Nuclear Configurations in the Spin-Orbit Coupling Model. I. Empirical Evidence

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An extreme one particle model of the nucleus is proposed. The model is based on the succession of energy levels of a single particle in a potential between that of a three-dimensional harmonic oscillator and a square well. (1) Strong spin orbit coupling leading to inverted doublets is assumed. (2) An even number of identical nucleons are assumed to couple to zero angular momentum, and, (3) an odd number to the angular momentum of the single odd particle. (4) A (negative) pairing energy, increasing with the j value of the orbit is assumed. With these four assumptions all but 2 of the 64 known spins of odd nuclei are satisfactorily explained, and all but 1 of the 46 known magnetic moments. The two spin discrepancies are probably due to failure of rule (3). The magnetic moments of the five known odd-odd nuclei are also in agreement with the model. The existence, and region in the periodic table, of nuclear isomerism is correctly predicted.

NUCLEI containing 2, 8, 20, 28, 50, 82 or 126 neutrons or protons are particularly stable.[1] These closed shells have been explained in different ways.[2,3] It has also been pointed out that the "magic numbers" can be explained on the basis of a single particle picture with the assumption of strong spin-orbit coupling.[4] The detailed evidence supporting this point of view will be discussed in this paper.

I. THE SHELLS

The single particle orbits for the neutrons and protons in a nucleus are determined by a potential energy which has a shape somewhat between that of a square well and a three-dimensional isotropic oscillator. The level order for these two potentials is closely related. In Table I the order of the levels is given. The first line contains the oscillator quantum number. The levels which would be degenerate for the oscillator are grouped together. The eigenfunctions of any such group have the same parity. The order of levels in each group is that calculated for the square well. The major quantum number in the second line counts the number of spherical nodes. The third line contains the number of neutrons or protons which completely fill all states up to each oscillator level. For the square-well potential, the grouping of energy levels indicated above exists only for the low lying eigenfunctions and explains the stability of the numbers 2, 8, and 20. Beyond $n=2$, the grouping is no longer pronounced; $3s$ and $1h$, and also $3p$ and $1i$, have approximately the same energy in a square well calculation.

TABLE I. Order of energy levels for a potential somewhat between that of a square well and of a three-dimensional isotropic oscillator.

$n=$	0	1	2	3	4	5	6
	$1s$	$1p$	$1d\ 2s$	$1f\ 2p$	$1g\ 2d\ 3s$	$1h\ 2f\ 3p$	$1i\ 2g\ 3d\ 4s$
N or $Z=$	2	8	20	40	70	112	

The grouping of oscillator levels does not explain the occurrence of the higher magic numbers; it is apparent that their stability must be due to a different cause. Assume that there exists a strong spin-orbit coupling such that the orbits with higher total angular momenta, $j=l+\frac{1}{2}$, have a higher binding energy. Since this coupling should depend on the orbital angular momentum, l, and be higher for large l values, it is greatest for the first level in each of the oscillator groups. For this level, the state with $j=l+\frac{1}{2}$ will be lowered in energy towards the group with lower n, the state with $j=l-\frac{1}{2}$ raised, so that a gap occurs at this point. Table II contains the order of levels obtained from those of the square well by spin-orbit coupling. It is seen that the shells so obtained correspond exactly to the magic numbers.

In the middle of the shell, spin-orbit coupling may give rise to crossing of some levels.

II. ASSUMPTIONS

As will be shown in the detailed discussion, the following assumptions are adequate to explain the observed facts in all but a very few exceptional cases:

(1) The succession of energies of single particle orbits is that of a square well with strong spin orbit coupling giving rise to inverted doublets.

(1a) For given l, the level $j=l+\frac{1}{2}$ has invariably lower energy and will be filled before that for $j=l-\frac{1}{2}$.

(1b) Pairs of spin levels within one shell, which arise from adjacent orbital levels in the square well in such a way that spin-orbit coupling tends to bring their energy closer together can, and very often will, cross. Examples are $d_{3/2}$ and $s_{1/2}$; $f_{5/2}$, $p_{3/2}$; $p_{1/2}$, $g_{9/2}$; $g_{7/2}$, $d_{5/2}$; $s_{1/2}$, $h_{11/2}$; $p_{1/2}$, $i_{13/2}$. These level pairs may cross to have their energy order reversed from that of Table II.

(2) An even number of identical nucleons in any orbit with total angular momentum quantum number j will always couple to give a spin zero and no contribution to the magnetic moment.

[1] W. Elsasser, J. de phys. et rad. 5, 625 (1934); M. G. Mayer, Phys. Rev. 74, 235 (1948).
[2] E. Feenberg and K. C. Hammack, Phys. Rev. 75, 1877 (1949).
[3] L. W. Nordheim, Phys. Rev. 75, 1894 (1949).
[4] Haxel, Jensen, and Suess, Phys. Rev. 75, 1766 (1949); M. G. Mayer, Phys. Rev. 75, 1969 (1949).

(3) An odd number of identical nucleons in a state j will couple to give a total spin j and a magnetic moment equal to that of a single particle in that state.

(4) For a given nucleus the "pairing energy" of the nucleons in the same orbit is greater for orbits with larger j.

The last assumption leads to the prediction that the higher j value appears less often as the spin of odd nuclei than the energy order of Table II predicts. For instance, if the $3s_{1/2}$ level has slightly lower energy than $h_{11/2}$, but if the pairing energy of $h_{11/2}$ exceeds that of $s_{1/2}$ by more than this difference, the spin 11/2 would not occur in odd nuclei, but 1/2 would be observed instead.

There is some theoretical justification for assumptions 2, 3, and 4, and this will be discussed in the next paper.

Assumption 2 has the consequence that all even-even nuclei have spin zero. The main testing ground for the level assignment consists then in the spins and magnetic moments of the nuclei of odd A. According to the assumptions we will adopt for these nuclei the extreme one-particle picture, ascribing both spin and magnetic moments to the last odd proton or neutron.

TABLE II. Order of energy levels obtained from those of a square well potential by spin-orbit coupling.

Osc. no.	Square well	Spin term	No. of states	Shells	Total no.
0	$1s$	$1s_{1/2}$	2	2	2
1	$1p$	$1p_{3/2}$	4		
		$1p_{1/2}$	2	6	8
2	$1d$	$1d_{5/2}$	6		
		$1d_{3/2}$	4	12	
	$2s$	$2s_{1/2}$	2		20
3	$1f$	$1f_{7/2}$	8	8	28
		$1f_{5/2}$	6		
	$2p$	$2p_{3/2}$	4	22	
		$2p_{1/2}$	2		
		$1g_{9/2}$	10		50
4	$1g$	$1g_{7/2}$	8		
	$2d$	$2d_{5/2}$	6		
		$2d_{3/2}$	4	32	
	$3s$	$3s_{1/2}$	2		
		$1h_{11/2}$	12		82
5	$1h$	$1h_{9/2}$	10		
	2	$2f_{7/2}$	8		
		$2f_{5/2}$	6	44	
	$3p$	$3p_{3/2}$	4		
		$3p_{1/2}$	2		126
		$1i_{13/2}$	14		
6	$1i$, $2g$, $3d$, $4s$	$1i_{11/2}$			

III. MAGNETIC MOMENTS OF ODD A NUCLEI

If assumption 3 were exactly correct, the magnetic moments of all odd nuclei could be computed by the vector model from the known gyromagnetic ratios of proton and neutron. The two possible cases, $l=j-\frac{1}{2}$ and $l=j+\frac{1}{2}$ for given j value lead to two computed lines in a plot of magnetic moment μ against j for nuclei with odd neutron number and two (different) lines for nuclei with odd proton number. These theoretical lines will be referred to as "Schmidt lines."[5] The experimental values lie in between the Schmidt lines, but do not coincide with them. For each j value the magnetic moments seem to fall into two groups, one reasonably close to the line corresponding to $l=j+\frac{1}{2}$, the other scattered from near the line corresponding to $l=j-\frac{1}{2}$ to about halfway. It turns out that the assignment of levels made attributes to the first group an odd nucleon in a state $l=j+\frac{1}{2}$, to the second one $l=j-\frac{1}{2}$. In the later discussion l-values as derived from magnetic moments will be quoted only if the magnetic moment of the nucleus is rather close to one of the two Schmidt lines.

The deviation of the magnetic moments from the Schmidt lines may be taken as an indication of the crudity of the single particle model. However, there is no indication that the magnetic moments for nuclei with one particle more or less than a closed shell fit the Schmidt lines any better than others, which might have been expected, since one would be inclined to expect greater validity of the single particle model in these cases.

For a given value of j, the different possible l-values correspond to eigenfunctions of opposite parity. Determination of l consequently means determination of parity. Since the transition probabilities of β-decays depend on change of parity, these probabilities furnish a check on the somewhat shaky measurements of l-values by the magnetic moments (Section VII).

IV. DETAILED DISCUSSION OF SPINS AND MAGNETIC MOMENTS FOR ODD NUCLEI

Spins and l-values, insofar as the latter are unambiguous, are contained in Table III. It will be seen that there are only two serious discrepancies between expected and observed spins, and one between magnetic moments.

[5] T. Schmidt, Zeits. f. Physik **106**, 358 (1937); H. H. Goldsmith and D. R. Inglis, Brookhaven Publications.

TABLE III. Spins of Even-Odd Nuclei. When two isotopes occur, individual spin values are given only when they are different. Orbital values are listed only when the magnetic moments are unambiguous.

No. of neutrons or protons	Odd protons					Odd neutron			O = Neutron, X = Proton Spin					No. odd p or n	Levels
	Element	Mass no.	Orbit	Mass no.	Orbit	Element	Mass no.	Orbit	1/2	p 1 3/2	d 2 5/2	f 3 7/2	g 4 9/2		
1	H	1	s	3	s	He	3	s	$\otimes$					1	$1s$
3	Li	7	p							X				3	$p_{3/2}$
5	B	11	p			Be	9	p		$\otimes$				5	
7	N	15	p			C	13	p	$\otimes$					7	$p_{1/2}$
9	F	19	s						X					9	
11	Na	23								X				11	$d_{5/2}$
13	Al	27	d			Mg	25	d			$\otimes$			13	
15	P	31				Si	29		$\otimes$					15	$s_{1/2}$
17	Cl	35	d	37	d	S	33			$\otimes$				17	$d_{3/2}$
19	K	39	d	41	d	S	35			$\otimes$				19	
21	Sc	45	f									X		21	$f_{7/2}$
23	V	51										X		23	
25	Mn	55									X			25	
27	Co	59	g									X		27	
29	Cu	63		65						X				29	$p_{3/2}$
31	Ga	69								X				31	
33	As	75								X				33	
35	Br	79		81						X				35	$f_{5/2}$
37	Rb	85	$f(5/2)$	87	$p(3/2)$	Zn	67	f		X	$\otimes$			37	
39	Y	89							X					39	$p_{1/2}$
41	Cb	93	g			Ge	73						$\otimes$	41	$g_{9/2}$
43	Tc					Se	77		O					43	
45	Rh													45	
47	Ag	107	p	109	p	Kr	83	g	X				O	47	
49	In	113	g	115	g	Sr	87	g					$\otimes$	49	
51	Sb	121	$d(5/2)$	123	$g(7/2)$	Zr	91				$\otimes$	X		51	$g_{7/2}$
53	I	127		129	$(7/2)$	Mo	95		O		X	X		53	$d_{5/2}$
55	Cs	133	g	137	g							X		55	
57	La	139	g									X		57	
59	Pr	141									X			59	
61	Pm													61	
63	Eu	151		153	f	Cd	111	s	O		X			63	
65	Tb	159				Cd	s, Sn	s	O	X				65	$h_{11/2}$
67	Ho	165				Sn	117	s	O			X		67	
69	Tm					Sn	119	s	(X)					69	
71	Lu	175	g			Te	123		O			X		71	
73	Ta	181	g			Te	125		O			X		73	
75	Re	185		187		Xe	129	s	O		X			75	
77	Ir	191	$(1/2)$	193	$(3/2)$	Xe	131	d	X	$\otimes$				77	$d_{3/2}$
79	Au	197	d			Ba	135	d		$\otimes$				79	
81	Tl	203		205		Ba	137	d	X	O				81	$s_{1/2}$
83	Bi	209	h										X	83	$h_{9/2}$

(A) Nuclei with Less Than 20 Neutrons or Protons

For nuclei up to ${}_8O_8{}^{16}$ no discrepancies occur, and the level order $s_{1/2}$, $p_{3/2}$, $p_{1/2}$ is strictly adhered to. The nucleus with 9 protons ${}_9F_{10}{}^{19}$, has definitely an $s_{1/2}$ orbit. This seems to suggest that for a nucleus as small as this the potential well is steeper than an oscillator. Nevertheless, one would expect the 9th and 10th proton to pair in $d_{5/2}$ states (assumption 4), and ${}_{11}Na_{12}{}^{23}$ to have a proton configuration of $(d_{5/2})^3$ with a spin 5/2 or possible $(d_{5/2})^2 s_{1/2}$ and a spin 1/2. However, here the first serious discrepancy occurs. The spin of ${}_{11}Na_{12}{}^{23}$ is 3/2. The magnetic moment of this nucleus would indicate a $p_{3/2}$ rather than a $d_{3/2}$ orbit (Section VIII).

From then on, the filling of the shell is regular and indicates the level order $d_{5/2}$, $s_{1/2}$, $d_{3/2}$. Both ${}_{13}Al_{14}{}^{27}$ and ${}_{12}Mg_{13}{}^{25}$ have $d_{5/2}$ orbits. At 14 the $d_{5/2}$ levels should be filled. Fourteen, in fact, should be a magic number, if the spin-orbit coupling in light nuclei were sufficiently strong to overcome the spacing of the oscillator levels (see Section IX).

(B) Odd Nuclei with $20 < N$ or $Z \leq 50$

During the first 8 steps, the eight $f_{7/2}$ orbits should be filled up. Consequently, the spin of 7/2 should occur for all odd neutron or proton numbers between 20 and 28. Only odd proton spins are measured in this region, and with one exception they are all 7/2. This exception, the second serious discrepancy encountered, is ${}_{25}Mn_{30}{}^{55}$, which is expected to have a proton configuration $(f_{7/2})^5$, and consequently, a spin of 7/2; but has actually a spin of 5/2 and a magnetic moment corresponding to a d

level. The magnetic moment of ${}_{27}Co_{32}{}^{59}$ seems to indicate $g_{7/2}$ rather than $f_{7/2}$. The limits of error on this measurement are, however, so great that this assignment is uncertain. At N or $Z=28$ the $f_{7/2}$ levels are filled, and no spin of 7/2 should or does occur between 29 and 49. Beyond 28, the experimental indications are that the level order is

$$p_{3/2} \quad f_{5/2} \quad p_{1/2} \quad g_{9/2},$$

namely $p_{3/2}$ and $f_{5/2}$ have crossed but seem to have approximately the same energy. Both levels will be completely filled at N or $Z=38$, and in between 29 and 37 inclusive spins of 3/2 are measured for every odd number, with the exception of ${}_{37}Rb_{48}{}^{85}$ and ${}_{30}Zn_{37}{}^{67}$, with 37 protons and neutrons, respectively, which show $f_{5/2}$ orbits. The $f_{5/2}$ orbits fill in preferentially in pairs, in accordance with rule 4.

Between 38 and 50, only the levels $p_{1/2}$ and $g_{9/2}$ remain to be filled, and only spins of 1/2 and 9/2 are encountered. The magnetic moments, in the cases where they are measured, are consistent with $p_{1/2}$ and $g_{9/2}$. ${}_{50}Y_{39}{}^{89}$, with 39 neutrons, has spin 1/2; ${}_{32}Ge_{41}{}^{73}$ and ${}_{41}Cb_{52}{}^{93}$, with 41 neutrons or protons respectively, have $g_{9/2}$ orbits. That later on, at 43 neutrons and 47 protons spins of 1/2 occur again, indicates that $p_{1/2}$ and $g_{9/2}$ have closely the same energy, and the greater pairing energy (rule 4) is causing the $g_{9/2}$ orbital to fill in pairs. The isomerism in this region (see Section VI) bears out this contention.

All eigenfunction in this shell, with the exception of $g_{9/2}$, have odd parity.

Up to this point, spins and l-values for odd neutron or odd proton numbers are identical, except for the two isotopes of silver, $Z=47$, and ${}_{36}Kr_{47}{}^{83}$, which have $p_{1/2}$ and $g_{9/2}$ levels respectively.

(C) Odd Nuclei with Odd Nucleon Number between 50 and 82

The over-all predictions for this shell are: (1) No spins of 9/2 should occur. In practice none are encountered. (2) All eigenfunctions of the shell, except those for $h_{11/2}$, should be even. An exception is found in ${}_{63}Eu_{90}{}^{153}$, which has a magnetic moment indicating $f_{5/2}$, instead of the expected $d_{5/2}$, and is the one serious discrepancy encountered in the magnetic moments.

The $h_{11/2}$ orbits are filled in this region, and one would expect to encounter spins of 11/2. None are found in the ground states of the atoms. Isomeric states of spin 11/2 are, however, encountered frequently (see Section VI). It must be postulated that the $h_{11/2}$ orbits are filled preferentially by pairs, according to rule 4.

In detail, the situation for protons is this: The level order, leaving out for the moment the $h_{11/2}$ levels, is

$$g_{7/2} \quad d_{5/2} \quad d_{3/2} \quad s_{1/2}.$$

The beginning of the shell is completely regular. $d_{5/2}$ and $g_{7/2}$ have approximately the same energy; ${}_{51}Sb$ has two stable isotopes, one with a $d_{5/2}$, the other with a $g_{7/2}$ level. The stable isotope of ${}_{53}I_{74}{}^{127}$ has a $d_{5/2}$ level, the radioactive ${}_{53}I_{76}{}^{129}$ has $g_{7/2}$. 55 and 57 protons have spins and magnetic moments corresponding to $g_{7/2}$. At 58, $g_{7/2}$ should be filled and actually only spins 5/2 occur after 58. At 64 protons both $d_{5/2}$ and $g_{7/2}$ should be completely filled, and the spin of 3/2 for ${}_{65}Tb_{94}{}^{159}$ bears this out.

TABLE IV. Spins and magnetic moments for five odd-odd nuclei.

	Neutron state	Proton state	Spin	Exp.	Calc.
${}_{3}Li_{3}{}^{6}$	$p_{3/2}$	$p_{3/2}$	1	0.82	0.63
${}_{5}B_{5}{}^{10}$	$p_{3/2}$	$p_{3/2}$	3	1.80	1.88
${}_{7}N_{7}{}^{14}$	$p_{1/2}$	$p_{1/2}$	1/2	0.40	0.37
${}_{11}Na_{11}{}^{22}$	$d_{5/2}$	$d_{5/2}$	3	1.75	1.73
${}_{19}K_{21}{}^{40}$	$f_{7/2}$	$d_{3/2}$	4	-1.29	-1.70

From now on irregularities occur which must be attributed to the filling of the $h_{11/2}$ level. It seems that between 66 and 74 protons the $h_{11/2}$ levels suddenly fill in, to the extent of redissolving the previously formed $g_{7/2}$ and $d_{5/2}$ shells. The reason for this may be Feenberg's "wine bottle" effect.[2] The end of the shell is again regular. Spins of 3/2 appear twice, and the thallium isotopes at 81 protons have spins 1/2.

The situation for neutrons is somewhat different. At 51 neutrons, ${}_{40}Zr_{51}{}^{91}$, a spin of 5/2 is found. The spin of ${}_{42}Mo_{53}{}^{95}$, with 53 neutrons, is given as 1/2, but the measurement is too doubtful to be believed.* The outstanding feature is the occurrence of spin 1/2 for all odd neutron nuclei from 63 to 75. Actually, if $h_{11/2}$ and $s_{1/2}$ have approximately the same energy and the $h_{11/2}$ fill in regularly, one would expect a string of 7 spins of 1/2 to occur due to the very strong pairing effect of 11/2 compared to 1/2.

Towards the end of the shell the $s_{1/2}$ level seems to be filled, and $d_{3/2}$ orbits occur. The level order for odd neutrons seems to be

$$d_{5/2} \quad g_{7/2} \quad s_{1/2} \quad d_{3/2} \quad h_{11/2}.$$

The pairing energy for $h_{11/2}$ orbits tends to depress the energy of pairs of $h_{11/2}$ in this scheme to below that of the $s_{1/2}$ pairs.

In this whole region, the last odd proton, in contrast with the odd neutron, is usually found in an orbit of high angular momentum. The Coulomb repulsion, which tends to concentrate the charge on the outside, is probably responsible for this difference between odd proton and odd neutron numbers.

(D) Odd Nuclei with Neutrons or Protons between 82 and 126

Very few spins are known in this part of the isotope table. The shell begins and ends correctly with $h_{9/2}$ at

* *Note added in proof:* According to a private communication from J. E. Mack, the spins of ${}_{42}Mo_{53}{}^{95}$ and ${}_{42}Mo_{55}{}^{97}$ are either 5/2 or 7/2.

$_{83}Bi_{126}^{209}$, and ends with $p_{1/2}$ at $_{82}Pb_{125}^{207}$ with 125 neutrons. The $i_{13/2}$ orbits should be filled, but one might expect them to "hide," as the $h_{11/2}$ orbits do. All other orbits should have odd parity.

V. ODD-ODD NUCLEI

Spins and magnetic moments are known for the five odd-odd nuclei, and are listed in Table IV. The states of the odd neutron and proton are also given. To predict the spins of the nuclei one would have to understand the coupling between the angular momenta of the two odd nucleons, which is evidently not such as to make the total spin zero. The only empirical indication given by the first four of the nuclei in Table IV, which have the neutron and proton in the same state, is that the eigenfunction is symmetric in the proton and neutron.

Feenberg[6] has pointed out that the magnetic moments of these nuclei, calculated on the basis of the orbit assignments above, are in excellent agreement with the experimental values.

VI. EXCITED STATES OF ODD NUCLEI

(A) Electric Dipole Transitions

The states within one shell consist of all but the highest j value obtainable from an oscillator level with quantum number n, plus the highest j value obtained from level $n+1$; this latter has a j value larger by at least 2 than any other level in the shell. Oscillator levels of the same n value have identical parity.

Low-lying excited states in odd nuclei should be those in which the odd nucleon is raised to another level in the same shell. Since γ-ray transitions corresponding to electric dipole require change in parity, and spin change by not more than one, one can make the prediction: *γ-rays between low lying states are never electric dipole transitions.*

(B) Isomeric States for Odd Nuclei

Isomerism in odd nuclei should occur when the two lowest levels available for the odd nucleon have very different spin. It is seen that this level assignment first predicts this situation between 39 and 49, where $g_{9/2}$ and $p_{1/2}$ compete. The existence of "islands of isomerism" has been pointed out by Feenberg and Hammack[2] and by Nordheim.[3]

The compilation by Segrè and Helmholtz[7] actually lists isomeric states for $_{20}Ca_{29}^{49}$, $_{22}Ti_{29}^{51}$, and $_{33}As_{38}^{71}$ in which the odd nucleon is the 29th or 33rd. The first two cases have been reinvestigated by M. Goldhaber[8] and found not to show isomerism. $_{33}As_{38}^{71}$ is then the only nucleus in which an isomeric state exists which does not fit the level scheme.

In agreement with the level assignment, isomerism occurs very frequently for Z or N between 39 and 49. 26 odd nuclei with the odd value of N or Z between 39 and 49 are known to have isomeric states. Beyond Z or $N=50$ the competition is between $1g_{7/2}$ and $2d_{5/2}$ and should not lead to isomeric states. The first isomerism occurs at 63 neutrons, Cd^{111}. From then on, to the end of the shell at 82, isomerism is again frequent.

The assignment of levels given ascribes all cases of isomerism in odd nuclei with the odd Z or N between 39 and 49 to the small difference in energy between $p_{1/2}$ and $g_{9/2}$. The isomeric transition should consequently be magnetic 2^4th pole in all cases. The Λ-value for all transitions should be 5. This is not in agreement with experiment. The isomeric states of the two indium isotopes $_{49}In_{64}^{113}$ and $_{49}In_{66}^{115}$ agree perfectly with the assignment, but, for instance, the two silver isotopes, $_{47}Ag_{60}^{107}$ and $_{47}Ag_{62}^{109}$ have isomeric states which decay much faster and are characterized by $\Lambda=4$.

Between 50 and 82, $h_{11/2}$ has the opposite parity from the other levels. All isomerism between 63 and 81 which has odd Λ-values has to be ascribed to the occurrence of this state. A clear-cut example of this is $_{56}Ba_{81}^{137}$, with 81 neutrons. The ground state is measured to be $d_{3/2}$. The isomeric transition has a lifetime corresponding to $\Lambda=5$, which requires spin change of at least 4 and change of parity, and agrees with the assignment $h_{11/2}$ for the isomeric level.

Other examples are the isomeric states of the 6 odd tellurium isotopes from $A=121$ to $A=131$ with neutron numbers from 67 to 77.[9] All 6 isotopes have isomeric transitions corresponding to magnetic 2^4 poles, or spin change of 4 and change in parity. In the lighter three of the isotopes this transition is followed by a γ-ray, which at least for Te^{123} and Te^{125} is consistent with magnetic dipole, or no change in parity, spin change of 1. For Te^{123} and Te^{125} the spins of the ground state are measured to be 1/2. If it is accepted that this is to be interpreted as $s_{1/2}$, one finds that the levels in ascending order are

$$s_{1/2} \quad d_{3/2} \quad h_{11/2},$$

just as given in Section IV, (c). For the three heavier tellurium isotopes the following γ-ray is absent, which may be understood by assuming that the $s_{1/2}$ level is now filled, and the ground state is $d_{3/2}$.

VII. β-DECAY FOR ODD NUCLEI

The transition probabilities for β-decay depend on change of spin and parity. The shell model gives a method of assigning spin and parity to an odd nucleus in the cases in which no measurements exist. The extent to which a β-transition between the ground states of parent and daughter nucleus is permitted can then be calculated. It is also possible to predict the properties of some of the low excited levels. The over-all agreement between theory and experiment is very satisfactory. To

[6] E. Feenberg, Phys. Rev. **76**, 1275 (1949).

[7] Emilio Segrè and A. C. Helmholz, Rev. Mod. Phys. **21**, 271 (1949).

[8] M. Goldhaber, private communication.

[9] R. D. Hill, Phys. Rev. **76**, 186 (1949); J. C. Bowe and Gertrude Scharff-Goldhaber, Phys. Rev. **76**, 437 (1949).

demonstrate this would require an extensive table. Such tables have been published by Nordheim[3] and Feenberg and Hammack.[2] Although the shell assignments in those two papers are entirely different, the resulting predictions are surprisingly similar. A detailed compilation of the level assignment for β-decay on the basis of the jj shell model is being prepared by S. Moszkowski.

The most unambiguous part of the predictions is that of parity. The β-transitions thereby contribute materially to the confirmation of the assignment of orbital angular momenta, which determine the parity.

The few startling contradictions, notably C^{14} and P^{32} occur mostly in even A nuclei.

VIII. THE TWO SPIN DISCREPANCIES

In Section IV, two major disagreements were encountered, ${}_{11}Na_{12}{}^{23}$, which should have a proton configuration $(d_{5/2})^3$, is found to have a spin of 3/2. The other case is ${}_{25}Mn_{30}{}^{55}$ with protons in a state $(f_{7/2})^5$ which has a spin of 5/2. In both cases the magnetic moments are not such as to indicate a violation of rule 1, the order of spin-orbit coupled levels; Na behaves more like a $p_{3/2}$ than a $d_{3/2}$ state, and Mn more like $d_{5/2}$ than $f_{5/2}$. It is tempting to ascribe the violation to a breakdown of rule 3. Three protons in a state $d_{5/2}$ can couple to give a spin 3/2, although as a rule the energy for the state with spin 5/2 is lower. The state with spin 3/2 is a definite configuration and permits a calculation of the magnetic moment; this value lies closer to the measured magnetic moment of ${}_{11}Na_{12}{}^{23}$ than either of the Schmidt lines. The same situation holds for ${}_{25}Mn_{30}{}^{55}$. The values are given in Table V. In the second column is the experimentally measured magnetic moment. The third column has the values calculated for $(d_{5/2})^3$ $J=3/2$ and $(f_{7/2})^5$, $J=5/2$, respectively. The last two columns give the two Schmidt line values.

TABLE V. Experimental and calculated values of magnetic moments.

	μ exp.	μ calc.	μ $l=j-\frac{1}{2}$	μ $l=j+\frac{1}{2}$
${}_{11}Na_{12}{}^{23}$	2.21	2.8	3.78	0.12
${}_{25}Mn_{30}{}^{55}$	3	4	4.8	0.87

IX. THE MAGNITUDE OF THE SPIN-ORBIT COUPLING

A calculation on the basis of meson theory by Gaus[10] indicates that the splitting between the levels with $j=l-\frac{1}{2}$ and $j=l+\frac{1}{2}$ is proportional to $(2l+1)A^{-2/3}$. The splitting responsible for the occurrence of the magic numbers should consequently not be very different for the different shells. A formula of this type would also explain the fact that the shell structure seems to be more pronounced for neutrons than for protons, since the neutron shells fill at smaller A values.

The empirical evidence indicates that the spin orbit coupling induces a splitting of the two j values, $l+\frac{1}{2}$ and $l-\frac{1}{2}$ by 2 Mev in the region of higher l-values. The value of at least 2 Mev was estimated from α-decay energies[11] for the extra stability of 126 neutrons, due to the $i_{13/2}$ and $i_{11/2}$ splitting. A similar value was previously estimated for the extra stability of 82 neutrons.[1]

The shell at 28 is not too strongly marked. It has been suggested,[12] however, that ${}_{26}Fe_{30}{}^{56}$ may owe its extreme abundance to the original creation of ${}_{28}Ni_{28}{}^{56}$ with subsequent decay to ${}_{26}Fe_{30}{}^{56}$. Nothing seems to be known about the lifetime of ${}_{28}Ni_{28}{}^{56}$.

Recently, some evidence was obtained indicating extra stability for 14 neutrons or protons. The energy released by the β-decay[13] of ${}_{13}Al_{16}{}^{29}$ is greater than would be estimated from the decay energy of neighboring nuclei with the same isotopic spin. In this process, a $d_{3/2}$ neutron is transformed into a $d_{5/2}$ proton. The excess energy is about 1.7 Mev.

The splitting of the $1p_{3/2}$ and $1p_{1/2}$ levels in Li^7 is 0.48 Mev, if one interprets the excited state to be $p_{1/2}$. Inglis[14] has pointed out that this assignment encounters some experimental difficulties. The extrapolation of the formula of Gauss would lead to about 2 Mev for the splitting rather than 1/2 Mev. A perturbation calculation on the symmetric eigenfunction for Russel-Saunders coupling shows, however, that the splitting in Li^7 is only one-third of that obtained for a single nucleon in the P-shell. A splitting of 1.5 Mev for a single nucleon is about the extrapolated value.

There is no adequate theoretical reason for the large observed value of the spin orbit coupling. The Thomas splitting has the right sign, but is utterly inadequate in magnitude to account for the observed values. A proper type of meson potential can be made to predict splitting qualitatively similar to the Thomas splitting, and therefore qualitatively similar to the observed, but greater in magnitude than the Thomas splitting, although usually somewhat less than the observed value.

[10] H. Gaus, private communication by C. F. von Weizsäcker.

[11] Perlman, Ghiorso, and Seaborg, Phys. Rev. **74**, 1730 (1948).

[12] O. Haxel, private communication from C. F. von Weizsäcker.

[13] Seidlitz, Bleuler, and Tendam, Phys. Rev. **76**, 861 (1949).

[14] D. R. Inglis, Phys. Rev. **74**, 1876 (1948).