less than the drift mobility for re-entrant energy surfaces like those possible for degenerate energy bands[6] and, in fact, may even change sign.

Further evidence that the energy bands are not spherical is furnished by the large and anisotropic magneto resistance effects in germanium single crystals.[7]

We are indebted to Drs. J. Bardeen and F. Seitz for helpful discussions of the theory.

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l-Type Doubling in OCS and HCN

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THE two components of the rotation-vibration states of a linear molecule in an excited perpendicular vibration level are separated by *l*-type doubling, l being the quantum number of angular momentum so that $l=V, V-2, \cdots 0$ or 1). The doubling is predicted by theory[1] to be $\Delta\nu=qJ(J+1)$ where $q=q_0(V_s+1)=(B_e^2/\omega_s)[1+4\sum_{s'}\xi_{ss'}^2\lambda_s/(\lambda_{s'}-\lambda_s)](V_s+1)$ when $|l|=1$, ω_s being the degenerate vibration frequency and $\xi_{ss'}$ the Coriolis coupling factors. Recent measurements by Shulman and Townes[2] have shown that q is, indeed, proportional to (V_s+1), thereby verifying one phase of the theory of *l*-type doubling. Calculations on the quantities q_0 for comparison with experimental values have been impeded largely because of the complicated nature of $\xi_{ss'}$. We shall report in this letter on the evaluation of q_0 for two molecules, namely OCS and HCN.

The quantities $\xi_{ss'}$ have been determined by A. H. Nielsen[3] for the linear XYZ molecule. He gives the following values

$$\xi_{21}=-[M_1M_3/\sigma I^{(e)}]^{\frac{1}{2}}(Z_1^0-Z_3^0)\cos\gamma-[M_2\Sigma/\sigma I^{(e)}]^{\frac{1}{2}}Z_2^0\sin\gamma$$

and

$$\xi_{23}=[M_1M_3/\sigma I^{(e)}]^{\frac{1}{2}}(Z_1^0-Z_3^0)\sin\gamma-[M_2\Sigma/\sigma I^{(e)}]^{\frac{1}{2}}Z_2^0\cos\gamma,$$

where M_2 is the central atom, $\sigma=M_1+M_3$, $\Sigma=M_1+M_2+M_3$, Z_i^0, the equilibrium values of the coordinates and

$$\left.\begin{matrix}\sin\gamma\\ \cos\gamma\end{matrix}\right\}=\pm 2^{-\frac{1}{2}}\{1\pm[(k_1-k_3)^2(4k_4^2-(k_1-k_3)^2)]^{-\frac{1}{2}}\}.$$

The constants k_i are defined as follows: $k_1=\mathfrak{K}_1/\mu_1^{\frac{1}{2}}$, $k_3=\mathfrak{K}_3/\mu_3^{\frac{1}{2}}$, $k_4=\mathfrak{K}_4/(\mu_1\mu_3)^{\frac{1}{2}}$ with $\mu_1=M_1M_3/(M_1+M_3)$ and $\mu_3=M_2\sigma/\Sigma$ and $\mathfrak{K}_1=\{K_1(M_3/\sigma)^2+K_2(M_1/\sigma)^2\}$, $\mathfrak{K}_3=K_1+K_2$, $\mathfrak{K}_4=\{-K_1(M_3/\sigma)+K_2(M_1/\sigma)\}$, where K_i are the valence force constants of $2V=K_1\delta Q_1^2+K_2\delta Q_2^2$ in which $\delta Q_1=Z_2-Z_1$, and $\delta Q_2=Z_3-Z_2$. One may, moreover, quickly show that $\sum_{s'}\xi_{ss'}^2=1$.

The force constants K_1 and K_2 used in this calculation have been taken from the tables of Herzberg[4] and are the following: $K_1=8\times10^5$ dynes/cm, $K_2=14.2\times10^5$ dynes/cm for OCS and $K_1=5.8\times10^5$ dynes/cm, $K_2=17.9\times10^5$ dynes/cm for HCN. Computation of the quantities $\xi_{ss'}$ for these molecules is now a simple matter. These in turn lead to values of $q_0=3.17$ Mc/sec. for OCS which is in complete agreement with the measured value[1] and $q_0=113.6$ Mc/sec. for HCN which is about 2 percent in excess of the measured value 111.8 Mc/sec. given by Shulman and Townes. When one considers that the K_1 and K_2 were determined from the band centers and not from the harmonic frequencies this agreement must be regarded as satisfactory.

Shulman and Townes have, moreover, reported a higher order effect in the *l*-type doubling of HCN. This manifests itself in a deviation of the quantity $q/J(J+1)$ which is proportional to J and of the order of magnitude $q(B_e/\omega_s)$. We have attempted to calculate this effect which involves a fourth-order perturbation calculation. We find a correction to the $(l/l\pm2)$ components of the energy matrix of a little more than $2B_e(B_e/\omega_s)^3(V_s+1)J^2(J+1)^2$. The following argument which is of a symmetry nature, may be used to support this. The correction required must be a correction to the (l/l) or the $(l/l\pm2)$ elements, and since for linear molecules $l=K$, also to the (K/K) and $(K/K\pm2)$ elements of the energy matrix. The matrix components must contain terms proportional to J^3 if the deviation to $q/J(J+1)$ is to be proportional to J. Such terms can arise only from terms in the Hamiltonian of the molecule proportional to P_x and P_y to some odd power. Such terms have only $(K/K\pm1)$, $(K/K\pm3)\cdots$elements and, therefore, only $(l/l\pm1)$, $(l/l\pm3)\cdots$elements.

The differences between the deviations in the states $J=8$ and $J=10$, and $J=10$ and $J=12$ have been calculated by our method. One obtains 0.13 Mc/sec. and 0.16 Mc/sec. respectively compared to 0.18 Mc/sec. measured for both of these instances. While the data of Shulman and Townes certainly fit a linear relation better than the one derived here, it is suggested that the next approximation may bring the two into agreement and that to this approximation our expression is substantially correct.

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Proton Groups from the Alpha-Particles Bombardment of Beryllium*

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BOMBARDMENT of a 0.23 mg/cm² surface density metallic beryllium target with 21.94 Mev alpha-particles from the cyclotron has produced four proton groups. We have calculated Q-values for the reaction $Be^9(\alpha,p)B^{12}$ and three energy levels of B^{12} and have obtained a value for the mass of B^{12}.

Protons emerging from the target at 90° from the incident beam were counted by means of a dual proportional counter in a coincidence and discriminator circuit as described by Brolley, Sampson, and Mitchell.[1] The pulse discriminator was adjusted so that only pulses were counted which corresponded to the peak of the Bragg curve. Proton energies were calculated from aluminum absorption using unit absorber foils of 1.06 mg/cm² surface density. The alpha-particle beam energy was measured by aluminum absorption after scattering from a gold foil of 0.17 mg/cm² surface density. The residual range of the counter and the surface density of the beryllium target were measured with the aid of thorium C′ alpha-particles. The beryllium target was prepared by vacuum evaporation from a tungsten filament. Smith's calculations[2] were used in determining proton energies from aluminum absorption.

Proton groups, relative intensities and calculated Q-values and excitations are given in Table I.

The −7.02 Mev Q-value of our end group gives a calculated atomic mass of 12.01839 for B^{12} which may be compared with the value 12.01827±0.00009 calculated by Hornyak and Lauritsen.[3]

A low intensity proton group of 2.98 Mev energy was observed with variable intensity with different beryllium targets and is

TABLE I. Proton groups from $Be^9(\alpha,p)B^{12}$.

Proton energy Mev	Relative intensity	Q-value (Mev)	B^{12} energy level (Mev)
7.02	1	−7.02	0
6.06	2	−8.06	1.04
5.26	25	−8.93	1.91
3.25	40	−11.11	4.09

thought to be due to oxygen or carbon contamination. Work on the alpha-particle bombardment of these elements is being done in this laboratory.

We wish to thank Mr. Robert G. Cochran who has constructed the beam integrator used in this work. Mr. William Stefanich has devoted much time and skill in the operation of the cyclotron for which we are grateful.

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Operation of a 300-Mev Betatron*

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GEIGER counter yield trials of our new betatron were successful on the first attempt. After a few minutes of operation, the yield was read on an ionization chamber. The present yield which can be held at 315 Mev is about 1000 r/min. at one meter behind $\frac{1}{8}$-in. Pb at 6 PPS and 80-kv injection. Injecting with a rising voltage wave form has increased the output by a factor of about 5 over that using a flat-topped pulse. The field at the 122-cm orbit radius rises sinusoidally from -50 to $+9200$ gauss with a 60 c.p.s. wave form. The flux-forced central core changes from -14 to $+16$ kilogauss simultaneously. Compensation for the 9 percent orbital radiation loss is achieved by supplying a shaped flux pulse in a separate package of iron which links the orbit but not the flux-forcing circuit. This peaked flux pulse is also used to expand the electron beam out to the x-ray target.

The ease with which good yield was obtained is attributed largely to freedom from disturbing magnetic and electrostatic bumps at injection time. Contributing design features[1] are:

(1) Each of the six field magnets is constructed of identical $\frac{1}{2}$-in. bundles of laminations. This insures that each bundle is carried through the same magnetic cycle, resulting in azimuthal uniformity of residual magnetic field and of field lag due to eddy currents.

(2) The circular pole faces are continuous except for $\frac{1}{8}$-in. spacers between field magnets. This minimizes field lag at these joints.

(3) Large eddy-currents were prevented by 0.005-in. Kraft paper every $\frac{1}{4}$ in. in the stacking of the flux core and of the field magnets.

(4) The interior of the porcelain vacuum tube was coated with liquid bright palladium except at the bottoms of grooves $\frac{1}{16}$ in. wide and $\frac{1}{8}$ in. deep which serve as accelerating gaps. The coating resistance was quite uniform and about 20 ohms between ends of each 60° section.

The test for bumps in the magnetic field was a measurement of the time of zero field at all azimuths. An electron gun with a three-slit collimator 2 in. long was used in this test. This detector could be moved around in the gap while the pulse at zero field was observed on an oscillograph. The azimuthal variation of field at injection is less than ± 0.13 gauss except at the six-field magnet junctions where lags 5° in azimuth and as much as 0.5 gauss exist and except at a small region near the coil terminals where the field is 0.2 gauss early. The radial magnetic field at the orbit must be kept small. This was observed by taking the time difference between zero field with the detector against the bottom pole and zero field with the detector against the top pole. The radial field is less than ± 0.13 gauss except at field magnet junctions where bumps as high as 0.4 gauss exist. The residual magnetic field between 300-Mev pulses is 4.5 gauss. It is azimuthally uniform within ± 0.15 gauss. The radial variation of this residual field is similar to the pulsed field distribution.

The doughnut-shaped porcelain vacuum tube has an approximately elliptical cross section with minimum inside diameters about $4\frac{7}{8}$ in. and $9\frac{1}{8}$ in. It was fabricated in 20° sections which were rigidly joined in groups of three by a soldering technique. These 60° groups could be manipulated into place in the pole gap where the final seals were made by the atmospheric compression of Neoprene gaskets. The injector filament is a 1-in. long coil of 0.020-in. tungsten wire; other electrode sizes are proportionally enlarged from earlier types. The peak current injected is about one ampere.

The magnets[2] are all stacked with high quality transformer laminations. The flux magnet contains 275 tons and the six-field magnets contain 11 tons each. The field magnets are excited by two coils outside the pole rim and a coil in the gap, connected in series opposing so as not to excite the flux core. The energy stored in the capacitor bank is about 170,000 joules, of which about 85 percent is used to energize the field magnet gap and 15 percent is used to energize the flux magnet. Unidirectional pulsing is used and a synchronous mechanical switch is employed to reverse the connections to the capacitor bank.

The flux core has a bias winding of 2600 ampere-turns empirically distributed to minimize leakage flux. The choke coil in the bias circuit is also designed for particle analysis.

Meson tracks have been observed in photographic emulsions.

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Quantum Electrodynamics—Second-Order Corrections to the Current Operator

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WE have calculated the second-order corrections to the current operator for electrons interacting with non-charged, vector mesons, obtaining results for electrodynamics in the limit of vanishingly small meson mass. This procedure was adopted to avoid the difficulties encountered in the quantization of the Maxwell field and to obtain determinate expressions in the infra-red terms.

The S-matrix was developed from the integral theory based on causality, invariance, and unitarity, proposed by Stueckelberg and Rivier.[1] This S-matrix has the same form as that obtained from a differential theory. However, the potential function $D^c(x-y)$ appearing in the space time integrals is not defined at the origin. We have,[2]

$$D^c(x-y)\sim D^s(x-y)+\tfrac{1}{2}iD^1(x-y), \quad x\neq y.$$

As a starting point in the calculation, we use the customary integral representations for D^s and D^1 which lead to the well-known indeterminate (infinite) charge and mass "renormalizations." Then, using the method discussed by Stueckelberg and Rivier,[3] we redefine the products of the potential functions in each matrix element in such a way that they are everywhere finite, but arbitrary at the origin. This method gives the same finite part obtained by Schwinger through an integration by parts,[4] but gives a finite arbitrary constant in lieu of a divergent integral. Thus we obtain matrix elements which are finite but which contain arbitrary constants.

We determine these arbitrary constants in such a way that (1) there are no charge or mass renormalizations, and (2) the passage to vanishing meson mass introduces no divergence except that associated with the infra-red catastrophe.[5] These conditions suffice to determine all the arbitrary constants in the matrix elements, and we obtain a completely determined expression for the second-order correction to the current operator. In the limit in which the ratio of the mass of the meson to that of the electron becomes very small, our results agree with those of Schwinger,