except for the fact that in the calculation of the infra-red terms, his expression[6]

$$(\pi/\lambda^2)[\ln(\lambda/2k_{\min})+1]$$

is replaced by

$$(\pi/\lambda^2)\ln(\lambda/|\mu|),$$

in which μ is the meson mass.

[1] E. C. G. Steuckelberg and D. Rivier, Helv. Phys. Acta, **23**, 215 (1949).
[2] For an electron $D^{\pm}(x)$ corresponds to $\bar{S}(x)$, and $D^{1}(x)$ corresponds to $S^{(1)}(x)$ in Schwinger's notation.
[3] E. C. G. Steuckelberg and D. Rivier, Helv. Phys. Acta, **23**, 236 (1949).
[4] J. Schwinger, Phys. Rev., **76**, 790 (1949), Eqs. (A32) and (A65).
[5] Terms inversely proportional to the square of the meson mass appear in the matrix elements.
[6] J. Schwinger, reference 4, Eq. (1.107).

A Variational Principle for the Computation of Reflection Coefficients

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In recent years several writers have dealt with variational principles for the computation of phase shifts[1] and transmission cross sections.[2] It seems worth noticing that the same approach is useful for calculating the reflection coefficient of a potential barrier.

Let u satisfy the one-dimensional wave equation

$$d^2u/dx^2+k^2(x)u=0, \tag{1}$$

where $k(x)$ is a given continuous function, which has the constant real values k_0 for $x<0$ and k_1 for $x>d$ respectively, being unrestricted (possibly complex) inside the barrier $0<x<d$. We write u in the form

$$u=f(x)\exp(ik_0x)+g(x)\exp(-ik_0x), \tag{2}$$

$f(x)$, $g(x)$ being everywhere continuous, together with their first derivatives, and subject to the conditions

$$\begin{aligned} &f(x)=1, \quad g(x)=\rho \quad \text{for} \quad x\leqslant 0, \\ &f(x)=\tau\exp[i(k_2-k_0)x], \quad g(x)=0 \quad \text{for} \quad x\geqslant d. \end{aligned} \tag{3}$$

Let us define

$$I=\int_0^d u(d^2u/dx^2+k^2u)dx.$$

By means of (1), (2), (3) it is easily seen that

$$\delta I=2ik_0\delta\rho. \tag{4}$$

This is a variational principle for the reflection coefficient ρ. We can either choose the trial functions $f_t(x)$, $g_t(x)$ containing n coefficients $c_1\cdots c_n$ and then determine these coefficients from the equations $\partial I_t/\partial c_i=0$, $\partial I_t/\partial\rho=2ik_0$, as made by Huang,[3] or simply, following the approach of Kohn,[4] derive from (4) the formula

$$\rho=\rho_t-I_t/2ik_0. \tag{5}$$

The soundness of Eq. (5) may be tested by means of the two following examples. Let us take for u inside the barrier the WKB approximation $u=(k_0/k)^{\frac{1}{2}}\exp(i\int_0^x k(\xi)d\xi)$. This trial function is a rather rough one, because it corresponds to $\rho_t=0$. Nevertheless, inserting it in (5), we readily find

$$\rho=\frac{i}{8}\int_0^d dx(3k^{-3}k'^2-2k^{-2}k'')\exp\left(2i\int_0^x k(\xi)d\xi\right)$$

in agreement with a first approximation[5] derived by other methods, which is valid when the variation of k is very small inside a wave-length, and $k\neq 0$.

As a second example, suppose we have inside the barrier $k(x)=k_0+\gamma(x)$, where γ represents a periodic function, whose average value inside a period is zero. Moreover one must have everywhere $|\gamma(x)|\ll k_0$. In this case we see that the WKB approximation can be written without appreciable error as $u=\exp(ik_0x)$. Inserting this function in (5), with $k^2=k_0^2+2k_0\gamma$, one finds

$$\rho=i\int_0^d \gamma(x)\exp(2ik_0x)dx. \tag{6}$$

If the barrier contains many times the period a of γ, the preceding integral gives a non-negligible value of ρ only when $2k_0=n2\pi/\alpha$, i.e., when α is an integral multiple of the half-wave-length. This is a well-known result (Bragg reflection, Lippmann plates[6]). Equation (6) is also quantitatively in agreement with an elementary theory.

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Element 98*

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Definite identification has been made of an isotope of the element with atomic number 98 through the irradiation of Cm^{242} with 35-Mev helium ions in the Berkeley Crocker Laboratory 60-inch cyclotron. The isotope which has been identified has an observed half-life of about 45 minutes and probably has the mass number 244. The observed mode of decay of the 98^{244} is through the emission of alpha-particles, with energy about 7.1 Mev, which agrees with predictions, and other considerations involving the systematic of radioactivity in this region indicate that it should also be unstable toward decay by electron-capture.

The chemical separation and identification of the new element was accomplished through the use of ion exchange adsorption methods employing the resin Dowex-50. The element 98 isotope appears in the eka-dysprosium position on elution curves containing 4.6-hour Bk^{243} (formed by a d,n reaction from deuteron impurity in the same bombardment) and the bombarded Cm^{242} as reference points; that is, it precedes berkelium and curium off the column just as dysprosium precedes terbium and gadolinium. The experiments so far have revealed only the tripositive oxidation state of eka-dysprosium character but practically no attempts at oxidation to possible IV and V states have been made as yet.

The successful identification of so small an amount of an isotope of element 98 was possible only as a result of the accurate predictions of the chemical[1] and radioactive[2] properties.

It is suggested that element 98 be given the name californium (symbol Cf) after the university and state where the work was done. This name, chosen for the reason given, does not reflect the observed chemical homology of element 98 to dysprosium (No. 66) as the names americium (No. 95), curium (No. 96), and berkelium (No. 97) signify that these elements are the chemical homologs of europium (No. 63), gadolinium (No. 64), and terbium (No. 65), respectively; the best we can do is point out, in recognition of the fact that dysprosium is named on the basis of a Greek word meaning "difficult to get at," that the searchers for another element a century ago found it difficult to get to California.

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