studies and the latter in studying atmospheric ionization at ground level. These increases in ionization are considered to be due to radioactive matter brought down with the rain. Between 0935 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Doan and Wait and McNish the 35 percent increase in the soft component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhaven (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

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An Identity in Quantum Electrodynamics

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It has been recently proved by Dyson[1] that all divergencies in the S-matrix of electrodynamics may be removed by a renormalization of mass and charge. Dyson defines certain fundamental divergent operators Γ_μ, S_F', D_F' and gives a procedure for the calculation of their finite parts $\Gamma_{\mu 1}$, S_{F1}', D_{F1}' by a process of successive approximation. It is then shown that

$$\Gamma_\mu=Z_1^{-1}\Gamma_{\mu 1}(e_1),\quad S_F'=Z_2S_{F1}(e_1),\quad D_F'=Z_3D_{F1}(e_1),$$
$$e_1=Z_1^{-1}Z_2Z_3^{\frac{1}{2}}e,$$

where Z_1, Z_2, and Z_3 are certain infinite constants and e_1 is the renormalized electronic charge. Dyson conjectured that $Z_1=Z_2$, and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part W, may be associated a set of proper vertex parts V^i obtained by inserting a photon line in one of the electron lines of W. Now consider the operators $\Lambda_\mu(V^i, p, p)$ in which the two external electron momentum variables p have been set equal, and the external photon variable made to vanish. Then $\Lambda_\mu(V^i, p, p)$ may be obtained from $\Sigma(W, p)$ by replacing S_F by $S_F\gamma_\mu S_F$ at one electron line of W. Because of the identity

$$-(1/2\pi)\partial S_F/\partial p_\mu=S_F\gamma_\mu S_F,$$

on summing $\Lambda_\mu(V^i, p, p)$ over all vertex parts V^i associated with W, one finds

$$\Sigma_{V^i}\Lambda_\mu(V^i, p, p)=-(1/2\pi)(\partial\Sigma(W, p)/\partial p_\mu).$$

(One can verify that any closed loop in W gives zero total effect.) Finally summing over all proper electron self-energy parts W, one finds

$$\Lambda_\mu(p, p)=-(1/2\pi)(\partial\Sigma^*(p)/\partial p_\mu).$$

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$\Lambda_\mu=Z_1^{-1}[(1-Z_1)\gamma_\mu+\Lambda_{\mu C}],\ \Sigma^*=Z_2^{-1}[(Z_2-1)S_F^{-1}+S_F^{-1}S_C/2\pi].$$

We have

$$-(1/2\pi)Z_2^{-1}\{(Z_2-1)2\pi\gamma_\mu+\gamma_\mu S_C+(\gamma_\lambda p_\lambda-iK_0)(\partial S_C/\partial p_\mu)\}$$
$$=Z_1^{-1}\{(1-Z_1)\gamma_\mu+\Lambda_{\mu C}(p, p)\}.$$

Now put

$$\gamma_\lambda p_\lambda=iK_0,\quad (p_\lambda)^2=-K_0^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi)Z_2^{-1}(Z_2-1)2\pi\gamma_\mu=Z_1^{-1}(1-Z_1)\gamma_\mu$$

which reduces immediately to $Z_1=Z_2$.

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The Partial Molal Entropy of Superfluid in Pure He^4 below the λ-Point

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In a recent article[1] (the notation of which is retained here, except that subscripts $4n$ and $4s$ refer to normal fluid and superfluid, respectively, in place of 1 and 2), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relation[2] which states that the total molal entropy S at any temperature is the mole fraction x_{4n} of normal fluid times the molal entropy S_λ at the λ-point

$$S=x_{4n}S_\lambda=(1-x_{4s})S_\lambda, \tag{1}$$

using the empirical relation for S as a function of temperature

$$S=S_\lambda(T/T_\lambda)^r \tag{2}$$

(with $r\sim 5.6$), and assuming that the partial molal enthalpy of superfluid, $\bar{H}_{4s}$, is independent of temperature (at essentially constant pressure), and independent of x_{4s} (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_{4s}=S_\lambda x_{4n}/(r+1). \tag{3}$$

However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below T_λ the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant S_λ, thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for S from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If $\bar{H}_{4s}$ is independent of x_{4s}, then $\bar{H}_{4n}$ must be also, and we have $\bar{H}_{4n}=H_{4n}$, where H_{4n} is the enthalpy of pure normal helium. We can write for the total molal enthalpy[3]

$$H=x_{4n}H_{4n}. \tag{4}$$

We will now proceed to derive an expression for $\bar{S}_{4s}$, in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since $F=H-TS$ and $\mu_{4s}=\bar{H}_{4s}-T\bar{S}_{4s}=-T\bar{S}_{4s}$ the condition for internal equilibrium, $F=\mu_{4s}$, gives

$$\bar{S}_{4s}=S-H/T. \tag{5}$$