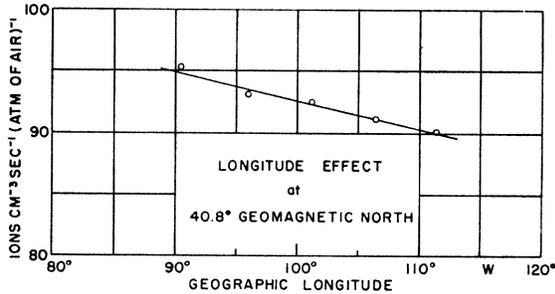


TABLE I. Percentage increase per degree change in latitude.

λ_{mag}	No added absorber		10 cm lead	
	Total ionization	Vertical radiation	Total ionization	Vertical radiation
A. Along longitude 80° W				
45°N	1.3	1.3	1.9	—
50°-64°N	0.16	0.18	0.25	—
B. Along longitude 115° W				
45°N	0.7	0.6	—	0.6
50°-64°N	0.2	0.1	—	0.18

FIG. 3. The longitude effect at 310 g/cm² air pressure as determined with the unshielded ionization chamber.

was applied when necessary. The result is plotted in Fig. 3. It is seen that there is in fact a longitude effect amounting to -0.24 percent per degree change in longitude. Extrapolated to 80° W and 115° W this curve would indicate an 8.2 percent decrease in going from 80° W to 115° W at this geomagnetic latitude and altitude. From Figs. 1 and 2 we find a corresponding difference of 6.2 percent for the ionization chamber and 8.2 percent for the counter telescope, each unshielded.

With the help of the theory of the behavior of charged particles in the magnetic field of a dipole as worked out by Vallarta *et al.* we have made an estimate of the longitude effect to be expected at this latitude due only to the eccentricity of the equivalent magnetic dipole of the earth. Thus one can calculate approximately the change in minimum momentum of the incident particles for a given change in longitude in terms of the corresponding change in latitude. Using the experimental data for the latitude effect as given in Fig. 1 the calculated value of the longitude effect at 40.8° geomagnetic north and 100° geographic west is -0.29 , a value consistent with the measured value of -0.24 percent per degree change in longitude.

We wish to express our gratitude to the ONR and the United States Air Forces for making these flights possible. We also wish to thank Major W. A. Gustafson, Captain Z. Shawhan, Captain George Freyer and other members of the crew of the B-29 for their cooperation and expert handling of the plane.

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Energy Band Structures in Semiconductors

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A METHOD of attack is proposed which for semiconductors has the potentiality of actually determining the shapes of the energy surfaces in the Brillouin zone in detail by proper comparison with experiment. The essential feature is that the most complicated family of energy surfaces arising from a degenerate

energy band in a cubic crystal can be described by three parameters. Similar results for the restricted case of tight binding have been discussed by Sommerfeld and Bethe.¹ We shall illustrate the more general case in terms of the wave functions near the bottom of an energy band which has its lowest energy H_0 at the center of the Brillouin zone and is triply degenerate with p -type wave functions,² type δ , which have the periodicity of the lattice. These are denoted by

$$\psi_z = xu_x(x^2, y^2, z^2) = xu_x(x^2, y^2), \text{ etc.}$$

A wave function near the lowest energy of the band may be written as

$$\Phi = [a_x \psi_x + a_y \psi_y + a_z \psi_z + \sum_{\alpha i} b_{\alpha i} \varphi_{\alpha i}] \exp(i\mathbf{P} \cdot \mathbf{r}/\hbar),$$

where the $\varphi_{\alpha i}$ are also periodic and satisfy $H\varphi_{\alpha i} = H_{\alpha}\varphi_{\alpha i}$. In order to solve the eigenvalue problem $(H - W)\Phi = 0$, a set of equations for the coefficients is found by multiplying by $\exp(-i\mathbf{P} \cdot \mathbf{r}/\hbar)\psi_z^*$, etc. and the resulting system treated by a method similar to that of Van Vleck³ so as to eliminate the b 's. This gives

$$[AP_x^2 + B(P_y^2 + P_z^2) - K]a_x + C[P_x P_y a_y + P_x P_z a_z] = 0, \quad (1)$$

and similar equations with permuted indices (x, y, z) , where $K = W - H_0$ is essentially the kinetic energy of motion and

$$A - \frac{1}{2}m = \sum_{\alpha i} (x | p_x | \alpha i) (\alpha i | p_x | x) / m^2 (H_0 - H_{\alpha}),$$

$$B - \frac{1}{2}m = \sum_{\alpha i} (x | p_y | \alpha i) (\alpha i | p_y | x) / m^2 (H_0 - H_{\alpha}),$$

$$C = \sum_{\alpha i} [(x | p_x | \alpha i) (\alpha i | p_y | y) + (x | p_y | \alpha i) (\alpha i | p_x | y)] / m^2 (H_0 - H_{\alpha}).$$

These equations are formally identical with equations for the frequency and polarization of an acoustical wave of displacement

$$\delta \mathbf{r} = \mathbf{G} \exp i(\mathbf{k} \cdot \mathbf{r} + \omega t)$$

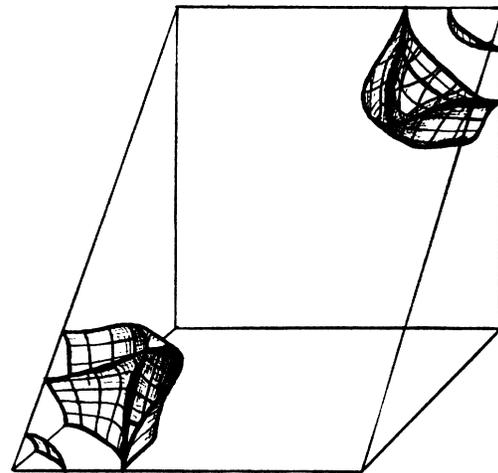
which gives

$$[c_{11}k_x^2 + c_{44}(k_y^2 + k_z^2) - \rho\omega^2]G_x + (c_{12} - c_{44})(k_x k_y G_y + k_x k_z G_z) = 0.$$

The secular equations for both the quantum and the mechanical cases give surfaces of three sheets in \mathbf{k} space for a given eigenvalue, the "polarization vector" (a_x, a_y, a_z) of the wave function being analogous to \mathbf{G} . The present treatment was suggested by the analogy of the surfaces Fig. 1 computed⁴ for the $3p$ band in NaCl to acoustical surfaces. The symmetry of (1) follows directly, of course, from the fact that it is quadratic in P and that the crystal is cubic.

A similar procedure for wave functions of the form² $x^2 - y^2$, $y^2 - z^2$ (type γ) leads to secular equation of two constants:

$$K^2 + 2FK + GP^4 + 3(F^2 - L(P_x^4 + P_y^4 + P_z^4)) = 0,$$

FIG. 1. Energy surface for degenerate band for $\mathbf{k} \approx 0$.

where F and G are similar to A, B, C . This formula can also be obtained directly from the condition that for $P||[111]$ the degeneracy of $x^2 + \omega y^2 + \omega^2 z^2$ and $x^2 + \omega^2 y^2 + \omega z^2$ ($\omega^3 = 1$) is not removed.

In the event that the minimum energy occurs at a lower symmetry point in the Brillouin zone, similar procedures can be adopted; however, more parameters may be involved.

Experimental results of G. L. Pearson⁵ and H. Suhl⁶ of the magneto-resistance of germanium indicate strongly that isotropic scattering and spherical energy surfaces cannot explain the data. It is to be hoped that the method of "deformation potentials"⁷ can be extended to the case of degenerate energy bands so as to give a not more than three parameter (A, B, C) theory which may be compared with experiment to determine the band shapes.

I am indebted to J. Bardeen, C. Herring, and F. Seitz for helpful discussions of group-theoretical aspects of this problem, and to P. M. Morse for suitable perspective.

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Microwave Measurements on the Stable Selenium Isotopes in OCS^e*

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FROM a re-examination of the $J=2 \rightarrow 3$ transition in OCS^e (previously studied by Strandberg¹), the relative masses of the stable Se isotopes have been determined, and an upper limit of 0.002×10^{-24} cm² assigned to the quadrupole moments.

A balanced bridge superheterodyne spectrometer² was used to obtain OCS^e rotational lines (which occur near 24,000 Mc) as narrow as 60 kc. This high resolution allowed very accurate frequency measurements to be made by the usual comparison with harmonics of a quartz crystal. The differences between isotopic lines and their statistical errors resulting from approximately seven measurements of each are given in Table I. Allowing for possible systematic errors, these separations are probably accurate to 0.015 Mc. They differ appreciably—in one case as much as 0.5 Mc—from an earlier measurement,¹ and hence modify considerably the isotopic masses previously obtained.

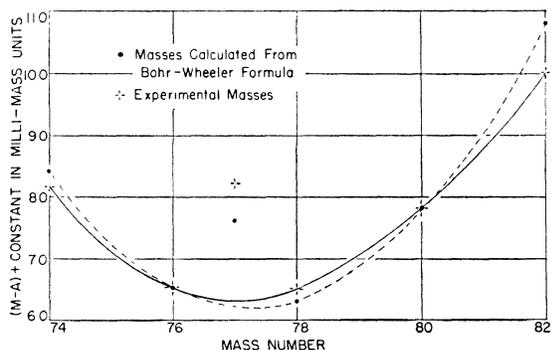


FIG. 1. Variation of masses of the stable Se isotopes as a function of mass number. The experimental masses are determined after assuming Se⁷⁶ and Se⁸⁰ are correctly given by the semi-empirical Bohr-Wheeler formula.

TABLE I. Observed frequency differences between the O¹⁶C¹²Se lines.

	$\Delta\nu$ (Mc/sec.)
O ¹⁶ C ¹² Se ⁷⁸ → OCS ^e ₇₄	320.43 ± 0.013
O ¹⁶ C ¹² Se ⁷⁸ → OCS ^e ₇₆	156.15 ± 0.006
O ¹⁶ C ¹² Se ⁷⁸ → OCS ^e ₇₇	76.95 ± 0.008
O ¹⁶ C ¹² Se ⁷⁸ → OCS ^e ₈₀	148.58 ± 0.008
O ¹⁶ C ¹² Se ⁷⁸ → OCS ^e ₈₂	290.10 ± 0.004
O ¹⁶ C ¹² S ³² → OCS ^e ₇₇ *	5.458 ± 0.007

* The absolute frequency of the OCS^e₇₇ line is 29,331.38 assuming the O¹⁶C¹²S³² frequency as previously measured to be 24,325.92.

TABLE II. Experimentally determined masses of stable Se isotopes compared with masses calculated from semi-empirical mass formula.

A (mass number)	Experimental masses	Masses calculated from Bohr-Wheeler mass formula
74	73.9481 ± 0.0006	73.9484
76	75.9465*	75.9465
77	76.9482 ± 0.0004	76.9476
78	77.9465 ± 0.0004	77.9463
80	79.9478*	79.9478
82	81.9500 ± 0.0006	81.9508

* These masses are assumed to be given exactly by the Bohr-Wheeler formula.

If zero-point vibrations were not present, the rotational frequencies of OCS^e and a known mass of one Se isotope would give the other five Se masses to an accuracy limited only by the errors of measurement. The 0.015 Mc frequency uncertainty would then correspond to an error of 0.00018 mass unit. Zero-point vibrations are of course unavoidable, but if two isotopic Se masses are taken to be known, the errors due to zero-point vibrations can be largely eliminated and the other four Se masses determined with rather good accuracy.^{2,3} Table II lists the masses and the maximum errors expected due to a combination of experimental uncertainty and the residual errors due to zero-point vibrations. These latter will be discussed more fully in a subsequent paper.

The experimental masses (assuming values of Se⁷⁶ and Se⁸⁰ given by the Bohr-Wheeler formula) are plotted in Fig. 1 and compared with predictions of the Bohr-Wheeler formula.⁴ It may be seen that agreement between the curvature of the mass defect and the odd-even mass difference is rather good, although there are some discrepancies larger than the errors allowed in Table II. This method of mass measurement is especially good for determination of the odd-even mass difference since the distance which the odd isotope, Se⁷⁷, lies off the curve of the even masses is unaffected by errors in the assumed Se⁷⁶ and Se⁸⁰ masses or by uncertainties due to zero-point vibrations. The odd-even mass difference is found to be 0.0018 mass units, rather than $\delta = 0.036/A^{3/2} = 0.0014$ from the Bohr-Wheeler formula.

Although the OCS^e lines were obtained as narrow as 60 kc, no noticeable asymmetry or splitting suggestive of hyperfine structure was observed in Se⁷⁷ or in any other Se isotopes. In addition, the Se⁷⁷ line-width was measured to be the same, within 5 kc, as that of Se⁷⁶. This agrees with previous observations,¹ and is a rather strong indication that the spins of the even isotopes are zero, and that of Se⁷⁷ one-half, since then no quadrupole hyperfine structure could be observed. If the Se⁷⁷ spin is not $\frac{1}{2}$, examination of the theoretical quadrupole coupling patterns for various spins gives an upper limit of 1 Mc to the quadrupole coupling constant $eQ\partial^2V/\partial z^2$. In comparison, S³³ shows a coupling constant of -28.5 Mc in the very similar molecule OCS. Estimating $\partial^2V/\partial z^2$ for Se in OCS^e by the same method⁵ as was used for OCS, an upper limit of 0.002×10^{-24} cm² may be assigned to the Se⁷⁷ quadrupole moment. Such a small quadrupole moment is puzzling in view of measurements on the optical hyperfine structure which indicate a spin for Se⁷⁷ of 5/2 or greater.⁶ It may be connected, however, with closure of a proton shell⁷ at Se⁷⁷.