function of the logarithm of N, the number of absorbing atoms per cm^3 in a cell 1.95 cm thick. The thick solid curve represents computations using the second approximation; the thin solid curve, computations of Holstein; the broken curve, computations of Zemansky using Milne's theory. The black circles represent laboratory measurements by Zemansky.

[3] M. W. Zemansky, Phys. Rev. **29**, 513 (1927).
[4] Alpert, McCoubray, and Holstein, Phys. Rev. **76**, 1257 (1949).
[5] H. Zanstra, Bull. Astronom. Insts. Netherlands **XI**, No. 401 (1949).
[6] M. W. Zemansky, Phys. Rev. **42**, 843 (1932).
[7] C. Kenty, Phys. Rev. **42**, 823 (1932).

scattering due to thermal motions. However, as Holstein[8] has pointed out, the use of such kinetic theory concepts as a mean free path and a diffusion coefficient in this calculation of σ is not completely satisfactory in describing accurately the diffusion of imprisoned resonance radiation, where the atomic absorption coefficient varies considerably with frequency. Holstein has circumvented these difficulties by analyzing the diffusion process in terms of the probability of a resonance quantum going a given distance between successive absorptions. The averaging of this probability over the resonance line takes account of incoherent scattering due to thermal motions, and this averaged probability is used in an equation of radiative equilibrium which is solved approximately by variational methods for the decay constant. However, Zemansky's method for calculating σ, which gives the right order of magnitude, is used in this paper.

The optical thicknesses obtained from (23) for the values of N and l involved in the laboratory measurements of the decay constant are large, ranging from ten to one hundred. In this range $\log\beta$ as calculated by the first approximation is less than one percent larger than $\log\beta$ as calculated by the second approximation. In Fig. 1, values of $\log\beta$ are plotted for various values of $\log N$ and for $l=1.95$ cm, as calculated for the second approximation, as calculated by Zemansky[6] using Milne's theory and as calculated by Holstein.[8] The measurements of Zemansky[3] for the same value of l are also plotted. Better agreement between measured and calculated decay constants have been obtained with the improved measuring techniques of Alpert *et al.*[4] The discrepancy between measured and calculated values of $\log\beta$ for large values of $\log N$ can be explained in part by the effects of collisional broadening which have been neglected in the computations.

It is a pleasure to express my appreciation to Dr. S. Chandrasekhar for suggesting this problem and for helpful discussions concerning it.

[8] T. Holstein, Phys. Rev. 72, 1212 (1947).

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The Growth of Circularly Polarized Waves in the Sun's Atmosphere and Their Escape into Space

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The theory, previously published, of plane waves in an ionized medium pervaded by static electric and magnetic fields is shown to predict wave amplification, and consequent electromagnetic noise, in certain frequency bands. It is then developed in detail for the case in which the static fields are both parallel to the direction of wave propagation and the perturbations are transverse to this direction.

It is shown that for any given frequency and electron drift velocity there are two trios of such waves, E_1 and E_2 waves, all circularly polarized; the E_1 and E_2 waves are oppositely polarized. It is found that any transverse perturbation temporally prescribed at a given plane can be split up into two such trios which can then be considered independently.

Necessary and sufficient conditions are then found under which a growing flux of energy carried by E_1 or E_2 waves can pass normally through the boundary between two different ionized media.

The theory is applied to show that under simple hypotheses about the drift of electrons in the atmosphere above a large sunspot strong circular waves can arise by growth of random transverse perturbations and can then escape from the sun. The consequences of two such hypotheses are compared with known observations of solar noise and used to interpret them.

It is concluded that the general hypothesis that electrons in a sunspot have a drift motion leads to results which are in good agreement with many facts about strong solar noise and which do not disagree with any others.

The ultimate intensity which a growing perturbation can attain is also discussed.

1. INTRODUCTION

In two previous publications[1,2] the general equations which specify the dispersion and polarization of plane waves in an ionized medium, pervaded by static electric and magnetic fields, have been derived. This theory may be conveniently referred to as the electro-magneto-ionic theory of wave propagation and more briefly as the E.M.I. theory. As a limiting case it includes the well-known magneto-ionic theory (M.I. theory) and its application is in general subject to the same conditions of validity as the latter.

We shall here apply the E.M.I. theory to the important solar phenomenon of emission of strong circularly polarized radio noise by a large sunspot.

As it appears that the Australian publications referred to above are not yet readily available in the United States and elsewhere, a summary of the E.M.I. theory is given here in the approximation which neglects the motions of the positive ions.

In Appendix 1 we also give the relativistic form of the

[1] V. A. Bailey, J. Roy. Soc. N.S.W. 82, 107 (1948).
[2] V. A. Bailey, Australian J. Sci. Res. A, 1, 351 (1948).