Solar Magnetic Moment and Diurnal Variation in Intensity of Cosmic Radiation*

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The effect of the earth's magnetic field upon the orbits of primary cosmic-ray particles is computed in detail for the special case of those particles arriving vertically anywhere on the earth's surface. A procedure is evolved for taking this deflection in conjunction with the previous deflection in the magnetic field of the sun, and thereby calculating for any given cosmic-ray energy spectrum an expected diurnal variation in total cosmic-ray intensity. Examples of this procedure are given for an inverse power spectrum applied to two different magnitudes of the solar magnetic moment. The sample calculations are made for several latitudes of observation, and the expected diurnal variation is then studied as a function of the solar magnetic moment and of the energy spectrum. The phase of the variation is found to be very sensitive to the solar moment, and so it is hoped that a successful experimental determination of the diurnal variation will settle the question of the existence and magnitude of a permanent solar magnetic moment.

I. INTRODUCTION

IF, as proposed by G. E. Hale,[1] the Sun has a polar field of 50 gauss (or a magnetic moment of 10^{34} gauss-cm^3, 1000 times that of the earth), then it will have a substantial effect on the trajectories of cosmic-ray particles, producing a latitude cut-off and diurnal variations of intensity. It is important to calculate exactly what this predicted effect is, and to look for it in experiments. Then one can hope to know whether or not there exists a permanent solar magnetic moment, and if so, of what magnitude. This question is significant both for solar and for cosmic-ray physics.

As yet it has been impossible to compute from the known properties of matter theoretical values for the magnetic moments of massive rotating bodies which agree with experiment. For this reason, Blackett[2] and others have considered the possibility of the existence of a relation (not envisaged in existing theory) between rotating mass and magnetic fields. This hypothesis has given good agreement with the observed fields of several stars, but much further information is required before it can be taken seriously. One of the necessities is the resolution of the uncertainty surrounding the magnitude, or even existence, of a general solar magnetic moment.

Considerable doubt was thrown upon Hale's conclusions from measurements of the Zeeman splitting of the solar spectrum lines[1] by the fact that several of his observers failed to detect any splitting. This doubt was augmented by the failure to find a completely successful explanation of his observation of different splittings for different spectrum lines. Then, in 1935 (a year of minimum sunspot activity), Thiessen made a similar observation, and concluded that the polar field was 53 ± 10 gauss.[3] During the next few years, Babcock found variations in the total solar field. These variations inspired Thiessen to repeat his observations in 1946 (a year of maximum sunspot activity), with the astonishing result that he failed to detect any solar field as large as 5 gauss.[4] It is clear that the inconsistencies among the experimental results prevent the determination of a reliable value for the permanent solar magnetic moment, and even throw doubt on its existence.

Similar inconsistencies are also present in experimental observations of the leveling off of cosmic-ray intensity with increasing latitude of incidence. Because of the decreasing cut-off effect of the earth's magnetic field toward the poles, a continuous rise in intensity with increasing latitude is to be expected. Actually there appears to be a saturation. No natural explanation is known for such an effect except the hypothesis that low energy particles are not present in the primary radiation. This exclusion could easily be understood on the basis of a permanent solar magnetic moment, as suggested by Janossy. Thus a solar field would result in a sharp knee in the curve representing intensity as a function of latitude. However, according to some experiments the intensity continues to rise gradually, whereas in some it even decreases, and in others merely behaves irregularly.[5] Thus once more doubt arises concerning the existence of a general solar magnetic moment.

The question of what the energy spectrum really looks like is fundamental to cosmic-ray studies. It is clear that if the energy cut-off exists and cannot be accounted for on the grounds of a permanent solar magnetic moment, then we must search for an explanation in the mechanism of creation, acceleration, and energy loss of the primary cosmic-ray particles. Hence the problem of the solar moment is important for the problem of cosmic-ray origin.

* Assisted by the Joint Program of the ONR and AEC.

[1] Hale, Seares, VanMaanen, and Ellerman, Astrophys. J. 47, 206 (1918).

[2] P. M. S. Blackett, Phil. Mag. 40, 125 (1949). This article also summarizes the earlier literature.

[3] A more complete summary of observations of solar fields is given by Blackett in reference 2.

[4] G. Thiessen, Observatory 66, 230 (1946). Also H. Von Kluber, Zeits. f. Astrophys. 24 (1947) and 121 (1947).

[5] M. G. E. Cosyns, Nature 137, 616 (1936); Millikan, Neher, and Pickering, Phys. Rev. 63, 234 (1943); Biehl, Montgomery, Neher, Pickering, and Roesch, Rev. Mod. Phys. 20, 360 (1948).

It is interesting to note that both types of observations mentioned above yield for the solar moment the same order of magnitude, 10^{34} gauss-cm^3, as far as the inconsistencies permit the deduction of values. In order to investigate the solar moment further, we must assume it to exist, and then try to deduce additional consequences which can be subjected to experimental test. One such effect is a diurnal variation in intensity of cosmic radiation, deduced originally by Janossy.[6] Further analyses have been made by Epstein,[7] by Vallarta and Godart in general terms,[8] and in greater detail by Rossi.[9] Many experiments have been performed in an attempt to detect the phenomenon,[10] but they have been unsuccessful.[11] However, these observations were all made on the ground, *where there should be practically no diurnal variation.* For the particles which contribute most to the diurnal variation are those of low energy (3 Bev). At ground level there is only a minute contribution to the intensity from the particles of that energy, either directly or through secondaries. Even at airplane altitudes (30,000 ft.), the effects of 3-Bev particles are presumably too weak to be significant for observations.[12]

In this paper we give, for two different values of the solar moment, the detailed calculation of the diurnal variation for the special case of primary radiation received vertically on the earth. Mr. W. G. Stroud made experimental determinations of the diurnal variation in vertical intensity at high altitudes (95,000 ft.) in the summer of 1949. Such experiments include an appreciable solid angle about the vertical. Fortunately this spread has only a small effect.[13] Thus such experiments, together with the present calculations, should lead to further information about the magnetic moment of the sun.

II. CALCULATED EFFECT

It is easy to see why a diurnal variation in intensity is to be expected if the sun does have a permanent magnetic moment. Moving charged particles, such as cosmic rays, will be deflected in the solar field, and so will be able to approach the earth from outer space only in certain allowed directions, which form a cone,[14] called the *solar allowed cone.* Therefore at any instant, particles of a given energy can arrive at the earth's surface vertically only over a certain region. This region is fixed in space, and so a given point on the earth will pass into it and then out again as the earth rotates. Thus, a diurnal variation in intensity of primary radiation at that point is produced.

In spite of the explanation given in the previous section, physicists still felt somewhat uncomfortable that there should be no indication of any diurnal variation at ground level. The effect expected for the primary radiation was so strong that it seemed that it should have some influence upon the intensity of the secondary particles. To try to account for this disturbing situation. Alfvén[15] suggested that the orbits forbidden by the solar field might be partially filled by particles scattered by the magnetic field of the earth. Further theoretical investigations showed that, because of this scattering, particles with any given energy that can approach the earth in some direction are never completely excluded from any other direction.[16] It is estimated that for a permanent solar moment of the order of 10^{34} gauss-cm^3, the intensity in "forbidden" directions is of the order of 90 percent of the full intensity. Therefore the expected effect is much smaller, and we shall certainly have to go to high altitudes to observe it.

The solar allowed cone centers about the tangent to the earth's orbit, and opens out backward along it. All positive particles arrive at full intensity from this direction. Hence, if there were no further deflection in the magnetic field of the earth, a maximum in vertical intensity of cosmic radiation would occur at 6 P.M. Because of the terrestrial field, a positive particle arriving from the direction of the axis of the solar allowed cone will be received vertically at a given point on the earth not at 6 P.M., but somewhat earlier. Therefore, the maximum total intensity occurs before 6 P.M.

In order to find the latitude at which the phenomenon can most easily be detected, we must study the variation as a function of latitude of observation. At very low latitudes, those particles which are appreciably affected by the solar field are prevented from reaching the earth by the terrestrial field. The amount of the resulting deflection of a particle depends upon its energy. Hence at other latitudes, where entry is possible, the individual variations of intensity with time of arrival at the earth for radiation of different energies are displaced with respect to one another. Thus a distortion, or smearing out, of the total variation of intensity with time is produced which is greatest for radiation incident at low latitudes, and so here no maximum in intensity exists. At the poles there is no distortion, but then the rotation

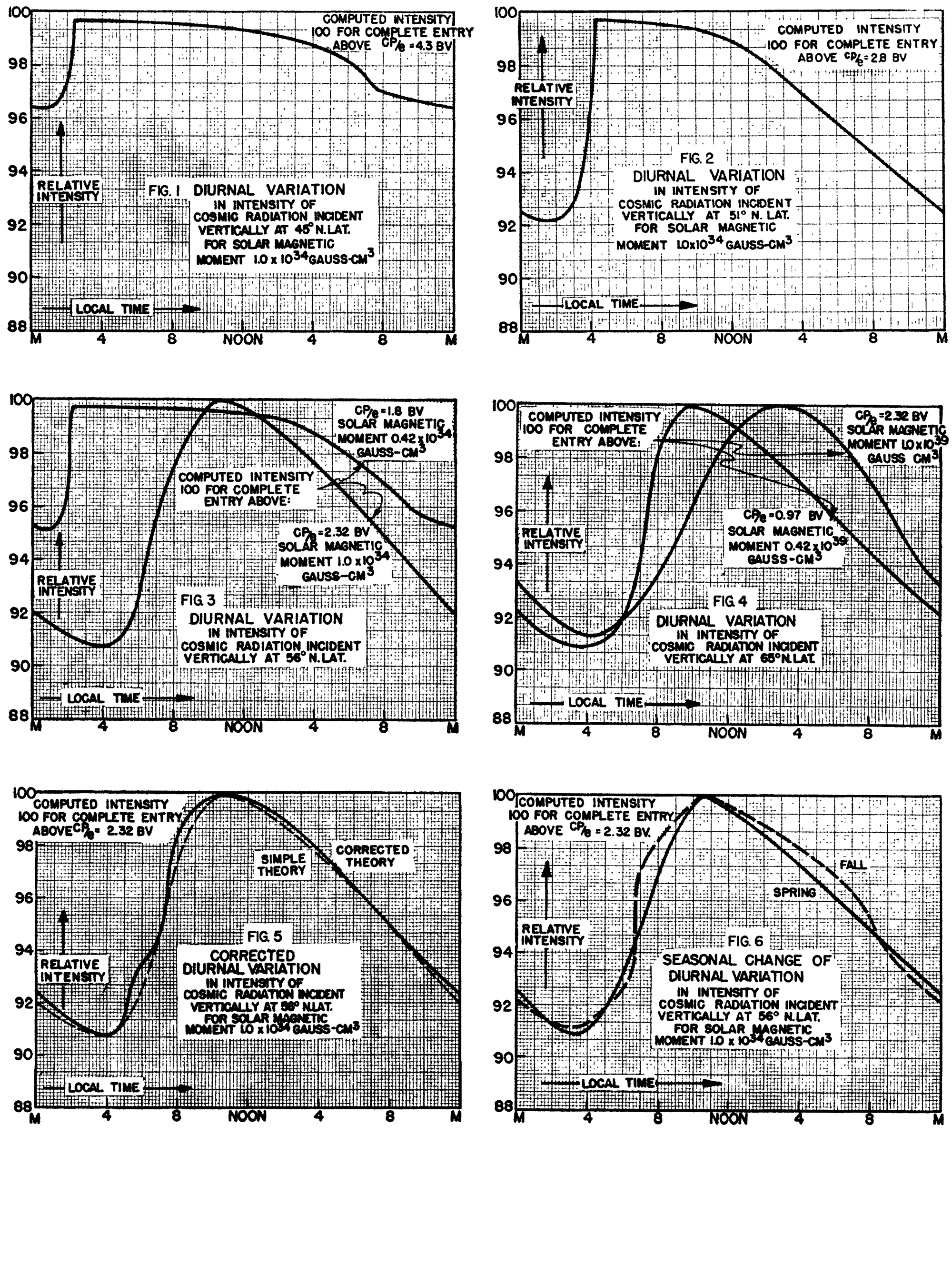
[6] L. Janossy, Zeits. f. Physik **104**, 430 (1937).
[7] P. S. Epstein, Phys. Rev. **53**, 862 (1938).
[8] M. Vallarta and O. Godart, Rev. Mod. Phys. **11**, 180 (1939).
[9] We are indebted to B. Rossi for a chance to see his unpublished calculations (1947).
[10] S. A. Korff, Rev. Mod. Phys. **11**, 211 (1939); V. Sarabhai and P. Nicolson, Proc. Phys. Soc. **60**, 509 (1948) give further references.
[11] The diurnal effect sought here has to be distinguished from the well known, and rather small, variations in sea-level cosmic-ray intensity which are associated with changes in the temperature of the atmosphere. Such changes receive a reasonable explanation in terms of changes in the height of the meson-producing layer and in the decay of mesons on their way from that elevation to the ground.
[12] Unpublished report from M.I.T. Laboratory.
[13] Malmfors, Arkiv. f. Mat. Astr. o. Fys. **30A**, No. 12 (1944) and **32**, No. 8 (1945).
[14] L. Janossy and P. Lockett, Proc. Roy. Soc. **A178**, 52 (1941); Lemaitre and M. Vallarta, Phys. Rev. **50**, 500 (1936); M. Vallarta, Nature **139**, 839 (1937); M. Vallarta, *Applied Mathematics Series* (Toronto Press, Toronto, 1938), No. 3.
[15] H. Alfvén, Phys. Rev. **72**, 88 (1947).
[16] Kane, Shanley, and Wheeler, Rev. Mod. Phys. **21**, 51–71 (1949).

of the earth has no effect, so that again there is no diurnal variation. We conclude that intermediate latitudes will be best for observing the effect.

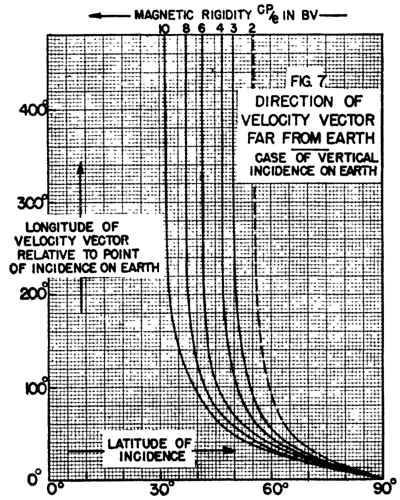
Sample calculated diurnal variations in vertical intensity are shown in Figs. 1–4. No allowance has been made for the difference between the magnetic axis of the earth and the geographic one, since the necessary correction is very special, having to be computed separately for incidence at different longitudes. This omission has only a slight effect upon the shape of the curve representing the diurnal variation, and leaves the phase and magnitude unaltered. The effect it does have is indicated in Fig. 5, where the corrected theory for incidence in the plane containing the two axes is compared with the approximate theory. Figure 6 shows the seasonal change. In view of the uncertainty about the existence or magnitude of the solar moment, it seemed hardly appropriate to allow for the supposed difference in orientation between the solar magnetic axis and axis of rotation.

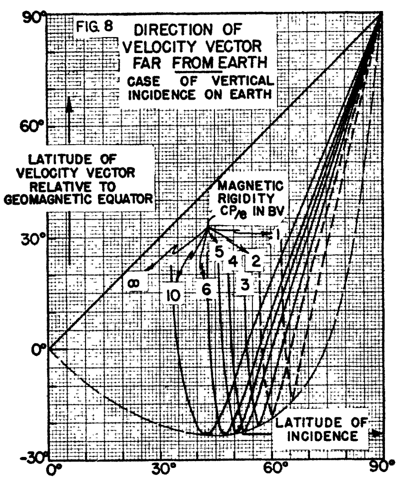
The relative intensity is based on a scale where 100 corresponds to the case of all energies above cut-off arriving at full intensity, in order that the magnitudes of the variations can be read directly as percentages.

The smaller value of the solar magnetic moment gives rise to virtually no diurnal variation at latitudes of observation of 45° and 51°, as is shown by qualitative reasoning in the next section. It is evident from the curves that both the magnitude and phase of the variation are sensitive functions of the solar moment. Consequently, there are excellent prospects that we can obtain information about the magnitude of this moment



FIGS. 1–6. Sample diurnal variations in total intensity, calculated for a differential cosmic-ray spectrum. $I \sim p^{-2.75}$.





FIGS. 7–8. Fundamental deflection curves, from which the diurnal variation in total intensity can be derived.

(if any) from successful observations of the diurnal variation in vertical intensity at high altitudes and appropriate latitudes.

To obtain the examples of the diurnal variation (Figs. 1–6) as deduced from our fundamental computations, we used for convenience a differential spectrum where the full intensity of radiation of a given energy was inversely proportional to the corresponding momentum raised to the 2.75 power. Theoretical studies of the means of accelerating cosmic-ray particles predict a power law form of spectrum, and the value used here is the best experimental value at the moment.[17] It was obtained for high energies by replacing the block diagrams of experimental observations by a smooth curve. This curve deviates from the blocks for energies less than 10 Bev, but the hypothesis of energy loss via neutrinos gives an excellent quantitative adjustment of the discrepancy down to, say, 5 Bev.[17] The experiment used averaged over all directions, and allowing for the exclusion of low energy particles from some directions would improve the agreement for still lower energies. Thus the experiment does not contradict the assumption of a power law for the primary radiation. However, it is not necessary to have an analytic form for the differential spectrum in order to deduce the diurnal variation from our computations. The procedure given in this paper can be applied straightforwardly using the integral spectrum given directly by experiment.

Because of our assumption of a power law, it is to be expected that determinations of the diurnal variation by experiments on particles subject to the experimental spectrum[17] will differ in a shape from our sample calculations. But the position of the maximum and sudden rise to it should remain unaltered. The principal difference will be a drastic decrease in the magnitude of the calculated variation. Thus a successful observation of the diurnal variation will indicate through its amplitude the nature of the energy spectrum, and through its phase, the magnitude of the solar magnetic moment. More generally, *high altitude observations of the diurnal changes in intensity of primary cosmic radiation are valuable for studying both the solar magnetic moment and the cosmic-ray energy spectrum.*

III. SUMMARY OF ANALYSIS

The fundamental result of our computations consists of two families of curves, plotted in Figs. 7 and 8. They give the two angles needed to describe the direction of motion at large distances from the earth for a particle which strikes the earth vertically with any energy at any latitude. From them the total diurnal variation in vertical intensity at any point on the earth's surface can be deduced to any desired degree of approximation for any type of particles, subject to any form of energy spectrum. The derivation of these fundamental curves and the process of deducing the diurnal variation from them are given in this section.

As a charged particle enters the solar system from outer space, it is deflected by the magnetic field of the sun. Then when it approaches near enough to the earth, it suffers further deflection. The problem of a charged particle acted upon simultaneously by two dipole magnetic fields is a very complex one. Fortunately we can make a simplification.

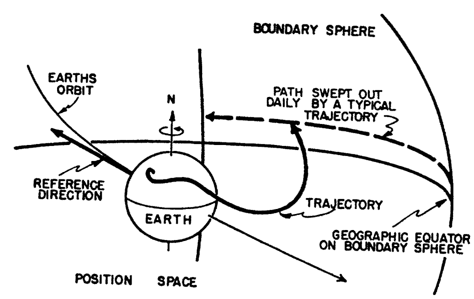


FIG. 9. The physical meaning of the integration. Diagram of position space (origin of diurnal variation).

[17] N. Hilberry, Phys. Rev. **60**, 7 (1941).

Consider the surface about the earth where the sun's magnetic field and that of the earth are equal. This surface can be approximated by a sphere whose radius is of the order of 22 earth-radii, which we shall call the *boundary sphere*. Since the earth's field falls off very rapidly with increasing distance, we can neglect its effect outside the boundary sphere. Conversely, its field increases rapidly as we decrease r, and so inside the sphere we can neglect the sun's field compared to that of the earth.

The problem of the intensity distribution has already been solved for the region outside the boundary sphere by Kane, Shanley, and Wheeler.[16] Therefore we have only to (A) determine the behavior of the particles inside the sphere, making use of the fact that here the earth may be considered an isolated system, and then (B) apply the existing calculations as boundary conditions at the surface of the sphere. We shall treat the case of positive particles.

A. Calculation of Asymptotic Velocity Vector

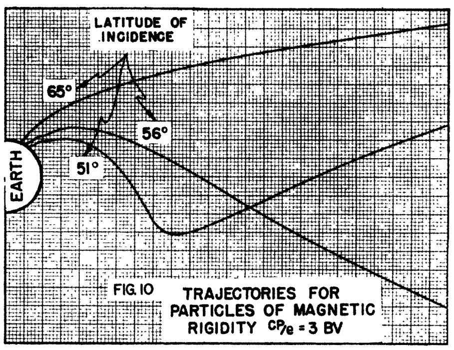
To find the paths followed by positive particles, we must integrate the differential equations of motion:[18]

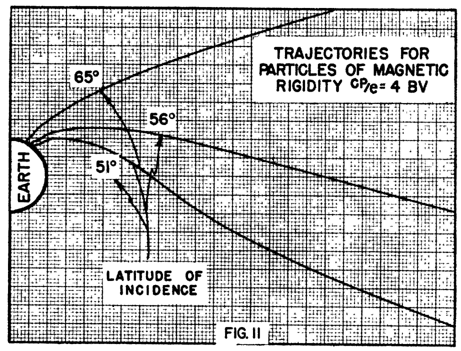
$$z''=\frac{1}{2}\frac{\partial}{\partial z}\left[1-\left(\frac{2g}{\rho}-\frac{\rho}{r^3}\right)^2\right], \tag{1}$$

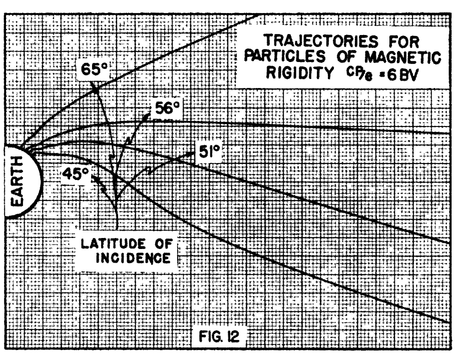
$$\rho''=\frac{1}{2}\frac{\partial}{\partial \rho}\left[1-\left(\frac{2g}{\rho}-\frac{\rho}{r^3}\right)^2\right], \tag{2}$$

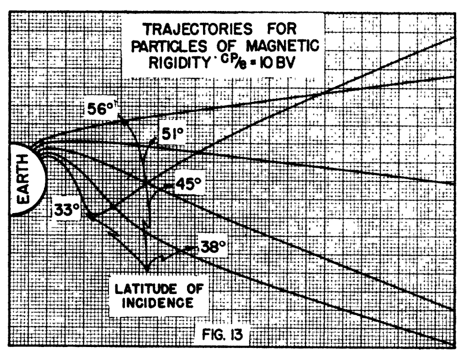
$$\rho^2\phi'=2g-\rho^2/r^3, \tag{3}$$

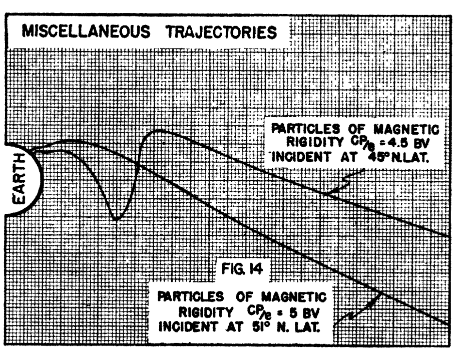
where primes represent differentiation with respect to arc length, r is the radial distance from the origin (center











FIGS. 10–14. The integrations. The scale used obscures the behavior of the trajectories near the earth.

[18] L. Janossy, *Cosmic Rays* (Oxford University Press, London, 1946).

of the dipole), ρ and z are cylindrical coordinates defining position in the meridian plane, ϕ is the coordinate of this plane, and g is the angular momentum of the particle at infinity with respect to the dipole, expressed in dimensionless units. However, we cannot obtain a solution directly, since we know only the final conditions (the point and direction of incidence). Therefore we replace the paths of the real particles by the identical trajectories of fictitious negatively charged particles ejected vertically from the earth.

We follow the paths of the negative particles by numerical integration of the second-order differential equations of motion out to the boundary sphere, using the method of central differences.[19] This process is equivalent to integration backward along the real orbits (see Fig. 9). The trajectories far from the earth are asymptotic to straight lines. Since the velocity is virtually constant in the general region of the boundary sphere, we call its value there the *asymptotic velocity vector*.

The simultaneous integrations[20] of Eqs. (1) and (2) for several conditions of incidence are summarized graphically in Figs. 10–14. These curves in ρz space are merely the projections on the meridian plane of the trajectories. The motion of the meridian plane was calculated from the curves by using Eq. (3) and then performing a first order numerical integration.[19]

To determine the asymptotic velocity vector is the next step. The three direction cosines of a trajectory are simply z', ρ', and $\rho\phi'$, and the meridian plane has moved through an angle ϕ. All these quantities either appear directly in the integration, or can be found by means of the formula for differentiation given by Hartree.[19]

Consider the unit geomagnetic sphere in velocity space. The latitude β of the asymptotic velocity, being measured from the equator, will be given by:

$$\beta = \text{arc } \sin z'. \tag{4}$$

If we take the longitude of the initial, or terrestrial, velocity vector to be 0°, then the longitude ψ of the asymptotic velocity relative to it will be just:

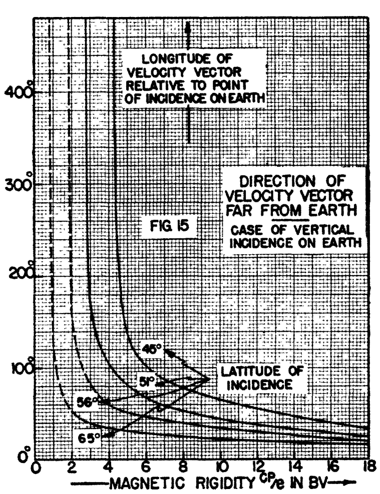
$$\psi = \phi + \text{arc } \tan \rho\phi'/\rho'. \tag{5}$$

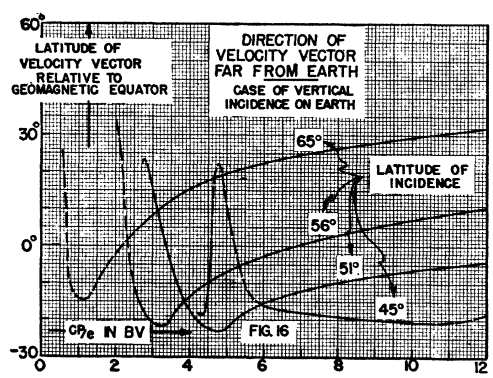
Our computations yield the values given in Table I for the latitudes and longitudes of the asymptotic vector on the geomagnetic sphere.

Figures 15 and 16, derived from Table I, show the dependence of the asymptotic velocity upon the mag-

TABLE I. Longitude ψ and latitude β of the asymptotic velocity vector.

Magnetic rigidity cp/e	Latitude of incidence	Velocity longitude ψ	Velocity latitude β
3 Bv	51°	175.0°	16.0°
	56°	82.6°	−20.9°
	65°	37.7°	8.0°
4	51°	100.0°	−17.1°
	56°	59.5°	−13.6°
	65°	34.2°	15.5°
4.5	45°	268.0°	−13.7°
5	51°	82.6°	−22.4°
6	45°	101.5°	−16.1°
	51°	66.4°	−15.3°
	56°	47.9°	−2.9°
	65°	33.0°	22.2°
8	45°	75.6°	−19.0°
10	33°	185.0°	20.4°
	38°	115.5°	−15.7°
	45°	65.8°	−21.0°
	51°	45.0°	−6.6°
	56°	35.3°	7.3°





FIGS. 15–16. Deflection curves used to calculate the diurnal variation in total intensity at a given latitude of incidence.

[19] D. R. Hartree, Manchester Phil. Soc. Mem. **77**, 91 (1933). To extend his method to two simultaneous equations, merely estimate both ρ_{n+1} and z_{n+1}, compute ρ_{n+1}'' and z_{n+1}'' from these estimates, and apply his process separately to ρ_n and z_n in order to complete the step in the integration. Iterate. It takes from 15 to 20 hours work on a hand computing machine to accomplish an integration.

[20] K. Dwight, senior thesis 1949, Princeton University, gives greater detail.

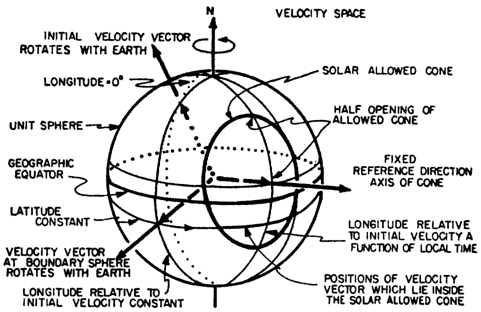


FIG. 17. The physical picture in velocity space. Diagram of velocity space (origin of diurnal variation).

netic rigidity of the particles for incidence at four different latitudes.

To find the position on the unit sphere of the asymptotic velocity for particles incident vertically at any point on the earth, it is convenient and practical to replot our values in the form of functions of latitude of incidence for various magnetic rigidities. Thus we have the fundamental families of deflection curves (see Figs. 7 and 8). By taking the intersections of these fundamental curves with lines of constant latitude, we can derive curves similar to those of Figs. 15 and 16 for any latitude of incidence.

The importance of the fundamental curves is due to their generality. First, they apply to particles of any mass. Second, to use them for the case of positive particles reaching the Southern Hemisphere, it is only necessary to reverse the signs of the latitudes—of incidence and of the asymptotic velocity vector. And finally, they apply to negative particles when the signs of all the angles are reversed. Thus we can find the asymptotic velocity vector of any type of particle received vertically anywhere on the earth.[20]

B. Derivation of Diurnal Variation from the Asymptotic Velocity

The solar allowed cone is one of directions, and not of positions, for the particle. It does not exist in position space, but only in velocity space, centering about the tangent to the earth's orbit, and opening out backward along it. We shall take this sense of the tangent for our *reference direction* (see Fig. 9). Therefore, on the unit sphere in velocity space, the allowed cone is represented by a circle drawn around that point which defines the reference direction (see Fig. 17). The opening of the cone is shown for two values of the solar moment in Fig. 18.[16]

The asymptotic velocity vector is fixed relative to the earth, and it is the rotation of the earth which produces a diurnal variation in intensity (see Fig. 9). Therefore we must consider the geographic unit sphere in velocity space, rather than the geomagnetic one, in order to compute the variation (see Fig. 17). We know the terrestrial velocity vector and the asymptotic velocity vector. These two vectors rotate together, with the earth, at the rate of 15° per hour, while the reference direction and allowed cone remain fixed in velocity space (see Fig. 17). As the velocity vectors rotate, their latitudes on the geographic sphere stay constant. Thus lines of constant latitude are the time-loci of the positions of the velocity vectors. In general, the time-locus of the asymptotic velocity will intersect the allowed cone, and so will determine the diurnal variation for particles with that given magnetic rigidity (see Fig. 17).

We can calculate just what this variation will be, breaking the process up into two parts. First we need to know the local time of day when the particles are reaching the earth from the center of their range of fully allowed directions. The time so defined will be called the *time-center* of the variation. Now the center of the intersection of the time-locus of the asymptotic velocity vector with the fully allowed cone has the same longitude as the reference direction (see Fig. 17). We consider this line of longitude to define a *reference time*, which is just 6 P.M. in local solar time.

The equation for the geographic longitude ψ^* shows that it can differ from the known geomagnetic longitude ψ by at most 0.6°,[20] which is completely negligible, and so we take $\psi^*=\psi$. This is the longitude of the asymptotic velocity relative to the terrestrial value, so that

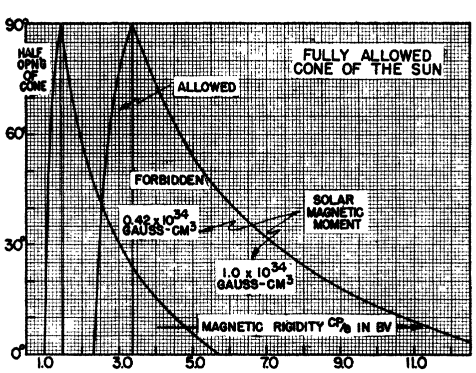


FIG. 18. The first graphical step in the derivation of the diurnal variation from the deflection curves. These curves were obtained from the table given in reference 16.

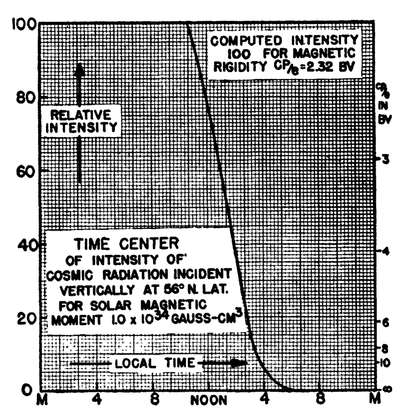


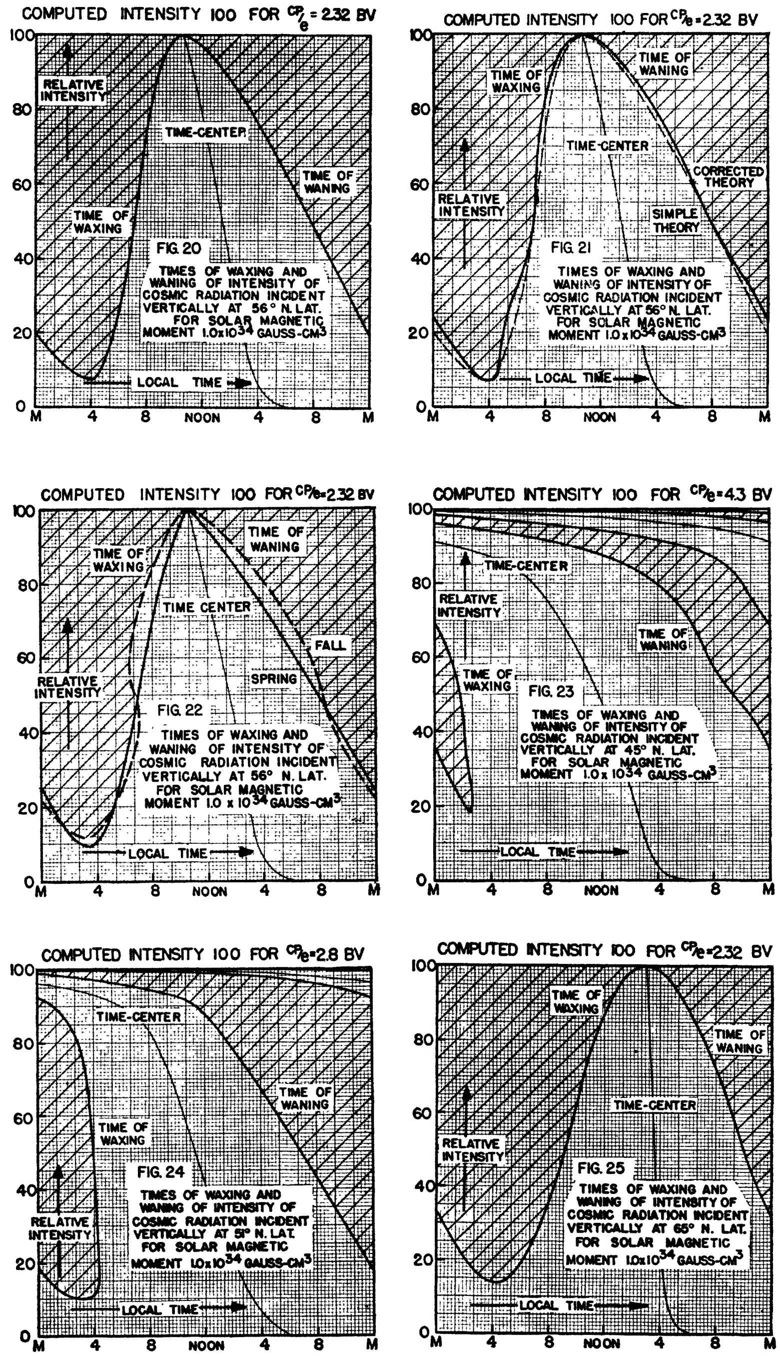
FIG. 19. The second graphical step. This graph illustrates the use of the integral spectrum.

the latter lags in time behind the former by $\psi^*/15=\psi/15$ hours. The lag is merely subtracted from the reference time in order to calculate the time-center:

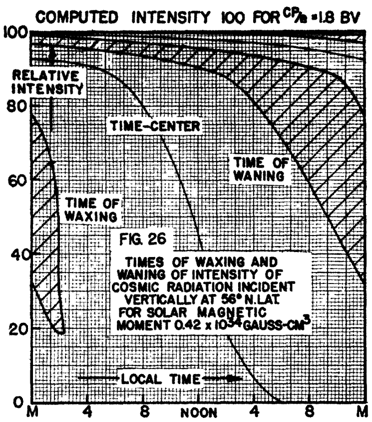
$$T=6 \text{ P.M.}-\psi/15 \text{ hours.} \tag{6}$$

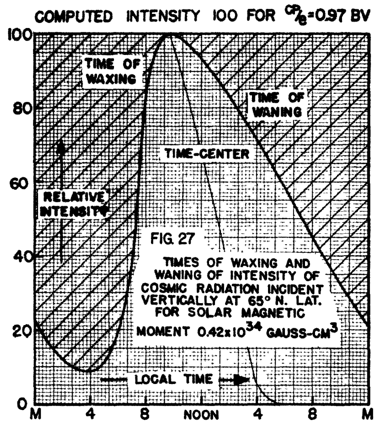
Since we will eventually want the total diurnal variation, it is more convenient to plot the time-center as a function of integrated intensity than as one of magnetic rigidity cp/e. The one is just as good a variable as the other, since we know from experiment the total relative intensity for all radiation with energy (momentum) greater than any given value. In Fig. 19 the time-center is plotted as a function of this integral spectrum for radiation incident at 56° N latitude. The corresponding magnetic rigidities are plotted along the right-hand edge of Fig. 19, based on the assumption of an inverse 2.75 power law for the differential spectrum relating intensity to momentum.

For the second part of the calculation of the variation, we must determine γ, the difference in geographic



FIGS. 20–25. Samples of the third graphical step in the derivation of the diurnal variation from the deflection curves.





FIGS. 26–27. Further examples of the third graphical step.

longitude between the center of the intersection of the time-locus of the asymptotic velocity with the allowed cone and the edge of this intersection (see Fig. 17). From solid geometry we obtain:[20]

$$\sin^2\gamma = \sec^2\beta^*(\sin^2\alpha - \sin^2\beta^* + 2\cos\alpha\sin\beta^*\sin\zeta - \cos^2\alpha\sin^2\zeta), \quad (7)$$

where α is the half-opening of the solar allowed cone (see Fig. 18), ζ is the geographic latitude of the reference direction, and β^* is the geographic latitude of the asymptotic velocity vector. If we define η, the angle in the plane of the ecliptic between the line of its intersection with the earth's geographic equator and the reference direction, in such a manner that $\eta=0$ when the earth is nearest the sun and increases during the year, then further geometry yields:[20]

$$\sin\zeta = 0.391\sin\eta. \quad (8)$$

Also, if μ is the angle between the magnetic axis of the earth and its axis of rotation, and λ is the geomagnetic longitude of incidence, then:[20]

$$\sin\beta^* = \cos\mu\sin\beta - \cos\beta\sin\mu\cos(\psi+\lambda). \quad (9)$$

Thus γ can be calculated from the fundamental deflection curves, the point of incidence, and the time of year.

Since the velocity vectors rotate at 15° per hour, the angle γ is equivalent to a time semiduration of the variation given by $\tau=\gamma/15$ hours. The *time of waxing* and *time of waning* are then computed by subtracting and adding, respectively, this semiduration of full intensity to the time-center. They represent the edges of the intersection of the time-locus with the allowed cone, are expressed in local solar time, and are functions of the energy of the radiation and the latitude of incidence.

The calculated times of waxing and waning are plotted in Fig. 20 as functions of intensity for radiation received at 56°N latitude. The shaded region in this graph indicates radiation which is "forbidden."

In Fig. 21 the times of waxing and waning obtained by the corrected theory (using β^*) for geomagnetic longitude of incidence $\lambda=0°$ [20] are compared with the results of the simple calculation (where β^* is replaced by β). All the significant properties remain unaltered (phase and magnitude). Therefore we shall henceforth make the simplifying substitution of β for β^* in our computations.

Equation (7) also indicates a dependence of the variation upon the season. We see that the calculations shown in Fig. 20 are valid for both the summer and winter solstices. The corresponding curves for the spring and fall equinoxes are shown in Fig. 22. Evidently their significant properties are not affected by the season.

Simple theory calculations for other latitudes of incidence are plotted in Figs. 23–25.

The computations given above were all for a solar magnetic moment of 1.0×10^{34} gauss-cm³. We can equally well apply the process of analysis for a moment of, say, 0.42×10^{34} gauss-cm³. These results are shown in Figs. 26 and 27 for radiation incident at 56° and 65°, respectively. In the former case there is a very important change in the shape of the curves of waxing and waning, one which alters the diurnal variation of total intensity in a most marked manner (see Fig. 3). When the cut-off values of the magnetic rigidity for incidence at 51° and 45° are compared with the openings of the allowed cone (see Fig. 18), it becomes clear that the lesser solar moment produces virtually no diurnal variation at these two latitudes.

Finally, we use the times of waxing and waning to determine the range of intensities (or energies) which correspond to "forbidden" radiation. This radiation, indicated by the shaded regions of the graphs, is assumed to arrive at 90 percent of its full intensity, to a first approximation. We remember that this intensity results from the scattering of particles into solar "forbidden" orbits,[16] which otherwise would contain no particles whatever. The percentile intensity in these orbits is actually a function of the energy of the particles. But because of the doubt concerning the existence of any permanent solar magnetic moment, there is no point in our attempting higher degrees of approximation at this stage. Therefore we take the average loss of intensity, 10 percent, to apply to radiation of any energy.

Hence, in order to compute the drop in total intensity for each moment of the day, we need only take 10 percent of the range of "forbidden" intensities given by the curves of waxing and waning for that instant of local solar time. This total drop is next subtracted frcm the integral intensity corresponding to the cut-off momentum, which is taken to be 100 on the scale of relative intensity. The intensities obtained by the subtraction are then plotted as functions of local time, thus providing us with the final descriptions of the diurnal variation in total intensity (see Figs. 1–6).

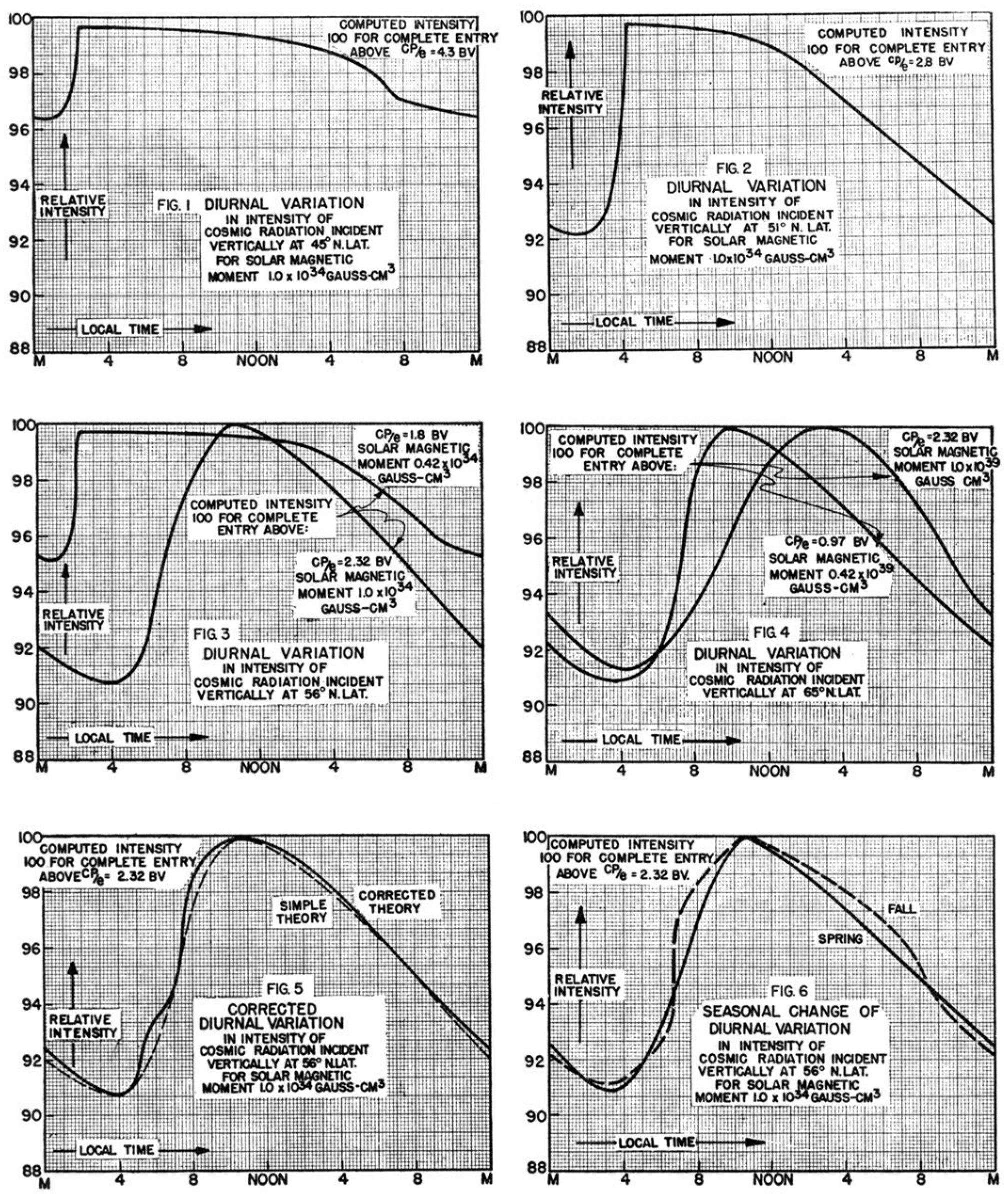
IV. DISCUSSION

It is instructive to compare the shapes and characteristics of Figs. 20–27 with the corresponding curves of total diurnal variation (see Figs. 1–6). Particular interest attaches to the position of the maximum, and to the region of the sudden rise to it. We see that at low latitudes of incidence the time-centers are so spread out by the strong terrestrial magnetic field that they tend to level off the variation in total intensity. As we pass to higher latitudes, the earth's field becomes weaker, so that particles of different momenta are dispersed less. Therefore the time-centers do not spread out so greatly, and consequently the amplitude of the variation increases. In this region of latitude, the time-centers and times of waxing and waning vary with intensity in such a manner that at a certain time of day there is an extremely wide range of momenta which change intensity at practically the same instant. This produces the sudden increase in intensity seen in our final curves.

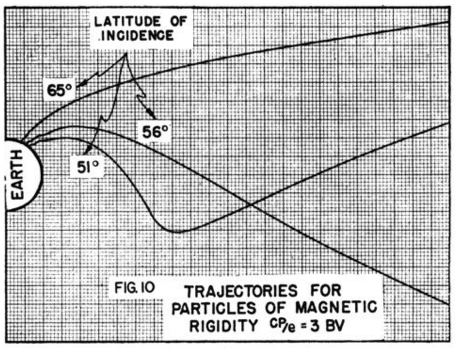
At still higher latitudes of incidence, the time-center curve straightens out so much that the curve for the time or waxing starts to round off, and this effect is reflected in the final results. As we continue to increase the latitude of observation, we see from Fig. 8 that the asymptotic velocity latitude will keep increasing for rigidities in the significant range $2.5 < cp/e < 6$ Bv, so that eventually, somewhere above 65° N latitude, the decreasing intersections with the solar allowed cone (see Fig. 17) will diminish the semidurations of full intensity. As a result, the amplitude of the total variation will start to decrease. The range of magnetic rigidities subject to changes in intensity becomes smaller and smaller, centering about 3.4 Bev. Finally, at the poles there is no variation in vertical intensity.

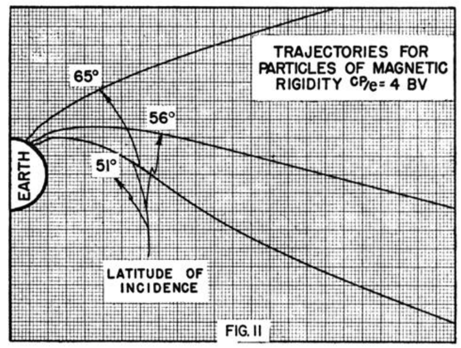
Thus we have a qualitative picture of the behavior of the diurnal variation in total intensity as a function of latitude of observation. By applying a process of interpretation to our fundamental curves giving the deflection due to the earth's magnetic field, we can make the description quantitative. By using better approximations for the individual variations and for the shape of the energy spectrum, we can derive more accurate descriptions whenever required. These descriptions provide us with a very powerful tool for testing the existence of a permanent solar magnetic moment and the nature of the spectrum, and therefore should prove to be quite valuable.

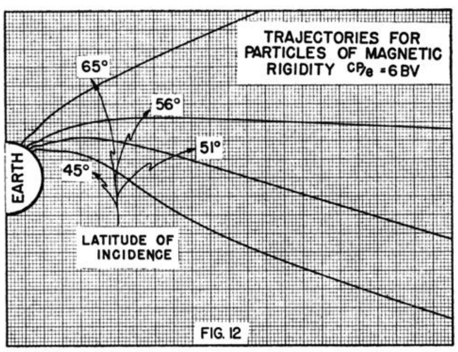
I wish to express my thanks to Mr. J. A. Wheeler, Ph.D., for his many invaluable suggestions, and to Messrs. W. G. Stroud and T. J. B. Shanley for helpful discussions.

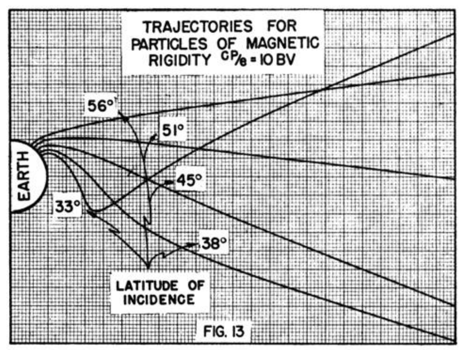


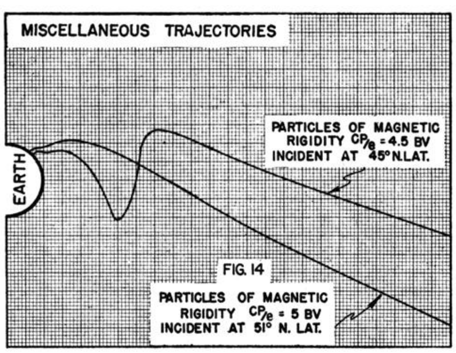
FIGS. 1–6. Sample diurnal variations in total intensity, calculated for a differential cosmic-ray spectrum. $I \sim p^{-2.75}$.



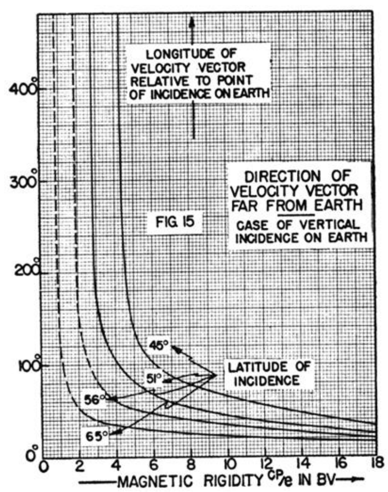


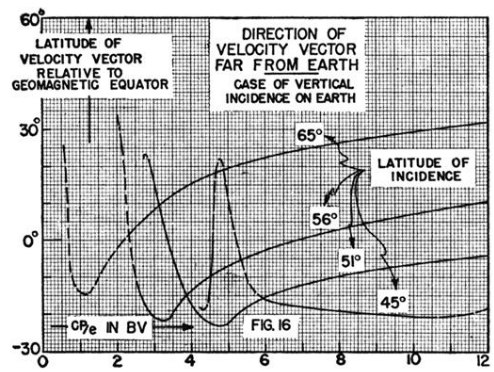






FIGS. 10–14. The integrations. The scale used obscures the behavior of the trajectories near the earth.





FIGS. 15–16. Deflection curves used to calculate the diurnal variation in total intensity at a given latitude of incidence.

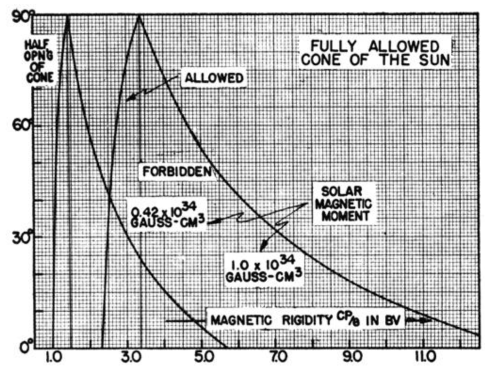


FIG. 18. The first graphical step in the derivation of the diurnal variation from the deflection curves. These curves were obtained from the table given in reference 16.

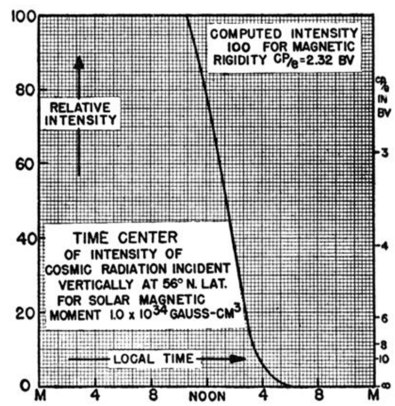
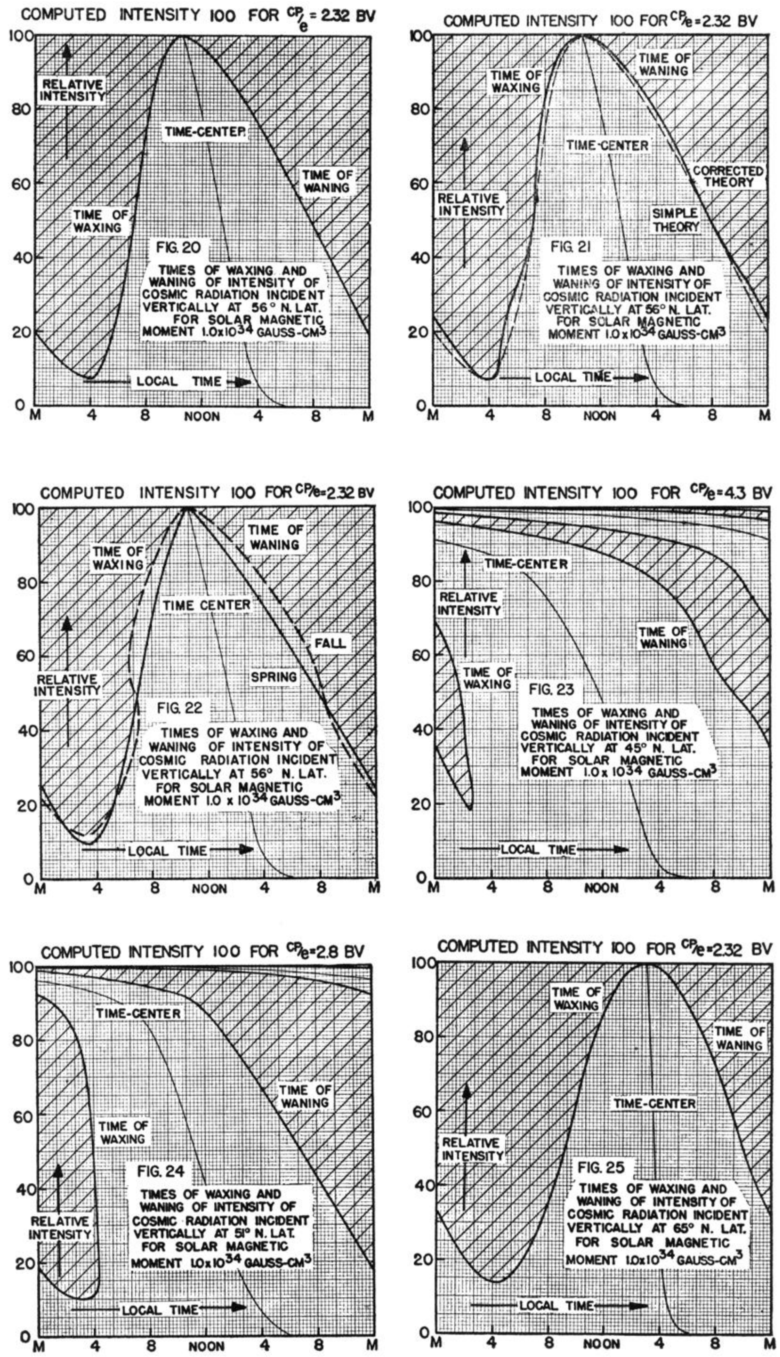
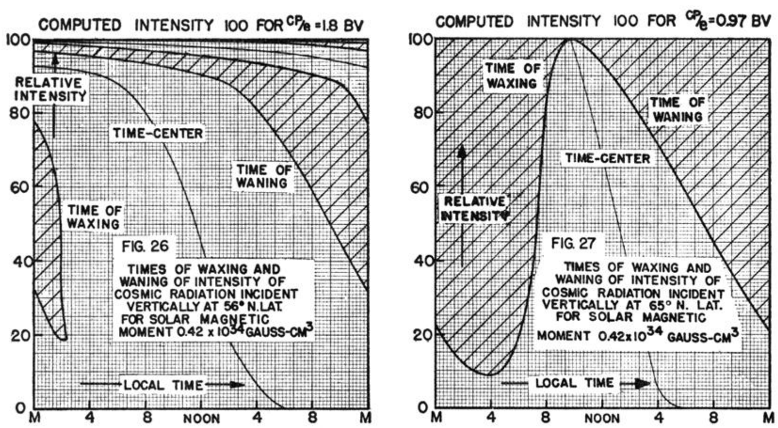


FIG. 19. The second graphical step. This graph illustrates the use of the integral spectrum.



FIGS. 20–25. Samples of the third graphical step in the derivation of the diurnal variation from the deflection curves.



FIGS. 26–27. Further examples of the third graphical step.

FIGS. 7–8. Fundamental deflection curves, from which the diurnal variation in total intensity can be derived.