

FIG. 1. The elastic scattering cross section Q for atomic hydrogen calculated by the Hulthén variational method for the cases (a), (b), (c), (d) described in the text.

exchange is ignored. This is particularly true for the antisymmetric case, as would be expected. The antisymmetry already ensures that the two electrons are kept apart.

These results may be seen by reference to Fig. 1, in which the collision cross sections derived in the various cases are illustrated.

A full account of this work will be published later.

Our thanks are due Miss K. Blunt for carrying out many of the extensive numerical calculations involved.

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On the Sudden Increase in Cosmic-Ray Intensity on November 19, 1949

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CONTINUOUS recording of cosmic-ray intensities by Geiger-Müller counters is being carried on at Ottawa and at Resolute (latitude $74^{\circ}41' N$; longitude $94^{\circ}55' W$; geomagnetic latitude 83°). The equipment and arrangement of counters at both stations are identical. Four counter trays are used, three in a vertical line with 6 in. of lead between (1) and (2) and $8\frac{1}{2}$ in. of lead between (2) and (3). The bottom tray (3) is surrounded with at least 4 in. of lead. A fourth counter tray, unshielded except for the building roof and $\frac{1}{8}$ in. of aluminum, is located nearby. At both stations coincidences are recorded between (1) and (2) and between (1) and (3) and the total counts in (3) are recorded. At Ottawa the total counts in all trays are recorded as well as the above coincidences.

At Ottawa on November 19 at 1048 GMT the intensity started to increase suddenly. It reached about 170 percent of the mean of the previous 12 hr. in about 6 min. The peak was narrow and sharp and reached roughly the same relative height in shielded counters, unshielded counters, and coincident pairs. The variation was so rapid at the peak that the necessity of counting over short intervals makes an accurate measure of its height impossible. The figures obtained vary from 50 to 90 percent with the different arrangements of counters. After the peak the intensity appears to decay roughly exponentially, though there are some significant irregularities in the decay curve. It returned to about 10 percent above the average in about an hour. A sudden increase on

November 19 was first reported by Dauvillier¹ who found an increment of 3.6 percent at Bagnères, France. Forbush, Stinchcomb, and Schein² report a 43 percent increase at Cheltenham and Godhavn, a larger increase at Climax, and none at Huancayo. The time given by Forbush, Stinchcomb, and Schein is about 1045 and the Ottawa peak was obviously coincident with this. The increase found by Dauvillier occurred somewhat later and reached a maximum about 1200 hr. At Resolute the intensity started to rise at about 1100 GMT and the curve shows a broad peak 12 to 15 percent in height centered about 1145 hours, or approximately fifty minutes after the Ottawa peak. It was recorded by the shielded counter and both coincidence channels. This probably represents the same increase found by Dauvillier.

The delay in time and the significant irregularities in the Ottawa curve suggest that after the initial increase in intensity there were irregular subsequent bursts. If the subsequent bursts were composed of particles, then they might be expected to strike the earth at different localities because of their different energies or directions. Figure 1 shows a plot for eight days of the results from a shielded and an unshielded counter tray and a coincidence unit (6 in. of lead) at Ottawa, and from a similar coincidence unit and a shielded tray at Resolute. An expanded plot from an unshielded tray at Ottawa is also shown.

Previous observations of sudden increases^{3,4} of this type have been associated with solar flares and have shown a decrease of 10 percent or more occurring some hours after the increase. After the increase on November 19 a sudden decrease did not occur. The decrease has been associated with the sudden commencement of a magnetic storm. An examination of magnetometer records taken at the same station at Ottawa does not show any strong sudden commencement but rather a gradually increasing magnetic disturbance starting about 1800 hr. On the days following November 19 the variations in intensity shown in Fig. 1 are probably meteorological.

On November 29 another unusual increase in intensity occurred at Ottawa. The unshielded trays showed an increase of about 35 percent lasting about 3 hr., centered about 1330 GMT. The shielded tray and the coincidence units indicated nothing irregular. A careful investigation eliminated any possibility of interference by radioactive sources in the neighborhood. Doan⁵ and Wait and McNish⁶ have reported increased ionization during rain, the former in an unshielded ionization chamber used for cosmic-ray

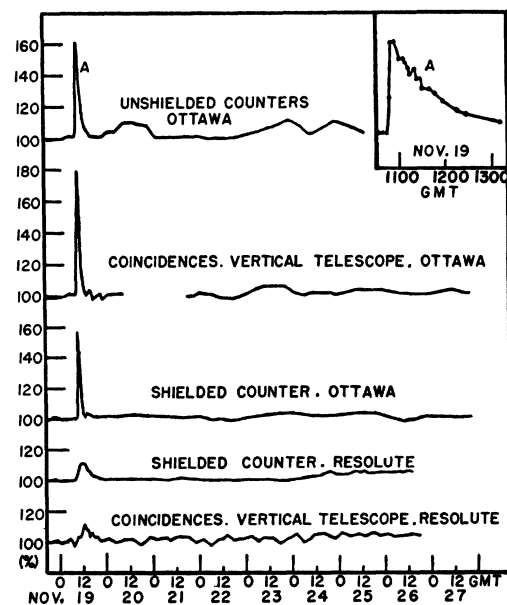


FIG. 1. Sudden increase in cosmic-ray intensity on November 19, 1949

studies and the latter in studying atmospheric ionization at ground level. These increases in ionization are considered to be due to radioactive matter brought down with the rain. Between 0935 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Doan and Wait and McNish the 35 percent increase in the soft component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhaven (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

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An Identity in Quantum Electrodynamics

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IT has been recently proved by Dyson¹ that all divergencies in the S -matrix of electrodynamics may be removed by a renormalization of mass and charge. Dyson defines certain fundamental divergent operators Γ_μ , $S_{F'}$, $D_{F'}$ and gives a procedure for the calculation of their finite parts $\Gamma_{\mu 1}$, $S_{F' 1}$, $D_{F' 1}$ by a process of successive approximation. It is then shown that

$$\Gamma_\mu = Z_1^{-1} \Gamma_{\mu 1}(e_1), \quad S_{F'} = Z_2 S_{F' 1}(e_1), \quad D_{F'} = Z_3 D_{F' 1}(e_1), \\ e_1 = Z_1^{-1} Z_2 Z_3^{1/2} e,$$

where Z_1 , Z_2 , and Z_3 are certain infinite constants and e_1 is the renormalized electronic charge. Dyson conjectured that $Z_1 = Z_2$, and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part W , may be associated a set of proper vertex parts V^i obtained by inserting a photon line in one of the electron lines of W . Now consider the operators $\Lambda_\mu(V^i, \not{p}, \not{p})$ in which the two external electron momentum variables \not{p} have been set equal, and the external photon variable made to vanish. Then $\Lambda_\mu(V^i, \not{p}, \not{p})$ may be obtained from $\Sigma(W, \not{p})$ by replacing S_F by $S_{F'} \gamma_\mu S_{F'}$ at one electron line of W . Because of the identity

$$-(1/2\pi) \partial S_{F'} / \partial \not{p}_\mu = S_{F'} \gamma_\mu S_{F'},$$

on summing $\Lambda_\mu(V^i, \not{p}, \not{p})$ over all vertex parts V^i associated with W , one finds

$$\Sigma_{V^i} \Lambda_\mu(V^i, \not{p}, \not{p}) = -(1/2\pi) (\partial \Sigma(W, \not{p}) / \partial \not{p}_\mu).$$

(One can verify that any closed loop in W gives zero total effect.) Finally summing over all proper electron self-energy parts W , one finds

$$\Lambda_\mu(\not{p}, \not{p}) = -(1/2\pi) (\partial \Sigma^*(\not{p}) / \partial \not{p}_\mu).$$

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$\Lambda_\mu = Z_1^{-1} [(1 - Z_1) \gamma_\mu + \Lambda_{\mu C}], \quad \Sigma^* = Z_2^{-1} [(Z_2 - 1) S_{F'}^{-1} + S_{F'}^{-1} S_C / 2\pi].$$

We have

$$-(1/2\pi) Z_2^{-1} \{ (Z_2 - 1) 2\pi \gamma_\mu + \gamma_\mu S_C + (\gamma_\lambda \not{p}_\lambda - iK_0) (\partial S_C / \partial \not{p}_\mu) \} \\ = Z_1^{-1} \{ (1 - Z_1) \gamma_\mu + \Lambda_{\mu C}(\not{p}, \not{p}) \}.$$

Now put

$$\gamma_\lambda \not{p}_\lambda = iK_0, \quad (\not{p}_\lambda)^2 = -K_0^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi) Z_2^{-1} (Z_2 - 1) 2\pi \gamma_\mu = Z_1^{-1} (1 - Z_1) \gamma_\mu$$

which reduces immediately to $Z_1 = Z_2$.

¹ F. J. Dyson, *Phys. Rev.* **75**, 1736 (1949).

The Partial Molal Entropy of Superfluid in Pure He⁴ below the λ -Point

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IN a recent article¹ (the notation of which is retained here, except that subscripts $4n$ and $4s$ refer to normal fluid and superfluid, respectively, in place of 1 and 2), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relation² which states that the total molal entropy S at any temperature is the mole fraction x_{4n} of normal fluid times the molal entropy S_λ at the λ -point

$$S = x_{4n} S_\lambda = (1 - x_{4s}) S_\lambda, \quad (1)$$

using the empirical relation for S as a function of temperature

$$S = S_\lambda (T/T_\lambda)^r \quad (2)$$

(with $r \sim 5.6$), and assuming that the partial molal enthalpy of superfluid, \bar{H}_{4s} , is independent of temperature (at essentially constant pressure), and independent of x_{4s} (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_{4s} = S_\lambda x_{4n} / (r + 1). \quad (3)$$

However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below T_λ the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant S_λ , thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for S from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If \bar{H}_{4s} is independent of x_{4s} , then \bar{H}_{4n} must be also, and we have $\bar{H}_{4n} = H_{4n}$, where H_{4n} is the enthalpy of pure normal helium. We can write for the total molal enthalpy³

$$H = x_{4n} H_{4n}. \quad (4)$$

We will now proceed to derive an expression for \bar{S}_{4s} , in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since $F = H - TS$ and $\mu_{4s} = \bar{H}_{4s} - T\bar{S}_{4s} = -T\bar{S}_{4s}$, the condition for internal equilibrium, $F = \mu_{4s}$, gives

$$\bar{S}_{4s} = S - H/T. \quad (5)$$