except for the fact that in the calculation of the infra-red terms, his expression[6]

$$(\pi/\lambda^2)[\ln(\lambda/2k_{\min})+1]$$

is replaced by

$$(\pi/\lambda^2)\ln(\lambda/|\mu|),$$

in which μ is the meson mass.

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[2] For an electron $D^{\pm}(x)$ corresponds to $\bar{S}(x)$, and $D^{1}(x)$ corresponds to $S^{(1)}(x)$ in Schwinger's notation.
[3] E. C. G. Steuckelberg and D. Rivier, Helv. Phys. Acta, **23**, 236 (1949).
[4] J. Schwinger, Phys. Rev., **76**, 790 (1949), Eqs. (A32) and (A65).
[5] Terms inversely proportional to the square of the meson mass appear in the matrix elements.
[6] J. Schwinger, reference 4, Eq. (1.107).

A Variational Principle for the Computation of Reflection Coefficients

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In recent years several writers have dealt with variational principles for the computation of phase shifts[1] and transmission cross sections.[2] It seems worth noticing that the same approach is useful for calculating the reflection coefficient of a potential barrier.

Let u satisfy the one-dimensional wave equation

$$d^2u/dx^2+k^2(x)u=0, \tag{1}$$

where $k(x)$ is a given continuous function, which has the constant real values k_0 for $x<0$ and k_1 for $x>d$ respectively, being unrestricted (possibly complex) inside the barrier $0<x<d$. We write u in the form

$$u=f(x)\exp(ik_0x)+g(x)\exp(-ik_0x), \tag{2}$$

$f(x)$, $g(x)$ being everywhere continuous, together with their first derivatives, and subject to the conditions

$$\begin{aligned} &f(x)=1, \quad g(x)=\rho \quad \text{for} \quad x\leqslant 0, \\ &f(x)=\tau\exp[i(k_2-k_0)x], \quad g(x)=0 \quad \text{for} \quad x\geqslant d. \end{aligned} \tag{3}$$

Let us define

$$I=\int_0^d u(d^2u/dx^2+k^2u)dx.$$

By means of (1), (2), (3) it is easily seen that

$$\delta I=2ik_0\delta\rho. \tag{4}$$

This is a variational principle for the reflection coefficient ρ. We can either choose the trial functions $f_t(x)$, $g_t(x)$ containing n coefficients $c_1\cdots c_n$ and then determine these coefficients from the equations $\partial I_t/\partial c_i=0$, $\partial I_t/\partial\rho=2ik_0$, as made by Huang,[3] or simply, following the approach of Kohn,[4] derive from (4) the formula

$$\rho=\rho_t-I_t/2ik_0. \tag{5}$$

The soundness of Eq. (5) may be tested by means of the two following examples. Let us take for u inside the barrier the WKB approximation $u=(k_0/k)^{\frac{1}{2}}\exp(i\int_0^x k(\xi)d\xi)$. This trial function is a rather rough one, because it corresponds to $\rho_t=0$. Nevertheless, inserting it in (5), we readily find

$$\rho=\frac{i}{8}\int_0^d dx(3k^{-3}k'^2-2k^{-2}k'')\exp\left(2i\int_0^x k(\xi)d\xi\right)$$

in agreement with a first approximation[5] derived by other methods, which is valid when the variation of k is very small inside a wave-length, and $k\neq 0$.

As a second example, suppose we have inside the barrier $k(x)=k_0+\gamma(x)$, where γ represents a periodic function, whose average value inside a period is zero. Moreover one must have everywhere $|\gamma(x)|\ll k_0$. In this case we see that the WKB approximation can be written without appreciable error as $u=\exp(ik_0x)$. Inserting this function in (5), with $k^2=k_0^2+2k_0\gamma$, one finds

$$\rho=i\int_0^d \gamma(x)\exp(2ik_0x)dx. \tag{6}$$

If the barrier contains many times the period a of γ, the preceding integral gives a non-negligible value of ρ only when $2k_0=n2\pi/\alpha$, i.e., when α is an integral multiple of the half-wave-length. This is a well-known result (Bragg reflection, Lippmann plates[6]). Equation (6) is also quantitatively in agreement with an elementary theory.

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Element 98*

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Definite identification has been made of an isotope of the element with atomic number 98 through the irradiation of Cm^{242} with 35-Mev helium ions in the Berkeley Crocker Laboratory 60-inch cyclotron. The isotope which has been identified has an observed half-life of about 45 minutes and probably has the mass number 244. The observed mode of decay of the 98^{244} is through the emission of alpha-particles, with energy about 7.1 Mev, which agrees with predictions, and other considerations involving the systematic of radioactivity in this region indicate that it should also be unstable toward decay by electron-capture.

The chemical separation and identification of the new element was accomplished through the use of ion exchange adsorption methods employing the resin Dowex-50. The element 98 isotope appears in the eka-dysprosium position on elution curves containing 4.6-hour Bk^{243} (formed by a d,n reaction from deuteron impurity in the same bombardment) and the bombarded Cm^{242} as reference points; that is, it precedes berkelium and curium off the column just as dysprosium precedes terbium and gadolinium. The experiments so far have revealed only the tripositive oxidation state of eka-dysprosium character but practically no attempts at oxidation to possible IV and V states have been made as yet.

The successful identification of so small an amount of an isotope of element 98 was possible only as a result of the accurate predictions of the chemical[1] and radioactive[2] properties.

It is suggested that element 98 be given the name californium (symbol Cf) after the university and state where the work was done. This name, chosen for the reason given, does not reflect the observed chemical homology of element 98 to dysprosium (No. 66) as the names americium (No. 95), curium (No. 96), and berkelium (No. 97) signify that these elements are the chemical homologs of europium (No. 63), gadolinium (No. 64), and terbium (No. 65), respectively; the best we can do is point out, in recognition of the fact that dysprosium is named on the basis of a Greek word meaning "difficult to get at," that the searchers for another element a century ago found it difficult to get to California.

It is a pleasure to acknowledge the special help in the bombardment of the curium samples of Professor J. G. Hamilton, G. B. Rossi, T. M. Putnam, Jr., M. T. Webb, and the operating crew of the 60-inch cyclotron in the Crocker Laboratory. The successful handling in a safe manner of the radioactivity involved was made possible through the use of the excellent protective equipment provided by Nelson Garden, the members of his Health Chemistry Group, and Mr. C. M. Gordon. We wish also to thank E. K.

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Radioactivity in Hafnium*

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A LONG-LIVED radioactivity in Hf^{181} activated by slow neutrons was first observed[1] by Hevesy and Levi and reported as having a half-life of 55 days. Subsequent spectrometric studies of similar specimens irradiated in the Oak Ridge pile showed[2] the presence of four internally converted gamma-rays. A level scheme utilizing these gamma-rays, but with divergent energy values, was proposed[3] by Wiedenbeck and Chu.

Continued investigation of other specimens now allows a more accurate evaluation of previously reported energies and reveals many electron lines not observed previously. These electron energies as obtained by internal conversion and by photo-emission from lead are presented in two groups. In Table I are those electron lines whose identification seems to be established beyond question. In Table II the remaining 13 electron lines are listed. These latter lines are so numerous and so closely spaced as to energy that their interpretation at the present time is not unique.

TABLE I. Electron energies identified.

Electron energy	Identification	Energy sum
25.7 kev	K(Lu)	89.1 kev
65.2	K^1	132.4
68.5	K^2	135.7
77.8	L(Lu)	88.1
85.5	M(Lu)	87.9
121.7	$L_{1,2}{}^1$	132.8
123.0	$L_3{}^1$	132.9
124.5	L^2	135.6
130.3	M^1	133.0
132.1	N^1	132.7
133.5	M^2	136.2
277.3	K^3	344.5
278.9	K(Lu)	342.3
331.4	L(Lu)	341.7
340.0	M(Lu)	342.4
414.0	K^4	481.2
469.5	L^4	480.6
478.0	M^4	480.7
543.8	K^5	612.0

A short-lived activity (10.2 hr.) previously unreported, appeared to exist in the pure specimen. The half-life associated with the beta-decay of Hf^{181} is here followed through five octaves and found to be 45 days. The associated gamma-rays following beta-emission would be in tantalum ($Z=73$). The K–L–M differences observed for most of the strong electron lines are characteristic of tantalum as shown in Table I, yielding five gamma-rays whose energies are 132.7, 135.7, 344.5, 481.0, and 612 kev and assigned arbitrary numbers in the order of increasing energy. These energies fit almost perfectly on the amended level scheme of Wiedenbeck and Chu as shown in Fig. 1. By the use of a coincidence magnetic beta-spectrometer (to be described elsewhere) Dr. R. G. Shreffler has shown conclusively that the 132.7 kev and the 135.7 kev lines are in sequence with the 344.5 kev line but the coincidence rate is greater for the latter two than for the first and last lines. The 481 kev line is found to be in coincidence with the 132.7 kev line but not with the 344.5 kev line. This behavior is exactly as would be expected in the branching scheme of Fig. 1. The K/L conversion ratios are about one for γ_1 and only about one-fifth for γ_2.

TABLE II. Electron energies not positively identified.

Electron energy	
127.0 kev	146.0 kev
135.0	148.7
137.0	193.8
139.0	197.5
141.0	199.7
142.5	203.2
144.5	

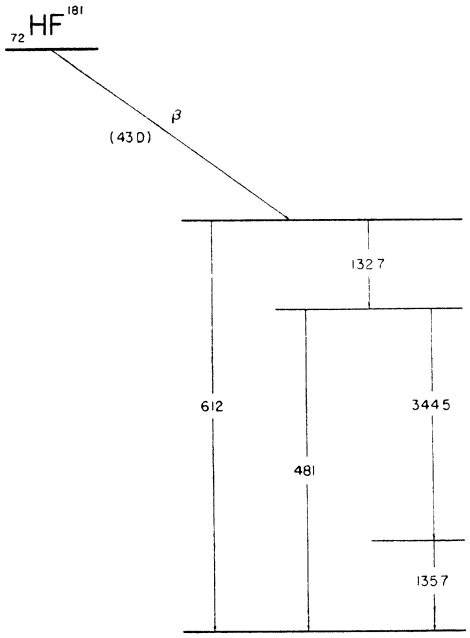

FIG. 1. Energy levels in Ta^{181}, following beta-emission from Hf^{181}.

Hf^{175} could be produced in the pile by neutron capture in Hf^{174} (0.18 percent) or by the $(n,2n)$ reaction of Hf^{176} (5.3 percent). This isotope would decay by K-capture to Lu^{175} ($Z=71$) and has been reported[4] to have a half-life of 70 days, and to emit a gamma-ray whose energy by absorption in lead was 350 kev. Two groups of the electron lines shown in Table I have K–L–M differences characteristic of lutecium and yield gamma-energies of 88.7 and 342.1 kev. Many of the electron lines in Table II can equally well be interpreted as due to gamma-rays in lutecium. There is no evidence however, of a 70 day half-life component in the specimens here studied. On aging beyond the 45-day activity a remaining relatively strong activity persists whose half-life appears to be greater than one year. This problem can ultimately best be resolved by the use of enriched stable isotopes.

* This investigation was made possible by the joint support of the AEC and ONR.
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Excitation Curve for Protons in the Reaction $F^{19}(d,p)F^{20}$ *

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THE total cross section for the production of protons in the reaction

$$F^{19}+H^2 \rightarrow F^{20}+H^1 \qquad (1)$$