expansion of the field, since the particle is actually spread out over a finite region in the space of its *mean-position variable*. From this point of view the explicit appearance of the magnetic-moment interaction and the accompanying spin-orbit interaction in the Hamiltonian is to be expected. In fact, the term in div**E**, which has previously been regarded as of rather mysterious origin, can now also easily be understood since it comes from the fact that the electric charge in the new representation is also spread out over a finite region. In a static potential, the particle then moves according to a suitable average of the potential over this region. But in lowest order such an averaging process is known to lead to a term proportional to the Laplacian of the potential and this is just the character of the term in div**E**.

(In employing a finite number of terms in a Hamiltonian such as (35) it must be remembered that wave functions, transition matrix elements, and expectation values of operators computed from the Hamiltonian are only correct to terms of the order in $(1/m)$ to which terms in the Hamiltonian are retained. Thus in the case of (36) it would not be consistent to retain only terms of order $(1/m)$ in the Hamiltonian and then to employ terms of this order in second-order perturbation theory where they generate terms of order $(1/m)^2$.)

The method employed above for reducing the Hamiltonian to non-relativistic two-component form for the case of interaction with an external electromagnetic field can be employed generally for interaction of a Dirac particle with any type of external field, such as various types of meson fields. With meson fields one obtains, in different cases, various types of spin interaction with the meson field and also various types of spin-orbit coupling terms such as have been employed in discussions of spin-orbit coupling in nuclei. The discussion of the reduction to non-relativistic form in the case of the many-particle theory is left for a later communication.

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The Nucleon Magnetic Moment in Meson Pair Theories

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The contribution to the nucleon magnetic moment from an interaction of the nucleon with a spinor or scalar meson pair field is calculated. In both cases it is found to be logarithmically divergent.

THE covariant formulation of the pseudoscalar meson theory[1] together with the concepts of mass and charge renormalization[2] were applied by Case[3] to the computation of the anomalous magnetic moment of nucleons. He showed that finite results are then obtained. This is essentially due to the fact that one is now able to isolate and incorporate[2] the divergences into the mass and charge of the nucleon field thus leaving a convergent expression for the magnetic moment. This separation of the reactive terms represents an improvement over the previous treatment of this and related problems.[4] On the other hand, divergence difficulties are still encountered in the magnetic moment calculation based on the vector meson theory with tensor coupling.[5]

It is the aim of this note to report on the application of the renormalization program to the computation of nucleon magnetic moments by assuming a meson pair interaction (containing no derivatives of the fields) between the heavy particles. Two cases have been considered for the meson field, namely, the scalar (or pseudoscalar, which here amounts to the same) and the

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[1] S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3**, 1 (1948); Y. Myiamoto, Prog. Theor. Phys. **3**, 124 (1948).

[2] See F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949).

[3] K. M. Case, Phys. Rev. **76**, 1 (1949); compare also J. M. Luttinger, Helv. Phys. Acta **21**, 483 (1948); M. Slotnick and W. Heitler, Phys. Rev. **75**, 1645 (1949); S. D. Drell, Phys. Rev. **76**, 427 (1949).

[4] Finite results for the nucleon magnetic moment were previously obtained by J. M. Jauch (Phys. Rev. **63**, 334 (1943)) in the conventional theory by using the lambda-limiting process; the nucleon field was treated non-relativistically.

[5] K. M. Case, Phys. Rev. **75**, 1440 (1949).

spinor one. The assumption that interactions of the type here investigated might partially contribute to the magnetic moment of nucleons is not without physical interest; indeed, the present stage of the theory of beta-decay as well as of the nuclear capture of negative mu-mesons does not seem to exclude[6] such couplings as are dealt with in this paper.

The result obtained from an interaction of the nucleon field with a spinor or a (bilinearly coupled) scalar meson field is that, even after renormalization is effected, the magnetic moment still diverges. This is not unexpected. It is a well known fact[7] that the meson pair theories give rise to nucleonic interactions strongly singular at small distances and it is reasonable to expect from these theories results more singular than those from the linear theories.

1. Let us first consider the case of a pair interaction of the nucleon field ψ with a scalar meson field ϕ_α. We shall use the four-dimensional isotopic spin formalism introduced by Case.[3] ϕ_α is a set of four real scalar fields; ϕ_1 and ϕ_2 describe each positively and negatively charged mesons; ϕ_3 describes the neutral mesons which are superposed to the charged ones in a symmetrical theory; ϕ_4 is another neutral meson field. The first three components of the isotopic spin τ_α are the conventional matrices τ_1, τ_2, τ_3, the eigenvalue $+1$ of τ_3 corresponding to a neutron, -1 to a proton; τ_4 is the unity operator in isotopic spin space.

In the interaction representation, the nucleon and scalar fields obey the uncoupled equations of motion and satisfy the following commutation rules (in units $\hbar$ and c):

$$\begin{aligned} &[\phi_\alpha(x), \phi_\beta(x')] = i\delta_{\alpha\beta}\Delta(x-x'); \\ &\{\psi_\mu(x), \bar{\psi}_\nu(x')\} = -iS_{\mu\nu}(x-x'); \\ &\{\psi_\mu(x), \psi_\nu(x')\} = \{\bar{\psi}_\mu(x), \bar{\psi}_\nu(x')\} = 0. \end{aligned} \tag{1}$$

The index μ corresponds to both spin and isotopic spin components; also:*

$$S(x) = S_P(x)\tau_P + S_N(x)\tau_N, \tag{2}$$

$$\tau_P = \tfrac{1}{2}(1-\tau_3); \quad \tau_N = \tfrac{1}{2}(1+\tau_3). \tag{3}$$

The interaction hamiltonian density for the system meson-nucleon field in an external electromagnetic field A_μ is the following:

$$H = H' + H_1^{\text{ext}} + H_2^{\text{ext}}, \tag{4}$$

$$H' = \tfrac{1}{2} f_{\alpha\alpha} \bar{\psi}\psi\phi_\alpha^2 + f_{i4}\bar{\psi}\tau_i\psi\phi_i\phi_4 \tag{5}$$

$$H_1^{\text{ext}} = -ie\bar{\psi}\gamma_\mu\tau_P\psi A_\mu, \tag{6}$$

$$\begin{aligned} H_2^{\text{ext}} = &-eA_\mu\{\phi_1(\partial\phi_2/\partial x_\mu) - \phi_2(\partial\phi_1/\partial x_\mu)\} \\ &+ (e^2/2)\{A_\mu^2 + (n_\lambda A_\lambda)^2\}(\phi_1^2 + \phi_2^2). \end{aligned} \tag{7}$$

[6] J. Tiomno and J. A. Wheeler, Rev. Mod. Phys. 21, 144, 153 (1949).

[7] See S. Hjalmars, Arkiv. f. Fysik 1, No. 3 (1949); O. Brulin, Arkiv. f. Fysik 1, No. 4, 5 (1949), and references therein.

* The functions Δ and S_P are defined as in J. Schwinger, Phys. Rev. 74, 1439 (1948) and refer to particles with mass κ and M_P, respectively.

H' is the interaction of the nucleon with the pair field. The f's are the coupling constants; the summation over α runs from 1 to 4, over i from 1 to 3. H_1^{ext} and H_2^{ext} describe the interaction of the nucleon and meson currents, respectively, with the external electromagnetic field. The charge conjugated expressions have been omitted from (4) which is always permissible provided that one replaces the vacuum expectation value of expressions of the type $\bar{\psi}(x)\psi(x)$ by zero.

The Hamiltonian (5) can be expressed in the following compact form:

$$H' = \tfrac{1}{2} f_{\alpha\beta}\bar{\psi}\tau_\alpha\tau_\beta\psi\phi_\alpha\phi_\beta, \tag{8}$$

where the summation over α and β runs from 1 to 4; $f_{\alpha\beta} = f_{\beta\alpha}$. Gauge invariance requires further that $f_{11} = f_{22}$; $f_{14} = f_{24}$. The several possibilities regarding the charge of a meson pair are specified by the following choice of coupling constants:

Charged pair theory:

$$f_{14} = f_{24} \neq 0; \quad \text{the other } f\text{'s} = 0.$$

Symmetrical pair theory:

$$\begin{gathered} f_{14} = f_{24} \neq 0; \quad f_{11} = f_{22} \neq 0; \quad f_{33} \neq 0; \\ \text{the other } f\text{'s} = 0. \end{gathered} \tag{9}$$

Neutral pair theory:

$$f_{11} = f_{22} \neq 0; \quad f_{44} \neq 0; \quad \text{other } f\text{'s} = 0.$$

The effective interaction Hamiltonian from which the nucleon magnetic moment can be evaluated in the order ef^2 is:

$$H_{\text{eff}} = H_1 + H_2, \tag{10}$$

where:

$$\begin{aligned} H_1 = \frac{(-i)^2}{2} \int\int dx dy P\{ &-ie\bar{\psi}(x_0)\gamma_\mu\tau_P\psi(x_0)A_\mu(x_0), \\ &\tfrac{1}{2} f_{\alpha\beta}\bar{\psi}(x)\tau_\alpha\tau_\beta\psi(x)\phi_\alpha(x)\phi_\beta(x), \\ &\tfrac{1}{2} f_{\gamma\epsilon}\bar{\psi}(y)\tau_\gamma\tau_\epsilon\psi(y)\phi_\gamma(y)\phi_\epsilon(y)\}; \end{aligned} \tag{11}$$

$$\begin{aligned} H_2 = \frac{(-i)^2}{2} \int\int dx dy P \\ \times \Bigg\{ &-eA_\mu(x_0)\left[\phi_1(x_0)\frac{\partial\phi_2(x_0)}{\partial x_\mu^0} - \phi_2(x_0)\frac{\partial\phi_1(x_0)}{\partial x_\mu^0}\right] \\ &\tfrac{1}{2} f_{\alpha\beta}\bar{\psi}(x)\tau_\alpha\tau_\beta\psi(x)\phi_\alpha(x)\phi_\beta(x), \\ &\tfrac{1}{2} f_{\gamma\epsilon}\bar{\psi}(y)\tau_\gamma\tau_\epsilon\psi(y)\phi_\gamma(y)\phi_\epsilon(y)\Bigg\}; \end{aligned} \tag{12}$$

the integrations are over the four-dimensional space and P is the chronological ordering operator introduced by Dyson.[2]

We need to evaluate the expectation value of (10) for the state defined by the meson vacuum and one nucleon present in the field. The meson vacuum expectation values of the P-brackets occurring in (11) and (12) are:

$$\langle P\{\phi_\alpha(x)\phi_\beta(x),\,\phi_\gamma(y)\phi_\epsilon(y)\}\rangle_0 = \tfrac{1}{4}(\delta_{\alpha\gamma}\delta_{\beta\epsilon}+\delta_{\alpha\epsilon}\delta_{\beta\gamma})\Delta_F{}^2(x-y);$$

$$\left\langle P\left\{\phi_1(x_0)\frac{\partial\phi_2(x_0)}{\partial x_\mu{}^0}-\phi_2(x_0)\frac{\partial\phi_1(x_0)}{\partial x_\mu{}^0},\ \phi_\alpha(x)\phi_\beta(x),\,\phi_\gamma(y)\phi_\epsilon(y)\right\}\right\rangle_0$$
$$=\tfrac{1}{8}\{(\delta_{1\alpha}\delta_{2\gamma}-\delta_{1\gamma}\delta_{2\alpha})\delta_{\beta\epsilon}+(\delta_{1\alpha}\delta_{2\epsilon}-\delta_{1\epsilon}\delta_{2\alpha})\delta_{\beta\gamma}+(\delta_{1\beta}\delta_{2\gamma}-\delta_{1\gamma}\delta_{2\beta})\delta_{\alpha\epsilon}+(\delta_{1\beta}\delta_{2\epsilon}-\delta_{1\epsilon}\delta_{2\beta})\delta_{\alpha\gamma}\}$$
$$\times\left\{\Delta_F(x_0-x)\frac{\partial\Delta_F(x_0-y)}{\partial x_\mu{}^0}-\Delta_F(x_0-y)\frac{\partial\Delta_F(x_0-x)}{\partial x_\mu{}^0}\right\}\Delta_F(x-y);$$

$\Delta_F(x)$ is the Feynman function.**

The one-nucleon expectation value of the P-brackets containing ψ, $\bar{\psi}$ can be evaluated along the lines indicated by Dyson[2] and Case.[3] The expressions thus obtained give the following expectation value for (10):

$$\langle H_{\text{eff}}\rangle = H_1{}^a+H_1{}^b+\langle H_2\rangle,$$

where:

$$H_1{}^a=-\frac{ie}{64}f_{\alpha\beta}f_{\gamma\epsilon}A_\mu\int\int dxdy\,\text{Trace}\times\{S_F(x_0-x)\gamma_\mu\tau_PS_F(x-x_0)\tau_\alpha\tau_\beta\}\times\{\delta_{\alpha\gamma}\delta_{\beta\epsilon}+\delta_{\alpha\epsilon}\delta_{\beta\gamma}\}\Delta_F{}^2(x-y)\bar{\psi}(y)\tau_\gamma\tau_\epsilon\psi(y)\quad(13)$$

describes the vacuum polarization by meson pairs emitted by the nucleon and vanishes on account of:***

$$\text{Trace}\{S_F(x_0-x)\gamma_\mu\tau_PS_F(x-x_0)\tau_\alpha\tau_\beta\}=0.\quad(14)$$

Furthermore:

$$H_1{}^b=\frac{ie}{64}f_{\alpha\beta}{}^2A_\mu\int\int dxdy\bar{\psi}(x)\tau_\alpha\tau_\beta S_F(x_0-x)\times\gamma_\mu\tau_PS_F(y-x_0)(\tau_\alpha\tau_\beta+\tau_\beta\tau_\alpha)\psi(y)\Delta_F{}^2(x-y)\quad(15)$$

is the analog of the Lamb-shift in the electromagnetic problem. Finally:

$$\langle H_2\rangle=\frac{ie}{8}f_{14}f_{24}A_\mu\int\int dxdy\bar{\psi}(x)\tau_3S_F(y-x)\psi(y)\times\left\{\Delta_F(x_0-x)\frac{\partial\Delta_F(x_0-y)}{\partial x_\mu{}^0}-\Delta_F(x_0-y)\frac{\partial\Delta_F(x_0-x)}{\partial x_\mu{}^0}\right\}\Delta_F(x-y)\quad(16)$$

corresponds to the interaction of the meson current with the external electromagnetic field; $\langle H_2\rangle$ has opposite signs for neutron and proton and vanishes in a neutral pair theory (see relations (9) above).

Both (15) and (16) give a logarithmically divergent contribution to the magnetic moment. The divergence comes from the occurrence of the extra factor $\Delta_F(x-y)$ in the integrand as compared to the expression of the linear theory; the number of Δ_F-functions in (15) and (16) corresponds physically to the emission and reabsorption of a meson pair by the nucleon. The divergent character of $H_1{}^b$ and $\langle H_2\rangle$ can best be seen by transforming them into momentum space. $H_1{}^b$ is found to be proportional to the following integral:

$$I=A_\mu\int\int d^4kd^4k'\frac{\bar{\psi}(x_0)\tau_\alpha\tau_\beta\{i\gamma_\lambda(P_\lambda'-k_\lambda-k_\lambda')-M\}\gamma_\mu\tau_P\{i\gamma_\nu(P_\nu'-k_\nu-k_\nu')-M\}\tau_{\alpha\beta}\psi(x_0)}{\{(P_\lambda'-k_\lambda-k_\lambda')^2+M^2\}\{(P_\lambda-k_\lambda-k_\lambda')^2+M^2\}\{k_\lambda{}^2+\kappa^2\}\{k_\lambda'^2+\kappa^2\}};$$

$$\tau_{\alpha\beta}=\tfrac{1}{2}(\tau_\alpha\tau_\beta+\tau_\beta\tau_\alpha),$$

where $M=M_P=M_N$. I can be expressed in the following form:

$$I=2A_\mu\int\frac{d^4k'}{k_P'^2+\kappa^2}\int_0^1dx\int_0^1J_\mu ydy,$$

where:

$$J_\mu=\int d^4k\frac{\bar{\psi}(x_0)\tau_\alpha\tau_\beta\{i\gamma_\lambda(Q_\lambda'-k_\lambda)-M\}\gamma_\mu\tau_P\{i\gamma_\nu(Q_\nu-k_\nu)-M\}\tau_{\alpha\beta}\psi(x_0)}{\{(k_\lambda+C_\lambda)^2+B^2\}^3}$$

$$Q_\lambda'=P_\lambda'-k_\lambda';\quad Q_\lambda=P_\lambda-k_\lambda';\quad C_\lambda=y\{k_\lambda'-[(P_\lambda'-P_\lambda)x+P_\lambda]\};$$

$$B^2=\kappa^2(1-y)+[k_\lambda'^2-2k_\lambda'((P_\lambda'-P_\lambda)x+P_\lambda)]y-C_\lambda{}^2.$$

With $k_\lambda+C_\lambda=u_\lambda$, the integration over k becomes:

$$J_\mu=J_{1\mu}+J_{2\mu},$$

** See Case, reference 3, Eq. (23).

*** A more general proof of the vanishing of the vacuum polarization for certain types of coupling is given by Case, reference 3, Appendix A.

where:

$$J_{1\mu}=\int d^4u\bar{\psi}(x_0)\frac{\tau_\alpha\tau_\beta\{i\gamma_\lambda(Q_\lambda'+C_\lambda)-M\}\gamma_\mu\tau_P\{i\gamma_\nu(Q_\nu+C_\nu)-M\}\tau_{\alpha\beta}\psi(x_0)}{(u^2+B^2)^3},$$

$J_{2\mu}$ is a logarithmically divergent integral; it can be discarded since it is independent of the nucleon momentum transfer and does, therefore, give no contribution to the magnetic moment. The integration in $J_{1\mu}$ is easily carried out and it is found that $H_1{}^b$ is proportional to:

$$A_\mu\int_0^1 dx\int_0^1 ydy\int\frac{d^4k'}{k_P'^2+\kappa^2}\frac{\bar{\psi}(x_0)T_\mu\psi(x_0)}{B^2}$$
$$T_\mu=\tau_\alpha\tau_\beta\{i\gamma_\lambda(Q_\lambda'+C_\lambda)-M\}\times\gamma_\mu\tau_P\{i\gamma_\nu(Q_\nu+C_\nu)-M\}\tau_{\alpha\beta}. \quad (17)$$

The analysis for $\langle H_2\rangle$ can be carried out in a similar manner.

After elimination of the renormalization terms—those independent of the nucleon momentum transfer—we are left with a logarithmically divergent magnetic moment which, apart from numerical coefficients, is roughly of the form:

$$(-\sum_\alpha f_{\alpha\alpha}{}^2+f_{14}{}^2)M\int_0^\infty\frac{k'^3dk'}{(k'^2+\kappa^2)(k'^2+M^2)}$$

for the proton:

$$-f_{14}{}^2M\int_0^\infty\frac{k'^3dk'}{(k'^2+\kappa^2)(k'^2+M^2)}$$

for a neutron.

We see that the divergence of the magnetic moment is not removed by separation of the renormalization terms.

2. For an interaction of the nucleons with a spinor field φ, (5) and (7) are replaced by:

$$H'=\tfrac{1}{2}g_{\alpha\beta}\bar{\psi}\tau_\alpha\tau_\beta\psi\bar{\varphi}\tau_{\alpha\beta}'\varphi;\quad \tau_{\alpha\beta}'=\tfrac{1}{2}(\tau_\alpha'\tau_\beta'+\tau_\beta'\tau_\alpha');$$
$$H_2{}^{\text{ext}}=-ie\bar{\varphi}\gamma_\mu'\tau_e\varphi A_\mu,$$

$g_{\alpha\beta}=g_{\beta\alpha}$ are the coupling constants; the dash indicates that τ' refers to the light spinor particles. τ_e and τ_ν are analogous to τ_P and τ_N in (3).

$H_1{}^a$ also vanishes in the present case. $H_1{}^b$ and $\langle H_2\rangle$ are now as follows:

$$H_1{}^b=-\frac{ie}{64}g_{\alpha\beta}g_{\gamma\epsilon}A_\mu\int\int dxdy\bar{\psi}(x)\times\tau_\alpha\tau_\beta S_F(x_0-x)\gamma_\mu\tau_P S_F(y-x_0)\tau_\gamma\tau_\epsilon\psi(y)\times\text{Trace}\{\tau_{\gamma\epsilon}'S_F'(x-y)\tau_{\alpha\beta}'S_F'(y-x)\};\quad (18)$$

$$\langle H_2\rangle=\frac{e}{64}g_{\alpha\beta}g_{\gamma\epsilon}A_\mu\int\int dxdy\bar{\psi}(x)\times\tau_\alpha\tau_\beta S_F(x-y)\tau_\gamma\tau_\epsilon\psi(y)\text{Trace}\{S_F'(x_0-y)\times\gamma_\mu'\tau_e S_F'(x-x_0)\tau_{\alpha\beta}'S_F'(y-x)\tau_{\gamma\epsilon}'\},\quad (19)$$

where:

$$S_F'(x)=S_F{}^e\tau_e+S_F{}^\nu\tau_\nu.$$

Thus, for a proton:

$$H_{1P}{}^b=-\frac{ie}{64}g_{\alpha\alpha}{}^2A_\mu\int\int dxdy\bar{\psi}_P(x)S_F{}^P(x_0-x)\times\gamma_\mu S_F{}^P(y-x_0)\psi_P(y)\text{Trace}\{S_F{}^e(y-x)\times S_F{}^e(x-y)+S_F{}^\nu(y-x)S_F{}^\nu(x-y)\};$$

$$\langle H_{2P}\rangle=\frac{ie}{16}g_{14}g_{24}A_\mu\int\int dxdy\bar{\psi}_P(x)S_F{}^N(y-x)\psi_P(y)\times\text{Trace}\{S_F{}^e(x_0-y)\gamma_\mu'S_F{}^e(x-x_0)S_F{}^\nu(y-x)\},$$

and for a neutron:

$$H_{1N}{}^b=-\frac{ie}{64}g_{14}g_{24}A_\mu\int\int dxdy\bar{\psi}_N(x)S_F{}^P(x_0-x)\times\gamma_\mu S_F{}^P(y-x_0)\psi_N(y)\text{Trace}\{S_F{}^e(y-x)S_F{}^\nu(x-y)\};$$

$$\langle H_{2N}\rangle=-\frac{ie}{16}g_{14}g_{24}A_\mu\int\int dxdy\bar{\psi}_N S_F{}^P(y-x)\psi_N(y)\times\text{Trace}\{S_F{}^e(x_0-y)\gamma_\mu'S_F{}^e(x-x_0)S_F{}^\nu(y-x)\}.$$

The same conclusion is also reached here, namely, that from both (18) and (19) a logarithmically divergent magnetic moment is derived, as in the scalar pair theory.

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