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Dipole Transitions in the Nuclear Photo-Effect*

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In this paper we apply sum rules to calculate under certain approximations the integrated cross section and the mean energy for photon absorption by heavy nuclei. In Section II we calculate the summed oscillator strength for dipole transitions as $\Sigma_n f_{on}=(NZ/A)(1+0.8x)$ where x is the fraction of attractive exchange force for the neutron-proton potential. For $N=Z$ this gives the cross section integrated over photon energy $\int_0^\infty \sigma dW=0.015A(1+0.8x)$ Mev-barns. In Section III we calculate the mean energy $\bar{W}$ for photon absorption. The mean energy is 4/3 the average kinetic energy of a nucleon for pure ordinary forces, or about 19 Mev; and is greatly increased by attractive exchange force. The harmonic mean energy

$$W_H=\Sigma_n f_{on}/[\Sigma_n f_{on}/(E_n-E_o)]$$

is much too low using the model of uncorrelated nucleons, but is reasonable if we use the alpha-particle model of the nucleus. In Section IV we develop a sum rule for quadrupole transitions, and show that quadrupole transitions can account for only about six percent of the experimentally observed integrated cross section. In Section V we apply the sum rules for dipole transitions to the photo-disintegration of the deuteron, and compare the results from sum rules with those from direct calculations of the cross section. In Section VI we determine the asymptotic behavior of the cross section for dipole transitions at high photon energies. This is determined by the nature of the singularities in the neutron-proton potential. In the last section we discuss the G.E. experiments on the cross section for photo-disintegration, and its energy dependence, and find that our calculations explain the general features of the experiments. It is not necessary to assume the Goldhaber-Teller model of dipole vibrations by the entire nucleus.

I. INTRODUCTION

THE recent G.E. experiments[1,2] on the cross section and energy dependence of the nuclear photo-effect indicate that there are strong dipole transitions. At low excitation energy (a few Mev) it is commonly assumed that dipole and quadrupole transitions are about equally strong, the dipole transitions being greatly reduced by correlations between the motions of the nucleons.[3] Following this assumption, calculations of quadrupole transitions in the nuclear photo-effect were made some time ago by Weisskopf,[4] and others. However, correlations between the nucleons should cease to exist when very highly excited states (excitation energy more than about 20 Mev) are considered. Indeed, Goldhaber and Teller[5] have suggested that radiative transitions lead to rather sharply defined states in which there is anticorrelation between neutrons and protons, i.e., all protons move together against all neutrons.

In this paper we shall calculate dipole transitions in the nuclear photo-effect, without making Goldhaber and Teller's assumption of dipole vibrations of the whole nucleus. We wish to see which results are peculiar to their model, and which are characteristic for all dipole transitions.

Calculations of the photoelectric cross section for a nuclear transition from the ground state to a particular excited state demands knowledge of the wave functions of both the ground state and excited state. Very little is known of the wave function for the ground state of a heavy nucleus, and much less is known of the wave functions for the excited states. In this paper we shall sum over all excited states, using closure for the matrix elements, so that our results will depend only on the wave function assumed for the ground state.

In Section II we calculate $\sum_n f_{on}$ for heavy nuclei, where f_{on} is the oscillator strength for a dipole transition from ground state o to excited state n. Feenberg[6] and Siegert[7] have shown that attractive exchange forces increase the summed oscillator strength above the customary value for pure ordinary forces. We shall calculate this increase for two assumed shapes of the neutron-proton potential. From the summed oscillator strength we find $\int \sigma dW$, where W is the photon energy, and σ is the cross section for photon absorption. (σ is the sum of all the partial cross sections for the various nuclear reactions that may occur subsequent to the photon absorption: $\gamma-n$, $\gamma-p$, $\gamma-\gamma$, etc.)

In Section III we calculate $\sum_n(E_n-E_o)f_{on}$, where E_n-E_o is the energy difference between the ground and excited state. The ratio

$$\sum_n(E_n-E_o)f_{on}/(\sum_n f_{on})=\int \sigma W dW \Big/ \int \sigma dW=\bar{W},$$

the mean energy for photons absorbed in the photoelectric effect. We also discuss the harmonic mean energy $\sum_n f_{on}/[\sum_n f_{on}/(E_n-E_o)]$. In Section IV we develop a sum rule for quadrupole transitions, and show that quadrupole transitions alone are unable to account for the results of the G.E. experiments. In Section V we apply the sum rules for dipole transitions to the photoelectric cross section of the deuteron, and compare the results with those from direct calculations of the cross section. In Section VI we discuss the asymptotic form for the dipole cross section at high energies. In Section VII we discuss the G.E. experi-

* Supported in part by the ONR.
[1] J. L. Lawson and M. L. Perlman, Phys. Rev. 74, 1190 (1948).
[2] G. C. Baldwin and G. S. Klaiber, Phys. Rev. 73, 1156 (1948).
[3] H. A. Bethe, Rev. Mod. Phys. 9, 71 (1937), Sections 87 to 90.
[4] V. F. Weisskopf, Phys. Rev. 59, 318 (1941).
[5] M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).
[6] E. Feenberg, Phys. Rev. 49, 328 (1936).
[7] A. J. F. Siegert, Phys. Rev. 52, 787 (1937).

ments on the nuclear photo-effect, and compare the experimental results with the calculations of this paper.

We shall make several important approximations for the nuclear ground state. First, we shall use the Hartree-type approximation of one nucleon moving in the potential due to all the other nucleons; i.e., we assume that there are no correlation effects among the nucleons. We shall also neglect surface effects due to the finite size of the nucleus. Further, we are considering the case of pure central forces, and we are assuming that the forces on a nucleon in a nucleus are just the sum of the forces due to all the other nucleons. This neglects the possibility of "many-body forces."

We shall calculate the results from the dipole sum rules for both square and Yukawa wells for the neutron-proton potential; and we shall use the fraction of the neutron-proton force that is exchange as a parameter. In principle this parameter could be determined by comparison of our calculations with experimental data, but both the data and the calculations are at present too uncertain to make this practical. The nuclear radius also enters into our calculations. We shall use nuclear radius $=r_oA^{\frac{1}{3}}$, and calculate for both $r_o=1.50\times10^{-13}$ cm, and $r_o=1.37\times10^{-13}$ cm.[8]

II. SUM RULE FOR OSCILLATOR STRENGTH

The oscillator strength f_{on} for a photoelectric dipole transition between state o and state n is defined as

$$f_{on}=[2M(E_n-E_o)/\hbar^2]\left|\int \psi_o{}^*z\psi_n d\tau\right|^2=|z_{on}/\lambdabar_{on}|^2. \quad (1)$$

Here z is the component of the displacement along the direction of polarization of the photon; and $\lambdabar_{on}$ is the wave-length divided by 2π, for a nucleon of mass M and energy (E_n-E_o).

The photoelectric cross section for absorption of a photon of energy $W=E_n-E_o$ is proportional to the oscillator strength:

$$\sigma_{on}=(2\pi^2e^2\hbar/Mc)f_{on}, \quad (2)$$

where f_{on} is the oscillator strength per unit energy in the final state.

For the case of electrons in an atom, the Thomas-Reiche-Kuhn sum rule[9] states that the sum of the oscillator strengths equals the number of electrons:

$$\sum_n f_{on}=Z. \quad (3)$$

$\sum_n$ means sum over discrete levels and integrate over the continuum.

When we consider the displacement from the center of mass of the nucleus,[3] each proton behaves as if its charge were $e(N/A)$, where N is the number of neutrons in the nucleus; and each neutron as if its charge were $-e(Z/A)$. The summed oscillator strength becomes

$$\sum_n f_{on}=Z(N/A)^2+N(-Z/A)^2=NZ/A. \quad (4)$$

Thus, for the case $N=Z=A/2$, the summed oscillator strength is $Z/2$. The remainder goes into transitions in which the nucleus as a whole vibrates due to the electric field and this vibration does not lead to photon absorption.

Using Eqs. (2) and (4) for the case $N=Z$, we have the integrated cross section for photon absorption

$$\int \sigma dW=\frac{\pi^2e^2\hbar}{Mc}\frac{A}{2}=0.015A \text{ Mev-barn.} \quad (5)$$

This result agrees with that given by Goldhaber and Teller[5] in their theory of dipole vibrations of the entire nucleus.

Feenberg[6] and Siegert[7] have shown that the sum rules are modified for exchange forces. We want first to find $(E_n-E_o)(\sum_i z_i)_{on}$, where $\sum_i$ means summation over all protons in the nucleus. We multiply the Schrödinger equations for $\psi_o{}^*$ and ψ_n by ψ_n and $\psi_o{}^*$, respectively, subtract, multiply by the coordinate $\sum_i z_i$, and integrate over-all space for all nucleon coordinates. ψ_o is the wave function of the ground state for the whole nucleus.

$$\begin{aligned}(\hbar^2/2M)\nabla^2\psi_o{}^*+E_o\psi_o{}^*-V\psi_o{}^*=0 \quad &|\psi_n,\\ (\hbar^2/2M)\nabla^2\psi_n+E_n\psi_n-V\psi_n=0 \quad &|\psi_o{}^*,\end{aligned} \quad (6)$$

$$\begin{aligned}(E_n-E_o)\psi_n\psi_o{}^*+(\hbar^2/2M)(\psi_o{}^*\nabla^2\psi_n-\psi_n\nabla^2\psi_o{}^*)\\ -\psi_nV\psi_o{}^*+\psi_o{}^*V\psi_n=0,\end{aligned} \quad (7)$$

$$\begin{aligned}(E_n-E_o)(\textstyle\sum_i z_i)_{on}=\frac{\hbar^2}{2M}\int \textstyle\sum_i(\psi_n\nabla^2\psi_o{}^*-\psi_o{}^*\nabla^2\psi_n)z_i d\tau\\ +\int \textstyle\sum_i(\psi_n z_i V\psi_o{}^*-\psi_o{}^*z_iV\psi_n)d\tau.\end{aligned} \quad (8)$$

The first integral on the right is integrated by parts. The second integral on the right gives a contribution only for exchange forces between neutron and proton. For ordinary forces the potential and coordinate commute. Exchange between two protons does not contribute since we sum over all protons in the dipole moment. We shall consider an attractive exchange force of the same shape as the ordinary force, and of magnitude a fraction x of the entire force between neutron and proton. We then write in the second integral $V=xV(r_{ij})P_{ij}$ where r_{ij} is the distance between proton and neutron and P_{ij} is the operator for exchanging the ith the proton with the jth neutron. Using the Hermitian character of zP_{ij}, Eq. (8) becomes:

$$\begin{aligned}(E_n-E_o)(\textstyle\sum_i z_i)_{on}=\frac{\hbar^2}{2M}\int \textstyle\sum_i\left(-\psi_n\frac{\partial\psi_o{}^*}{\partial z_i}+\psi_o{}^*\frac{\partial\psi_n}{\partial z_i}\right)d\tau\\ -\textstyle\sum_i\sum_j x\int \psi_nV(r_{ij})(z_i-z_j)P_{ij}\psi_o{}^*d\tau.\end{aligned} \quad (9)$$

[8] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).

[9] H. A. Bethe, *Handbuch der Physik* (1933), Vol. 24/1, p. 434.

We want N/A times the result summed over all protons combined with $-Z/A$ times this result summed over all neutrons. When we sum over all neutrons the roles of neutron and proton are reversed in the second integral on the right, and we have the negative of the result of Eq. (9). We find

$$\sum_n f_{on}=\sum_n\left\{\int\left[\frac{N}{A}\sum_i\left(\psi_o^*\frac{\partial\psi_n}{\partial z_i}-\psi_n\frac{\partial\psi_o^*}{\partial z_i}\right)\right.\right.$$
$$\left.-\frac{Z}{A}\sum_j\left(\psi_o^*\frac{\partial\psi_n}{\partial z_j}-\psi_n\frac{\partial\psi_o^*}{\partial z_j}\right)\right]d\tau$$
$$\left.-\sum_i\sum_j\frac{2M}{\hbar^2}x\int\psi_n V(r_{ij})(z_i-z_j)P_{ij}\psi_o^* d\tau\right\}$$
$$\times\left\{\int\left(\frac{N}{A}\sum_i\psi_n z_i\psi_o^*-\frac{Z}{A}\sum_j\psi_n z_j\psi_o^*\right)d\tau\right\},\tag{10}$$

$$\sum_n f_{on}=NZ/A-\frac{2M}{\hbar^2}x$$
$$\times\int\psi_o^*\sum_i\sum_j[\tfrac{1}{2}(z_i-z_j)+(N/A-\tfrac{1}{2})(z_i+z_j)]$$
$$\times(z_i-z_j)V(r_{ij})P_{ij}\psi_o d\tau.$$

We have two reasons to neglect the term $(N/A-\frac{1}{2})\times(z_i^2-z_j^2)$ as small when compared with $\frac{1}{2}(z_i-z_j)^2$. One is that $(N/A-\frac{1}{2})\ll\frac{1}{2}$, and the other that protons and neutrons have similar distributions in the nucleus, so $z_i^2-z_j^2$ is small. Making this approximation we have a result in agreement[10] with Siegert's.

$$\sum_n f_{on}=NZ/A-(2M/\hbar^2)x$$
$$\times\frac{1}{6}\int\psi_o^*\sum_i\sum_j r_{ij}^2 V(r_{ij})P_{ij}\psi_o d\tau.\tag{11}$$

We shall compute the last term of Eq. (11) on two different assumptions as to the potential $V(r)$. For purposes of illustration, let us first ignore the exchange operator P_{ij} and evaluate Eq. (11) for the square well potential.

$$\begin{aligned}V&=-V_o\quad r<b\quad V_o=s\pi^2\hbar^2/4Mb^2,\\ V&=0\qquad r>b\end{aligned}\tag{12}$$

where b is the intrinsic range and s is the depth parameter of Blatt and Jackson.[11] ψ_o is the wave function for all nucleons involving all their coordinates. When we consider the contribution of the ith proton and jth neutron, integration over the coordinates of all the other particles gives just unity. We shall neglect effects due to the edge of the nucleus, and use the model of a uniformly dense nucleus of radius $r_oA^{\frac{1}{3}}$.

$$|\psi_o|^2=\frac{1}{(4/3)\pi r_o^3A}.\tag{13}$$

For this wave function, each nucleon is "smeared out" over the whole nucleus; and each of the NZ neutron-proton pairs gives the same contribution. Using this potential and wave function in Eq. (11) we have

$$\sum_n f_{on}=NZ/A-\left(\frac{2M}{\hbar^2}\frac{1}{6}x\right)\left(s\frac{\pi^2}{4}\frac{\hbar^2}{Mb^2}\right)$$
$$\times\left(\frac{4}{3}\pi r_o^3A\right)^{-1}NZ\int_0^b r^2 4\pi r^2 dr$$
$$=(NZ/A)\left[1+\frac{\pi^2}{20}s(b/r_o)^3x\right].\tag{14}$$

The exchange effect on the summed oscillator strengths is the same order of magnitude as the first term in the bracket due to ordinary forces. For three-fourths of the interactions we should use the triplet well and for one-fourth of the interactions the singlet well.

Ignoring the exchange operator in Eq. (11), as is done in reaching this result, overestimates the effect of the exchange forces since the P_{ij} term has an interference effect. In calculating the effect of the exchange operator with interference we shall follow the treatment of the mixed density for a completely degenerate Fermi gas.[12] We use plane waves for each nucleon, with wave numbers chosen in accord with the Pauli principle. This neglects surface effects at the edge of the nucleus; as well as correlation effects among nucleons. For ground state wave function we use[13]

$$\psi_o=\prod_i\prod_j\exp(i\mathbf{k}_i\cdot\mathbf{r}_i)\exp(i\mathbf{k}_j\cdot\mathbf{r}_j).\tag{15}$$

Then

$$\psi_o^*P_{ij}\psi_o=\prod_i\prod_j\exp(-i\mathbf{k}_i\cdot\mathbf{r})\exp(i\mathbf{k}_j\cdot\mathbf{r}),\tag{16}$$

where $\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$ is the vector between the ith proton and jth neutron. We now wish to sum this over all protons and neutrons, for use in Eq. (11). Take the direction of $\mathbf{r}$ as the polar axis. The $\sum_i$ over protons is replaced by an integral over their wave numbers:

$$((4/3)\pi k^3)^{-1}\int_0^k\exp(-i\mathbf{k}_i\cdot\mathbf{r}_i)k_i^2\sin\theta dk_i d\theta d\varphi$$
$$=\frac{3}{(kr)^2}\left[\frac{\sin kr}{kr}-\cos kr\right].\tag{17}$$

Here k is the maximum wave number for protons in the

[10] Our exchange term appears at first to be twice that given by Siegert. This apparent difference arises from our different notation for summing over the nucleons.

[11] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

[12] H. A. Bethe and R. Bacher, Rev. Mod. Phys. **8**, 25 (1936).

[13] It is not necessary to write the determinantal wave function obeying the Pauli principle of antisymmetry between protons (or between neutrons) since we are concerned only with exchange between proton and neutron.

nucleus. We obtain the same result for the sum, or integration, over neutrons using k', the maximum wave number for neutrons.

To a first approximation, $Z=N$, and for this case

$$k=k'=(9\pi)^{\frac{1}{3}}/2r_o. \qquad (18)$$

In evaluating Eq. (11) we use Eq. (17) for the interference effect due to the exchange operator and otherwise follow our derivation of Eq. (14).

$$\sum_n f_{on}=NZ/A-\frac{2M}{\hbar^2}\frac{1}{6}x\frac{NZ}{(4/3)\pi r_o^3 A}\times\int_0^\infty\left[\frac{3}{(kr)^2}\left(\frac{\sin kr}{kr}-\cos kr\right)\right]^2 V(r)4\pi r^4 dr. \qquad (19)$$

The integral is evaluated for both square and Yukawa wells. For the former,

$$\sum_n f_{on}=(NZ/A)\left[1+\frac{\pi^2}{20}s(b/r_o)^3 x f_1(kb)\right],$$

where

$$f_1(kb)=\frac{45}{(kb)^4}\left[\frac{1}{2}+\frac{\sin 2kb}{4kb}-\frac{\sin^2 kb}{(kb)^2}\right]. \qquad (20)$$

For the Yukawa well $V=-V_o\exp(-\mu r)/r$,

$$\sum_n f_{on}=(NZ/A)\left[1+6\frac{M}{\mu\hbar^2}x\frac{V_o}{(\mu r_o)^3}f_2(kb)\right],$$

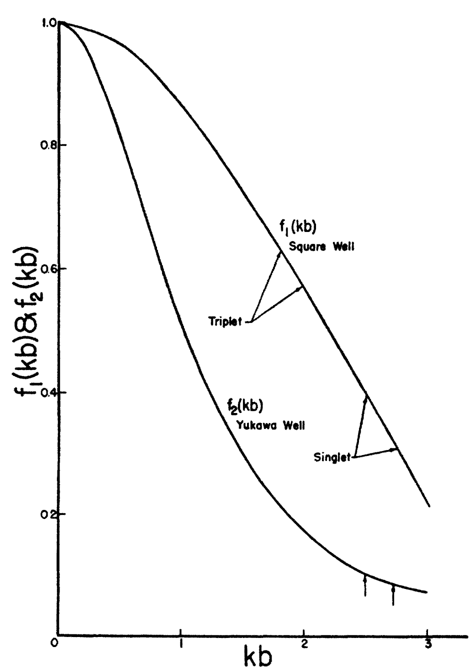

FIG. 1. Interference functions $f_1(kb)$ and $f_2(kb)$. See Eqs. (20) and (21) of the text. k is the maximum nucleon wave number, and b is the intrinsic range of the neutron-proton potential. The arrows show the values of kb used, using the nuclear constants of Eq. (22).

where

$$f_2(kb)=\frac{3}{8}\left(\frac{\mu}{k}\right)^4\left[-2+\left(1+\frac{\mu^2}{2k^2}\right)\log(1+4k^2/\mu^2)\right] \qquad (21)$$

and[11] $\mu^{-1}=b/2.12$. The interference functions $f_1(kb)$ for square well and $f_2(kb)$ for Yukawa well are plotted in Fig. 1. For k small, the interference function gives unity. In these equations r_o is the parameter for nuclear radius taken as 1.50×10^{-13} cm or 1.37×10^{-13} cm, and x is the fraction of attractive exchange force.

The following were assumed for the neutron-proton potential:[11, 14]

Square well:

triplet range $b=1.8\times10^{-13}$ cm,
depth $V_o=1.50(\pi^2/4)(\hbar^2/Mb^2)$,
singlet range $b=2.5\times10^{-13}$ cm,
depth $V_o=0.95(\pi^2/4)(\hbar^2/Mb^2)$. (22)
Yukawa well for both singlet and triplet:
$\mu^{-1}=1.18\times10^{-13}$ cm;
intrinsic range $b=2.12\mu^{-1}=2.5\times10^{-13}$ cm,
triplet depth $V_o=(1.45)3.56b\ \hbar^2/Mb^2$,
singlet depth $V_o=(0.95)3.56b\ \hbar^2/Mb^2$.

The numerical results for Eqs. (20) and (21) are given in Table I.

The summed oscillator strength, considering interference, is about $(NZ/A)(1+0.8x)$. That is, attractive exchange forces roughly double the summed oscillator strength.

Neglecting the interference effect, the result for the summed oscillator strengths is proportional to the assumed value of $(b/r_o)^3$. [See Eq. (14) for the square well.] Since the interference function decreases with increasing kb (and k depends on r_o), the result including interference is seen to be much less sensitive to the assumed value of b/r_o. Neglecting interference, the long tail of the Yukawa well gave a much larger contribution to the summed oscillator strength than did the square well. With interference this tail has very little effect, and the Yukawa well and square well give about the same contribution.

Corresponding to this summed oscillator strength we have the integrated cross section for photon absorption

$$\int\sigma dW=(2\pi^2e^2\hbar/Mc)\sum_n f_{on}=(2\pi^2e^2\hbar/Mc)(NZ/A)(1+0.8x). \qquad (23)$$

For the case $N=Z=A/2$ this gives the numerical result

$$\int\sigma dW=0.015A(1+0.8x)\ \text{Mev barns}. \qquad (24)$$

In the last section we show that this result is consistent with present experimental data.

[14] H. A. Bethe, Phys. Rev. 76, 38 (1949).

TABLE I. Summed oscillator strength (times A/NZ).

	Square well		Yukawa well	
r_o(cm)$\times 10^{13}$	1.50	1.37	1.50	1.37
Neglecting interference	$(1+1.48x)$	$(1+1.96x)$	$(1+6.5x)$	$(1+8.5x)$
With interference	$(1+0.80x)$	$(1+0.91x)$	$(1+0.69x)$	$(1+0.71x)$

III. MEAN ENERGY FOR PHOTON ABSORPTION

In this section we shall evaluate the mean energy for photon absorption:

$$\bar{W}=\sum_n f_{on}(E_n-E_o)/(\sum_n f_{on})$$

$$=\int \sigma W dW \bigg/ \int \sigma dW. \quad (25)$$

The numerator of this expression is given from Eq. (9) (integrate the first term on the right of Eq. (9) by parts).

$$\sum_n f_{on}(E_n-E_o)$$

$$=\frac{2M}{h^2}\sum_n \left|(E_n-E_o)\left[\frac{N}{A}(\sum_i z_i)_{on}-\frac{Z}{A}(\sum_j z_j)_{on}\right]\right|^2$$

$$=\frac{2M}{h^2}\sum_n\left|\frac{h^2}{2M}\int\left(2\frac{N}{A}\sum_i\psi_o^*\frac{\partial\psi_n}{\partial z_i}-2\frac{Z}{A}\sum_j\psi_o^*\frac{\partial\psi_n}{\partial z_j}\right)d\tau\right.$$

$$\left.-\sum_i\sum_j x\int\psi_n V(z_i-z_j)P_{ij}\psi_o^* d\tau\right|^2. \quad (26)$$

The first term in the second writing of Eq. (26) occurs for any force and the second term is due only to exchange forces. When squared we will have three terms: (1) the first term squared, which we shall call the ordinary term (it is the only term for pure ordinary forces); (2) the cross-product term; and (3) the pure exchange term. We shall evaluate these separately.

The ordinary term can be written using matrix notations where P is the z component of momentum:

$$\sum_{n\text{ ord}} f_{on}(E_n-E_o)$$

$$=\frac{2M}{h^2}\sum_n\frac{i\hbar}{M}\left(\frac{N}{A}(\sum_i P_i)_{on}-\frac{Z}{A}(\sum_j P_j)_{on}\right)$$

$$\times\left[-\frac{i\hbar}{M}\left(\frac{N}{A}(\sum_i P_i)_{no}-\frac{Z}{A}(\sum_j P_j)_{no}\right)\right]$$

$$=\frac{2}{M}[(N/A)^2(\sum_i P_i^2)_{oo}+(Z/A)^2(\sum_j P_j^2)_{oo}]. \quad (27)$$

In writing the last term we are neglecting correlation terms by taking only the diagonal elements in the double summation. That is, we are neglecting terms of the form $(P_iP_{i'})_{oo}$, where $i\neq i'$; and similar terms for two different neutrons, or a neutron and a proton. There will always be at least a small amount of correlation of momentum since the center of mass of the nucleus remains at rest in the ground state. If a particular proton has a momentum p_1, then the sum of the momenta of all the other nucleons is $-p_1$. If this momentum $-p_1$ is assumed to be distributed evenly among the other $A-1$ nucleons, the correlation terms can be computed to be $1/(A-1)$ of the diagonal terms that we shall calculate, and can therefore be neglected for heavy nuclei.

We can rewrite Eq. (27) using $p_i^2/2M=\frac{1}{3}T_i$ where T_i is the kinetic energy of the ith particle. (P is the z-component of momentum)

$$\sum_{n\text{ ord}} f_{on}(E_n-E_o)$$

$$=(4/3)[(N/A)^2(\sum_i T_i)_{oo}+(Z/A)^2(\sum_j T_j)_{oo}]$$

$$=(4/3)\bar{T}(NZ/A). \quad (28)$$

Here $\bar{T}$ is the expectation value of the kinetic energy for the average nucleon in the nucleus.[15] Using the uncorrelated plane-wave model of the nucleus with nucleons of wave number from 0 to $k_{\max}$ filling up phase space according to the Pauli principle,

$$\bar{T}=\frac{h^2\langle k^2\rangle_{\text{Av}}}{2M}$$

and

$$\langle k^2\rangle_{\text{Av}}=\left(\int_0^{k_{\max}}k^2k^2dk\right)\bigg/\left(\int_0^{k_{\max}}k^2dk\right)=\frac{3}{5}k_{\max}^2. \quad (29)$$

For the case of pure ordinary forces, the mean energy $\bar{W}$ for photon absorption is from Eqs. (12), (28), and (29),

$$\bar{W}=\sum_n f_{on}(E_n-E_o)/\sum_n f_{on}=(4/5)T_{\max}. \quad (30)$$

The value of $k_{\max}$ and $T_{\max}$ depend on the assumed nuclear radius. For parameter $r_o=1.37\times 10^{-13}$ cm, $(4/5)T_{\max}=19$ Mev; while for $r_o=1.50\times 10^{-13}$ cm, $(4/5)T_{\max}=16$ Mev. (The value of $\bar{W}$ depends on $1/r_o^2$; but is independent of A in our approximation where we neglect surface effects.)

The cross product (cp) term in $\sum_n(E_n-E_o)f_{on}$ is from Eq. (26)

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}$$

$$=-\sum_n 2\left[\int\left(-\frac{N}{A}\sum_i 2\psi_n\frac{\partial\psi_0^*}{\partial z_i}+\frac{Z}{A}\sum_j 2\psi_n\frac{\partial\psi_o^*}{\partial z_j}\right)d\tau\right]$$

$$\times\left[\sum_i\sum_j x\int\psi_n V(z_i-z_j)P_{ij}\psi_o^* d\tau\right]$$

[15] In taking the average, each proton is given weight N^2, and each neutron is given weight Z^2.

$$=4\frac{N}{A}x\int\sum_i\sum_j\frac{\partial\psi_o^*}{\partial z_i}V(z_i-z_j)P_{ij}\psi_o d\tau$$

$$-4\frac{Z}{A}x\int\sum_i\sum_j\frac{\partial\psi_o^*}{\partial z_j}V(z_i-z_j)P_{ij}\psi_o d\tau. \quad (31)$$

Using the plane-wave expression for ψ_o given in Eq. (15), we have $\partial\psi_o^*/\partial z_i=-k_{iz}\psi_o^*$ and $\partial\psi_o^*/\partial z_j=-ik_{jz}\psi_o^*$. Equation (31) becomes

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}$$

$$=4\frac{N}{A}x\int\sum_i\sum_j[-ik_{iz}\psi_o^*V(z_i-z_j)P_{ij}\psi_o$$

$$+ik_{jz}\psi_o^*V(z_i-z_j)P_{ij}\psi_o]d\tau$$

$$+4\left(-\frac{N}{A}+\frac{Z}{A}\right)x\int\sum_i\sum_j ik_{jz}\psi_o^*$$

$$\times V(z_i-z_j)P_{ij}\psi_0 d\tau. \quad (32)$$

We neglect the second term, since for actual nuclei $|Z/A-N/A|\ll N/A$; and we take $N/A=\frac{1}{2}$. Using $z_i-z_j=z$ and Eq. (16) for $\psi_o^*P_{ij}\psi_o$ the first term can be rewritten as

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}=2x\int\sum_i\sum_j\frac{\partial}{\partial z}(\psi_o^*P_{ij}\psi_o)Vzd\tau$$

$$=-2x\int\sum_i\sum_j\psi_o^*P_{ij}\psi_o\frac{\partial}{\partial z}(Vz)d\tau. \quad (33)$$

The sum over protons and neutrons is performed by integrating over their momenta, as in deriving Eq. (19). We have

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}=-\frac{NZ}{A}\frac{6}{r_0^3}x\int\left(\frac{1}{3}\frac{\partial V}{\partial r}r+V\right)$$

$$\times\left[\frac{3}{(kr)^2}\left(\frac{\sin kr}{kr}-\cos kr\right)\right]^2 r^2dr. \quad (34)$$

We shall evaluate this for both a square well and for a Yukawa well. For the square well of depth V_o and width b the integration gives

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}=(NZ/A)2V_0(b/r_0)^3xf_3(kb),$$

where

$$f_3(kb)=\frac{9}{(kb)^4}\left[-2\left(\frac{\sin kb}{kb}\right)^2+4\frac{\sin 2kb}{2kb}-2+kb\text{Si}(2kb)\right]. \quad (35)$$

where Si(x) is the integral sine function. For the Yukawa well, $V=-V_o\exp(-\mu r)/r$, Eq. (34) gives

$$\sum_{n\text{ cp}}(E_n-E_o)f_{on}=(NZ/A)18(V_o/\mu^2r_0^3)xf_4(kb),$$

where

$$f_4(kb)=-\tfrac{1}{4}(\mu/k)^4+\tfrac{1}{2}(\mu/k)^2-(\mu/k)^3\tan^{-1}(2k/\mu)$$

$$+\tfrac{1}{16}[(\mu/k)^6+8(\mu/k)^4]\log(1+4k^2/\mu^2) \quad (36)$$

and $\mu^{-1}=b/2\cdot 12$. The functions $f_3(kb)$ and $f_4(kb)$ are plotted in Fig. 2.

The third type of term is the exchange-exchange term which from Eq. (26) is

$$\sum_{n\text{ exch}}(E_n-E_o)f_{on}=\frac{2M}{\hbar^2}\sum_i\sum_j\tfrac{1}{3}x^2\int\psi_o^*V^2r^2\psi_o d\tau. \quad (37)$$

The square well potential gives

$$\sum_{n\text{ exch}}(E_n-E_o)f_{on}=(NZ/A)(b/r_o)^3x^2s(\pi^2/10)V_o. \quad (38)$$

For the Yukawa potential $V=-V_o\exp(-\mu r)/r$ we have

$$\sum_{n\text{ exch}}(E_n-E_o)f_{on}=\frac{NZ}{A}\frac{x^2}{4}\frac{2M}{\hbar^2}\left(\frac{b}{2.12r_o}\right)^3V_o^2. \quad (39)$$

These equations for the contribution of the cross-product and exchange-exchange terms are evaluated assuming the nuclear constants given in Eq. (22).

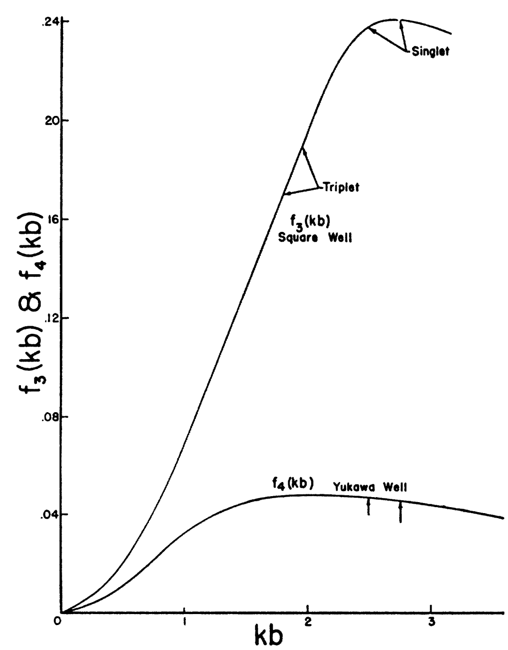

FIG. 2. Interference functions $f_3(kb)$ and $f_4(kb)$. See Eqs. (35) and (36) of the text. k is the maximum nucleon wave number; and b is the effective range of the neutron-proton potential. The arrows show the values of kb used, using the nuclear constants of Eq. (22).

Combining the ordinary, the cross-product and the exchange terms we find the mean energy for photon absorption $\bar{W}=\sum_n(E_n-E_o)f_{on}/\sum_n f_{on}$ for square well and $r_o=1.50\times10^{-13}$ cm (curve B)

$$\bar{W}=\frac{16+30x+101x^2}{1+0.80x}\text{ Mev;} \tag{40}$$

for square well and $r_o=1.37\times10^{-13}$ cm (curve A)

$$\bar{W}=\frac{19+42x+132x^2}{1+0.91x}\text{ Mev;} \tag{41}$$

for Yukawa well and $r_o=1.50\times10^{-13}$ cm (curve D)

$$\bar{W}=\frac{16+28x+36x^2}{1+0.69x}\text{ Mev;} \tag{42}$$

and for Yukawa well and $r_o=1.37\times10^{-13}$ cm (curve C)

$$\bar{W}=\frac{19+36x+48x^2}{1+0.71x}\text{ Mev.} \tag{43}$$

These four equations are plotted in Fig. 3.

In these four equations, the denominator is the value for $\sum_n f_{on}$ given in Table I, considering interference. The first term in the numerator is the ordinary term which was evaluated as 4/3 of the expectation value of the kinetic energy of an average nucleon. This term is sensitive to the assumed value for the parameter r_o in the nuclear radius $r_oA^{\frac{1}{3}}$; and does not depend on the well shape or character of the force between the nucleons. The second term in the numerator is the contribution

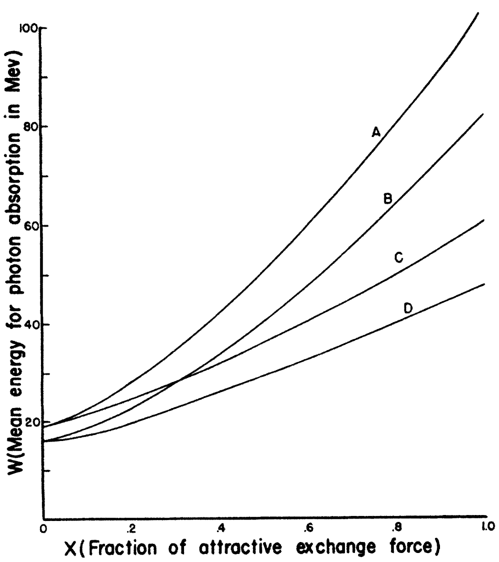

FIG. 3. Mean energy for photon absorption. The 4 curves are: A: square well, $r_o=1.37\times10^{-13}$ cm, see Eq. (41); B: square well, $r_o=1.50\times10^{-13}$ cm, see Eq. (40); C: Yukawa well, $r_o=1.37\times10^{-13}$ cm, see Eq. (43); D: Yukawa well, $r_o=1.50\times10^{-13}$ cm, see Eq. (42).

from the ordinary-exchange cross-product terms of Eqs. (35) or (36); and is therefore proportional to the fraction x of exchange force. The value of this term is sensitive to the ratio b/r_o, but it not sensitive to the well-shape. The third term in the numerator is the exchange-exchange term of Eqs. (38) or (39), and is therefore proportional to x^2. It is sensitive to the value of b/r_o; and also sensitive to the well-shape; being almost three times as large for the square well as for the Yukawa well. (The Gaussian and exponential wells, for the same effective range, give intermediate values between those for square and Yukawa wells.) With our model, which neglects surface effects at the boundary of the nucleus, $\bar{W}$ does not depend on A.

We would also like to calculate the harmonic mean energy for photon absorption

$$W_H=\sum_n f_{on}/[\sum_n f_{on}/(E_n-E_o)].$$

From Eq. (1) for the oscillator strength we have for a single proton

$$\sum_n f_{on}/(E_n-E_o)=\sum_n(2M/\hbar^2)|z_{on}|^2 = (2M/\hbar^2)(z^2)_{oo}. \tag{44}$$

If we take z_i as the component of displacement of the ith proton from the center of mass of the nucleus, and neglect all correlation terms:

$$\sum_n f_{on}/(E_n-E_o)=\frac{2M}{\hbar^2}\sum_n\left|\frac{N}{A}\sum_i(z_i)_{on}-\frac{Z}{A}\sum_j(z_j)_{on}\right|^2 = \frac{2M}{\hbar^2}\langle z^2\rangle_{oo}(NZ/A). \tag{45}$$

Here $\langle z^2\rangle_{oo}$ is the expectation value of the squared z component of displacement for the average nucleon in the nucleus. This result is independent of whether the force between nucleons is ordinary or exchange in character. Combining this result with that of Table I for $\sum_n f_{on}$ we have

$$W_H=\sum_n f_{on}/\sum_n f_{on}/(E_n-E_o)\cong\frac{\hbar^2}{2M\langle z^2\rangle_{oo}}(1+0.8x). \tag{46}$$

If we evaluate $\langle z^2\rangle_{oo}$ from our assumed values for the nuclear radius we find that W_H is only about 5 Mev for Cu^{63} and varies as $A^{-\frac{2}{3}}$, reaching a value of less than 1 Mev for uranium. This large nuclear absorption of comparatively low energy photons has not been observed. Let us instead determine $\langle z^2\rangle_{oo}$ from experiment.

Using Eq. (45) and the relation between oscillator strength and cross section we have

$$\int\frac{\sigma}{W}dW=\frac{2\pi^2e^2\hbar}{Mc}\sum_n f_{on}/(E_n-E_o) = \left(\frac{e^2}{\hbar c}\right)\frac{4\pi^2}{3}\frac{NZ}{A}\langle r^2\rangle_{oo}. \tag{47}$$

Using the experimental measurements discussed in the last section, we have $\langle r^2\rangle_{oo}=5.8\times10^{-26}$ cm^2 for the nucleus Ge^{76}. This compares with $\frac{3}{5}(r_oA^{\frac{1}{3}})^2=20\times10^{-26}$ cm^2 or 24×10^{-26} cm^2, depending on which value for r_o we assume.

These comparisons suggest that we should have a model where the mean square displacement in the ground state $\langle r^2\rangle_{oo}$ does not vary greatly with A, and is much smaller than the value found where r is measured from the center of the nucleus. We can obtain better agreement with experiment by considering as a tentative model a nucleus made up of sub-units, such as alpha-particles. For dipole transitions in a nucleus made of alpha-particles the distance r_i for the ith proton should be measured from the center of the alpha-particle of which the ith proton is a member.[16] Then $\langle r^2\rangle_{oo}$ will not vary with A, and will be much smaller than calculated above. We might assume that the alpha-particle in a heavy nucleus is a sphere of uniform density occupying four times the volume allotted to a single nucleon. Then $\langle r^2\rangle_{oo}$ would equal $\frac{3}{5}(r_o4^{\frac{1}{3}})^2=2.9\times10^{-26}$ cm^2 or 3.5×10^{-26} cm^2, depending on which value for r_o we use. This is of the same order of magnitude as the experimental results.

One might wonder whether the alpha-particle model would give very different results for $\sum_n f_{on}$ and $\bar{W}$ from those calculated on the plane-wave model. The result $\sum_n f_{on}=NZ/A$ for ordinary forces is independent of correlation among the nucleons. The increase due to attractive exchange forces came mainly from neutron-proton distances of order $k^{-1}\cong10^{-13}$ cm, due to interference effects and the short-range potential. For these small distances the plane-wave model should not be too bad. The result $\bar{W}=(4/3)\bar{T}_{oo}$ for ordinary forces would be changed somewhat, due to the different calculation of the average kinetic energy $\bar{T}_{oo}$. The contributions to $\bar{W}$ from the cross-product and exchange terms will not be greatly changed, since again the main contributions come from small neutron-proton distances.

IV. QUADRUPOLE TRANSITIONS

The electric moment for pure ordinary forces is[3]

$$D_{on}=(eh/M\omega)[\exp(i\mathbf{k}\cdot\mathbf{r})\nabla]_{on}. \quad (48)$$

For the quadrupole moment we take the second term, $i\mathbf{k}\cdot\mathbf{r}$, in the expansion of the exponential. This gives ky, where y is the component of $\mathbf{r}$ along the direction of propagation. The gradient is in the direction of polarization of the photon. Defining the oscillator strength as[3]

$$g_{on}=(2M\omega/e^2\hbar)(D_{on})^2, \quad (49)$$

we have the quadrupole oscillator strength as

$$g_{on}=(2k^2/M\hbar\omega)|(yP_z)_{on}|^2 = [2(E_n-E_o)/\hbar^2Mc^2]|(yP_z)_{on}|^2. \quad (50)$$

Then

$$\sum_n g_{on}/(E_n-E_o)=(2/\hbar^2Mc^2)\sum_n(yP_z)_{on}(P_zy)_{no} = (2/\hbar^2Mc^2)(yP_z^2y)_{oo}. \quad (51)$$

The integrated cross section for quadrupole transitions becomes, according to (2)

$$\int_0^\infty \sigma_q dW/W=4\pi^2(e^2/\hbar c)(1/Mc^2)(1/M)(yP_z^2y)_{oo}. \quad (52)$$

Assuming no correlation between y and p_z we write this as

$$\int_0^\infty \sigma_q dW/W=4\pi^2(e^2/\hbar c)(\tfrac{2}{3}T_{oo}/Mc^2)\tfrac{1}{3}\langle r^2\rangle_{oo}. \quad (53)$$

This result should be multiplied by the number of protons in the nucleus, if we assume no correlations among the protons, and the average kinetic energy $\bar{T}_{oo}$ should be used. For Ge^{76} we have the numerical result of 6 mb; or only about six percent of the experimental result for this quantity of over 100 mb as discussed in the last section. Thus quadrupole transitions represent only a small fraction of the nuclear photo-effect.

There is, however, good evidence that quadrupole transitions are as important as dipole transitions for low excitation energy. The measurements of internal conversion of gamma-radiation following beta- or alpha-decay indicate that dipole transitions, at an excitation energy of roughly 1 Mev, are very weak.[3] The small gamma-ray widths observed in most $(n-\gamma)$ reactions indicate that the oscillator strength for radiative transitions to the ground state are very small,[3] indicating that at 8 Mev excitation dipole transitions are not important. Earlier measurements[3] indicated that $(p-\gamma)$ reactions for light nuclei had gamma-ray width corresponding to quadrupole rather than dipole transitions. However recent measurements[17,18] give a much larger gamma-ray width, indicating dipole transitions. This is true, e.g., of the well-known reaction $Li^7(p,\gamma)$Be^8. The most striking, however, is the He$^3(p,\gamma)$ reaction,[18] where the gamma-ray width of about 1 kev corresponds to an oscillator strength of about one-half for the transition from the excited state of He^4 at 22 Mev down to the ground state. The sum of all dipole transitions from the ground state, for ordinary forces, is only $NZ/A=1$ for He^4, so this represents a very strong dipole transition. Further the angular distribution observed for the photons from this reaction proves that it is a dipole transition.

We have shown above that photo-nuclear reactions, which appear to occur principally at an excitation energy[2] of 20 to 30 Mev are predominantly dipole transitions.

Thus a model is needed which will suppress dipole transitions at excitation energies of, say, less than 10

[16] We can get this same result by considering the correlations among the particles in deriving Eq. (45).

[17] Fowler, Lauritsen, and Lauritsen, Rev. Mod. Phys. 20, 236 (1948).

[18] R. F. Taschek *et al.* (private communication).

Mev, but allow them at higher energies. Strong correlation between neutrons and protons, as, for example, in the alpha-particle model, will suppress dipole transitions at low energies. In a nucleus composed entirely of alpha-particles there will be no dipole transitions[3] due to vibration of the alpha-particles in the nucleus, as in the dipole approximation all the alpha-particles experience the same electric field and vibrate together. (We should put $N=0$ in our sum rule $\sum_n f_{on}=NZ/A$, giving no dipole transitions.) But at higher photon energies the photon can disrupt a particular alpha-particle by a dipole transition, thus transferring energy to the nucleus. That is, correlations will not prevent dipole transitions for photon energies great enough to excite the first excited state of the alpha-particle, or about 20 Mev. At very high photon energies, where the value of $\lambda\!\!\!^{-}$ for the photon is comparable to the size of the alpha-particle, higher multipole transitions will again be of importance.

We must make clear that correlations among the nucleons can never prevent dipole transitions altogether. Correlations can only prevent dipole transitions in some given energy range, but this decrease must be compensated for by stronger dipole transitions in some other energy range. The sum rule for ordinary forces $\sum_n f_{on}=NZ/A$ is not altered by correlations among the nucleons.

Thus the alpha-particle model gives qualitatively correct predictions for the integrated cross section $\int\sigma dW/W$, and for the energy dependence of ratio of quadrupole to dipole transitions. It is, of course, only a tentative model, and should not be taken too literally.

V. APPLICATION TO THE DEUTERON

Calculations of the photoelectric cross section for the deuteron for various photon energies have been performed on various assumptions as to size and shape of well, and the exchange character of the neutron-proton force. By comparison with these results we are able to check our calculations using sum rules. Also discussion of this simplest case of the photoelectric effect helps us to obtain some physical understanding of the change in the sum rule due to exchange forces.

For the deuteron, we shall use z as the component along the polarization of the displacement of the proton from the neutron, which is twice the proton displacement from the center of mass. We use the reduced mass $M/2$ for the proton; and the dipole oscillator strength is

$$f_{on}=\frac{2(M/2)}{\hbar^2}(E_n-E_o)\tfrac{1}{4}(z_{on})^2. \qquad (54)$$

Using Eq. (9) for $(E_n-E_o)z_{on}$ we find the summed oscillator strength

$$\sum{}_n f_{on}=\tfrac{1}{4}\left[1-\tfrac{2}{3}xM\hbar^{-2}\int\psi_o{}^*Vr^2P_{ij}\psi_o d\tau\right]. \qquad (55)$$

Pure ordinary forces ($x=0$) give $\sum f_{on}=\frac{1}{4}$ and give the integrated cross section

$$\int\sigma dW=\frac{2\pi^2e^2\hbar}{(M/2)c}\sum{}_n f_{on}=\pi^2e^2\hbar/Mc. \qquad (56)$$

This well-known result is in agreement with the result found by integration of the Bethe-Peierls formula, which holds for zero range of nuclear forces; and is also in agreement with the result for ordinary forces given by Breit and Condon[19] for a square well of finite range.

We have evaluated the contribution to the summed oscillator strength due to exchange forces for a square well and also for a Yukawa well using the nuclear constants given in Eq. (22). For the square well using the complete wave function and the constants of Eq. (22) we have

$$\sum{}_n f_{on}=\tfrac{1}{4}+0.10x. \qquad (57)$$

For the Yukawa well we use Hulthen's approximate wave function for the ground state[20] and obtain

$$\sum{}_n f_{on}=\tfrac{1}{4}+0.083x. \qquad (58)$$

Thus for $x=1$ (pure attractive exchange force) there is a 40 percent increase or a 33 percent increase in the summed oscillator strength for the square and Yukawa wells, respectively.

This increase for a square well is in qualitative agreement with Breit and Condon's[19] results for exchange forces. The result for the Yukawa well can be compared with the dipole cross section found for a Yukawa potential half exchange and half ordinary in character:[20]

$$\sigma=20.4[1-\gamma^2/(29+\gamma)^2]^2(\gamma-1)^{\frac{3}{2}}\gamma^{-3}\ \text{mb}. \qquad (59)$$

Here γ is the ratio of photon energy to deuteron binding energy. Numerical integration gives

$$\int_0^\infty\sigma dW=35.8\ \text{Mev}-\text{mb}=(\pi^2e^2\hbar/Mc)\cdot(1.2). \qquad (60)$$

This 20 percent increase due to half exchange force is in fair agreement with 17 percent as found from the sum rule calculation (Eq. 58). [Both Eqs. (58) and (59) are based on an approximate wave function for the ground state of the Yukawa potential.]†

The calculation of $\sum_n(E_n-E_o)f_{on}$ and $\bar{W}$ involves three terms as for the similar calculation for heavy nuclei:

$$\sum{}_n(E_n-E_o)f_{on}=\tfrac{1}{3}T_{oo}+2x\int\frac{\partial\psi_o{}^*}{\partial z}Vz\psi_o d\tau$$
$$+(M/\hbar^2)x^2\int\psi_o{}^*V^2z^2\psi_0 d\tau. \qquad (61)$$

[19] G. Breit and E. U. Condon, Phys. Rev. 49, 904 (1936).
[20] J. S. Levinger, Phys. Rev. 76, 699 (1949).
† E. Guth has called to our attention similar work on sum rules for the deuteron photoelectric effect by K. Way, Phys. Rev. 51, 552 (1937). K. Way found that the summed oscillator strength was increased for complete exchange force by a relative fraction about equal to αb, for a square well, a Gaussian well, or a velocity dependent potential. The work in this section for a square well and a Yukawa well is in good agreement with her result.

This is evaluated for square and Yukawa wells with the constants of Eq. (22), and the mean energy for photon absorption is found.

$$\bar{W}=\frac{17.2+27.5x+36x^2}{1+0.40x}\text{ Mev (square well)},$$
$$\bar{W}=\frac{16.4+21.6x+12.6x^2}{1+0.33x}\text{ Mev (Yukawa well)}. \quad (62)$$

Equation (62) for the Yukawa well with $x=\frac{1}{2}$ gives a mean absorption energy of 26 Mev, in approximate agreement with $\bar{W}=29.7$ Mev found by numerical integration of Eq. (59).

The harmonic mean energy W_H is calculated from Eq. (46). This gives $W_H=6\epsilon=13.2$ Mev for zero range of nuclear forces; $3.90\epsilon(1+0.40x)$ for the square well chosen; and $3.57\epsilon(1+0.33x)$ for the Yukawa well chosen. The first number agrees with the result from integration of the Bethe-Peierls formula; while the third result using $x=\frac{1}{2}$ agrees with that found from integration of Eq. (59).

We have found that the summed oscillator strength is increased by an attractive exchange force; and the mean energy for photon absorption is greatly increased by such a force. This can be understood qualitatively since an attractive exchange force for the ground (S) state means a repulsive exchange force for the final (P) state. This repulsive exchange force decreases the P wave function at small distances and thus the matrix element for small E_n. For large E_n, it shifts the first node of the radial wave function to larger values of r which reduces destructive interference and thus increases the matrix element. Since $\sum_n f_{on}/(E_n-E_o)$ is independent of whether the force is ordinary or exchange, $\sum_n f_{on}$ will be increased by an attractive exchange force, and higher moments will be increased even more.

This reference to S and P states is just a special case for the general case of parity change due to dipole photoelectric transitions in any nuclei; the same interpretation holds for the more general case. This physical argument implies that the presence of exchange forces will not affect cross sections for quadrupole transitions.

VI. ASYMPTOTIC EXPRESSION FOR PHOTOELECTRIC CROSS SECTION

For the deuteron we are able to calculate still higher moments such as $\sum_n f_{on}(E_n-E_o)^2$. This is of interest in finding the asymptotic behavior of the dipole term in the photoelectric cross section for high photon energies. If for a particular potential we find that $\sum_n f_{on}(E_n-E_o)^s$ is finite, but that $\sum_n f_{on}(E_n-E_o)^{s+1}$ is infinite, this shows that the asymptotic form of the dipole term in the photoelectric cross section decreases as W^{-t}, where W is the photon energy and $s+1<t\leqq s+2$. We shall calculate the higher moments for the deuteron photoelectric cross section for the case of a pure ordinary force, and for several well shapes.

The first moment $\sum_n f_{on}(E_n-E_o)=\frac{1}{3}T_{oo}$ is finite for wells of finite size. The mean kinetic energy for a square well is approximately

$$T_{oo}=-\frac{\hbar^2}{M}\int\psi_o{}^*\nabla^2\psi_o d\tau$$
$$=\frac{\hbar^2}{M}\frac{2\alpha}{1+\alpha b}\left[\int_0^b k^2\sin^2 kr dr-\int_b^\infty \alpha^2 e^{-2\alpha r}dr\right]. \quad (63)$$

For negligible range of nuclear forces $kb\cong\pi/2$, and the average of $\sin^2 kr\cong\frac{1}{2}$ so the first integral in the bracket gives a result proportional to $1/b$. Since the first moment diverges for zero range of nuclear forces, the exponent t for the asymptotic form for this case has the limits $1<t\leqq 2$. This agrees with $t=\frac{3}{2}$ given for this potential by the Bethe-Peierls formula.

We shall use the matrix relations:[9]

$$(E_n-E_o)z_{on}\sim i\hbar(\dot{z})_{on}\sim(i\hbar/M)(p_z)_{on},$$
$$(E_n-E_o)(p_z)_{on}\sim i\hbar(\dot{p}_z)_{on}\sim i\hbar(\partial V/\partial z)_{on} \quad (64)$$

in determining the higher moments. The second moment gives:

$$\sum_n f_{on}(E_n-E_o)^2=\frac{2M}{\hbar^2}\sum_n(E_n-E_o)^3(z_{on})^2$$
$$\sim\sum_n(E_n-E_o)|(P_z)_{on}|^2\sim\sum_n(P_z)_{on}\left(\frac{\partial V}{\partial z}\right)_{no}$$
$$\sim\left(P_z\frac{\partial V}{\partial z}\right)_{oo}\sim\left(\frac{\partial^2 V}{\partial z^2}\right)_{oo}\sim(\nabla^2 V)_{oo}. \quad (65)$$

For the Yukawa potential we have

$$\nabla^2 V=4\pi\rho+\mu^2 V. \quad (66)$$

Here ρ is the density of the source of the meson field, and will be taken as a δ-function around the origin. Both terms on the right of Eq. (66) give finite results in Eq. (65) for the second moment.

For the square well potential of depth V_o and range b

$$\nabla^2 V=V_o\delta'(r-b)+(2/r)V_o\delta(r-b). \quad (67)$$

Both terms on the right of this equation give finite results in Eq. (65).

The third moment:

$$\sum_n f_{on}(E_n-E_o)^3\sim\sum_n[(E_n-E_o)(p_z)_{on}]^2$$
$$\sim\sum_n[(\partial V/\partial z)_{on}]^2\sim[(\partial V/\partial r)^2]_{oo}. \quad (68)$$

For the Yukawa potential, the leading term in the third moment calculation is $(1/r^4)_{oo}$ which diverges at the origin. Then the asymptotic form for the Yukawa potential is W^{-t} where $3<t\leqq 4$. This agrees with the calculation for the Yukawa potential (half exchange and half ordinary)[20] which gives $t=7/2$.

For the square well potential, the third moment is proportional to $[\delta(r-b)\delta(r-b)]_{oo}$ which diverges at $r=b$. The square well, also, has an asymptotic form with $3<t\leqq 4$. However, this divergence is caused by the artificial singularity of the infinitely steep walls of the well. If the walls are made to have a high but not infinite slope the third moment converges.

Equation (68) shows that the third moment converges for any potential that has a finite derivative at each point. A smooth potential, such as the Gaussian, will have a smooth wave function, with very small high Fourier components, and will therefore have very small probability of dipole transitions with photons of very short wave-length.

It is of interest to note that the electronic photoelectric effect has just the same $W^{-7/2}$ asymptotic dependence as that for the nuclear photo-effect for the Yukawa potential. The asymptotic dependence is set by the singularities in the potential, and the Yukawa singularity at the origin of $\exp(-\mu r)/r$ is the same as $1/r$ for the Coulomb potential.*

In general the asymptotic dependence of the cross section at high energies depends just on the nature of the singularities of the nuclear potential. The results for singularities at the origin apply to heavy nuclei as well as to the deuteron: it is only necessary that there be a finite probability for two nucleons to come very close, then $[(\partial V/\partial r)^2]_{oo}$ will be infinite whenever V has a singularity as $r^{-\frac{1}{2}}$ or stronger at small distances.

For comparison with experimental results, we must know in what region, if any, the asymptotic form for dipole transitions holds. The photon energy must be large enough that the de Broglie wave-length λ of the emitted nucleon is small compared to the effective range of the potential. But if the photon wave-length becomes comparable to the size of the nuclear system, the dipole transition cross section will not correspond to the experimentally determined photoelectric cross section: we must consider retardation effects, quadrupole and higher moments, and perhaps mesonic effects. If the effective range of nuclear forces and the size of the nuclear system are about the same (as for the alpha-particle) there is a region of photon energies satisfying these criteria, since the depth of the nuclear potential well is much less than Mc^2.

For the deuteron case, the range of the neutron-proton potential is much less than the size of the deuteron ($\alpha b<1$). For wave-lengths of the emitted proton $\lambda_p \gg b$, Eq. (59) for dipole transitions for a Yukawa well gives results in approximate agreement with the Bethe-Peierls formula, which takes range $b=0$. (In this energy region nearly all the contribution is from the region outside the potential.) For $\lambda_p \ll b$, the interference term in Eq. (59) changes the $W^{-3/2}$ energy dependence of the Bethe-Peierls formula to a $W^{-7/2}$ dependence. The inequality $\lambda_p \ll b$ gives $\hbar^2/2ME \ll b^2$, where E is the proton energy; or proton energy $E \gg \hbar^2/2Mb^2 \cong V_o$, the depth of the neutron-proton potential for a square well. Dipole transitions are the principal term in the deuteron photo-disintegration provided $\lambda_\gamma \alpha \gg 1$, or $(\hbar c/W)(M\epsilon)^{\frac{1}{2}}/\hbar \gg 1$, where W is the photon energy and we have used the definition of α. This relation gives $W \ll (\epsilon Mc^2)^{\frac{1}{2}}=45$ Mev. Since the depth of the neutron-proton potential is comparable to 45 Mev, we find that for the deuteron there is actually no range of photon energies where the total cross section for photo-disintegration follows the asymptotic $W^{-7/2}$ form.[21]

* *Note added in proof:* Similar conclusions have been reached by L. I. Schiff, Bull. Am. Phys. Soc. **25**, No. 1, Abstract E2 (1950).

VII. EXPERIMENTAL DATA

The G.E. experiments provide information concerning the integrated cross section for photo-nuclear reactions, the energy dependence of the photo-nuclear cross section, and set an upper limit on elastic nuclear scattering of photons.

Lawson and Perlman[1] measured the C^{11} activity produced by the $C^{12}(\gamma,n)$ reaction. The photons were produced as bremsstrahlung in the betatron, and the photon intensity measured by reference to a pair spectrometer. They expressed the photon intensity as number of quanta/Mev-min., at a photon energy of 30 Mev. The ratio of (γ,n) processes/min.-atom, to the number of quanta/Mev. min., is given as 1.5×10^{-25} Mev-cm$^2\pm 20$ percent. This result was obtained both with electron energy of 100 Mev and 50 Mev.

Lawson and Perlman interpret their result assuming a sharply peaked curve of cross section *vs.* photon energy. We shall here express their result in a more general manner. Let $\gamma(W)$ be the photon spectrum from the betatron, normalized to 1 photon/Mev at $W=30$ Mev. Then their result can be written

$$\int_0^{W_{\max}} \sigma_{(\gamma,n)}(W)\gamma(W)dW = 0.15 \text{ Mev-barn} \pm 20 \text{ percent} \quad (69)$$

for $W_{\max}=100$ Mev, or 50 Mev. A rough approximation to $\gamma(W)$ is the normalized bremsstrahlung spectrum $\gamma(W)=30/W$. The upper limit for the integral can be infinity, since Lawson and Perlman found no difference for upper limit 50 or 100 Mev. Then the integrated (γ,n) cross section for C^{12} can be rewritten as

$$\int_0^{\infty} \sigma_{(\gamma,n)}dW/W = 0.005 \text{ barn} \pm 20 \text{ percent.} \quad (70)$$

Measurements of the relative yields of many photonuclear reactions, using the method of induced activities, were made with the same photon spectrum by Perlman and Friedlander.[22] Combining their relative

[21] We should also note that Eq. (59) holds exactly only for the special case of half ordinary and half exchange forces, with wells of the same shape. Further, Siegert's theorem, reference 7, has been extrapolated to high photon energies, where it may not hold.

[22] M. L. Perlman and G. Friedlander, Phys. Rev. **74**, 442 (1948) and Phys. Rev. **75**, 988 (1949).

yields with the absolute determination by Lawson and Perlman, we now have experimental data for $\int \sigma_p dW/W$ for many different nuclei, where σ_p refers to the partial cross sections for specific nuclear disintegrations studied by the method of induced activities. There are always some nuclear reactions that are missed by this method since they result in stable isotopes. To obtain information on $\int \sigma dW/W$ we therefore look for the nucleus which gives the largest experimental result for $(1/A)\int \sigma_p dW/W$, since for that nucleus the partial cross section σ_p is probably most nearly equal to the photon absorption cross section σ.[23] This integrated cross section is largest for $Ge^{76}(\gamma,n)$, for which $\int \sigma_p dW/W = 0.126$ barn. This result is used in Section III in estimating the mean square displacement $\langle r^2\rangle_{oo}$, and in Section IV to show that quadrupole transitions can account for only a small fraction of the experimentally determined cross section.

We want to obtain an experimental value for $\int \sigma dW$, since our sum rules for that quantity are not greatly affected by the nuclear model used. $\int \sigma dW = W_H \int \sigma dW/W$, where W_H is the harmonic mean energy for photon absorption. W_H can be obtained from Baldwin and Klaiber's measurements[2] of the energy dependence of the $Cu^{63}(\gamma,n)$ cross section. Using their value $W_H = 21$ Mev, the relative yield $Cu^{63}(\gamma,n)/C^{12}(\gamma,n)$ and Lawson and Perlman's measurements on $C^{12}(\gamma,n)$ we have for Cu^{63}

$$\int \sigma_{(\gamma,n)} dW = (33/2.3)(21)0.005 = 1.5 \text{ Mev-barns} \pm 30 \text{ percent.} \quad (71)$$

This result should be increased by 10 percent, since the $Cu^{63}(\gamma,2n)$ and $Cu^{63}(\gamma,2p)$ yields together represent 10 percent of the $Cu^{63}(\gamma,n)$ yield.[22] Further, the $Cu^{63}(\gamma,p)$ and $Cu^{63}(\gamma,np)$ reactions that lead to stable isotopes are probably appreciable.[24]

We conclude that for Cu^{63}, $\int \sigma dW \geqq 1.5$ Mev-barns ± 30 percent. This result agrees with that given by Lawson and Perlman, and is derived in detail here only to show that we did not need any assumption as to the shape of the $\sigma(W)$ curve, but only the harmonic mean energy W_H. Since W_H for photon absorption is not known reliably for any other nuclei (see below) this is at present the only experimental result for $\int \sigma dW$.

This experimental result is in good agreement with our Eq. (24) for dipole transitions. For Cu^{63} our calculation gives $\int \sigma dW = 0.95(1+0.8x)$ Mev-barn. For no exchange forces we have 0.95 Mev-barn, which is the same result as given by Goldhaber and Teller,[5] and is not too far outside the experimental error. Solving for the fraction of exchange force we have $x = 0.8 \pm 0.7$. The value $x = 0.55 \pm 0.05$ from the Berkeley neutron-proton scattering experiments[25] gives a result well within the experimental error. We conclude that the experimental value of $\int \sigma dW$ for Cu^{63} is strong evidence for dipole transitions, and weak evidence for exchange forces.

The energy dependence of photonuclear reactions has been measured by Baldwin and Klaiber[2] for $C^{12}(\gamma,n)$, $Cu^{63}(\gamma,n)$ and photo-fission, and by McElhinney and others[26] for $Cu^{63}(\gamma,n)$ and $Ta^{182}(\gamma,n)$. These measurements were made by differentiating the curve of yield of induced activity *vs.* maximum energy of the bremsstrahlung spectrum. Bothe and Gentner[27] have measured the relative yields for several (γ,n) reactions at 11- and 17-Mev photon energies.

All these measurements have large uncertainties in the curve of $\sigma_{(\gamma,n)}$ *vs.* photon energy. (For example, the two measurements on Cu^{63} disagree.) Further, the curve $\sigma_{(\gamma,n)}$ *vs.* W gives an accurate picture of the curve of photon absorption cross section σ *vs.* W only if the (γ,n) reaction is the predominant photonuclear reaction. We can determine this by comparing the experimental value of $\int \sigma_{(\gamma,n)} dW$ with the value $\int \sigma dW$ given by our sum rules. The comparison for $Cu^{63}(\gamma,n)$ given shows that for this nucleus the (γ,n) reaction is predominant. For $C^{12}(\gamma,n)$ and $Ta^{182}(\gamma,n)$ the integrated (γ,n) cross section is only about one-third of the expected value of $\int \sigma dW$, and, as shown by Goldhaber and Teller, the photo-fission cross section is much less than this fraction.

Evidence for the difference between measurements of the (γ,n) cross section and other measurements of nuclear photon absorption was found in recent preliminary results by McDaniel and Walker.[28] They measured the yield of neutrons from photo-disintegration of various nuclei for photons of 17.5 Mev, and obtained a smooth curve for relative neutron yield *vs.* A. In contrast, Wäffler *et al.*[29] found that the relative yield for induced activity produced by the (γ,n) reaction at this photon energy showed large deviations from a smooth curve. We believe that neutron production is a better measure of nuclear photon absorption than is the production of radioactivity in some particular photo-nuclear reaction because at least one neutron is emitted in *any* absorption process, except (γ,p), $(\gamma,2p)$ etc. reactions which are probably relatively rare. Measurements of integrated cross sections, and of cross sections *vs.* photon energy, by this method of measuring neutron production would be of great interest.

Thus, to date, only the measurements on $Cu^{63}(\gamma,n)$ reaction give a value of mean energy for photon ab-

[23] We divide by A since from the theory of this paper the integrated cross section is roughly proportional to A.

[24] However use of the better approximation $\gamma(W) = (4/3) \times (1 - W/W_{max})/W$ to the bremsstrahlung spectrum reduces by about 10 percent the integrated cross sections given in Eqs. (70) and (71).

[25] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. 75, 351 (1949); and R. Serber (private communication).

[26] McElhinney, Hanson, Becker, Duffield, and Diven, Phys. Rev. 75, 542 (1949).

[27] W. Bothe and Gentner, Zeits. f. Physik 112, 45 (1939).

[28] McDaniel and Walker (private communication).

[29] Huber, Lienhard, Scherrer, and Wäffler, Helv. Phys. Acta 17, 195 (1945) and 16, 33 (1943).

sorption which can be compared with our calculations of Section III. Baldwin and Klaiber[2] give $\bar{W}=22$ Mev for the (γ,n) reaction. Since other photo-nuclear reactions of Cu^{63} generally occur at higher photon energies than does the (γ,n), we can write $\bar{W}\geqq 22$ Mev. Our calculated value for $\bar{W}$ is sensitive to the value of x, to the shape of the neutron-proton potential, and to the nuclear density. Figure 3 shows that 22 Mev corresponds to values of the fraction of exchange force varying from 0.1 to 0.3 depending on the shape of well and nuclear density assumed. The value $x=0.55\pm0.05$ gives a very high value of $\bar{W}$ for a square well, but not too unreasonable a value for a Yukawa well; 38 Mev for a dense nucleus ($r_o=1.37\times10^{-13}$ cm), and 31 Mev for a less dense nucleus ($r_o=1.50\times10^{-13}$ cm). In principle better experimental measurements of $\bar{W}$ and less approximate calculations would provide good evidence on some combination of well shape, nuclear density, and the fraction of exchange force.

Similarly only the $Cu^{63}(\gamma,n)$ measurements give evidence as to the shape of the curve of the cross section for photon absorption *vs.* photon energy; and even in this case it has not been proved that competition between the (γ,n) and other photo-nuclear reactions is not appreciable. The sharply peaked curve of $\sigma_{(\gamma,n)}(W)$ found by Baldwin and Klaiber for this reaction can be taken as evidence for the Goldhaber-Teller theory[5] of dipole vibration of the entire nucleus. Goldhaber and Teller assume that only one excited nuclear level is of importance (the first excited level for the simple harmonic dipole vibration of the entire nucleus) so the width of the $\sigma(W)$ curve would be due entirely to the energy-spread of that excited level due to its short life. Goldhaber and Teller believe that the gamma-ray width of this level should be appreciable compared to the neutron width, so that there should be appreciable elastic scattering of gamma-rays in this narrow energy band. Gaertner and Yeater[30] have recently looked for gamma-rays of the resonance energy scattered at 90° from C^{12} or Cu^{63}. They set the upper limit for elastic nuclear scattering of photons as one percent of the (γ,n) cross section for C^{12}, and three percent for Cu^{63}.

The ratio of elastically scattered photons to nuclear disintegrations is $\Gamma_\gamma/(\Gamma_n+\Gamma_\gamma)\cong\Gamma_\gamma/\Gamma_n$, where Γ_γ is the gamma-ray width, and Γ_n the neutron width. Γ_γ can be found from the oscillator strength.[3] Assuming that there is a single level having the summed oscillator strength of $(NZ/A)(1+0.8x)=20$ for Cu^{63}, and that the resonance energy is about 22 Mev, then $\Gamma_\gamma=50$ kev. Gaertner and Yeater's upper limit for scattered photons then gives $\Gamma_n>1.7$ Mev for the neutron width for Cu^{63}. This neutron width is not inconsistent with the measurements of Baldwin and Klaiber, which give a "resonance width" the order of magnitude of 3 Mev. However, we could rule out the assumption that Baldwin and Klaiber's width is mostly instrumental.

[30] E. R. Gaertner and M. L. Yeater, Phys. Rev. **76**, 363 (1949).

A many-level picture of photon absorption, which is consistent with our calculations in this paper, would not give very different results from those on the Goldhaber-Teller single level theory.[31] Consider L levels, with average spacing D, so that LD= the width of the $\sigma(W)$ curve observed by Baldwin and Klaiber. At high neutron energies the neutron width from state n to the ground state, $\Gamma_{no}\cong D\xi/2\pi\cong D/2\pi$, since sticking probability $\xi\cong1$ at high neutron energies. The total neutron width from a level excited by photon absorption is

$$\Gamma_n=\sum\nolimits_r \Gamma_{nr}\cong\Gamma_{no}N_n(W-B)\cong(D/2\pi)N_n(W-B). \quad (72)$$

The summation goes over neutron energies between zero and $(W-B)$, where B is the neutron binding energy, and W is the photon energy. $N_n(W-B)$ is the number of levels of the nucleus in this energy range that can be reached by neutron emission.

The gamma-width $\Gamma_{\gamma o}$ for photon emission to the ground state from a single level is about $1/L$ that for the gamma-width Γ_γ for a single level theory. The excited state can decay by photon emission to many levels above the ground state, and if the final level is less than, say, 5 Mev above ground then the emitted photon will be hard to distinguish experimentally from a photon of the resonance energy. The effective gamma-width

$$\Gamma_{\gamma\,\mathrm{eff}}=\sum_{r=0}^{5\,\mathrm{Mev}}\Gamma_{\gamma r}\cong\Gamma_{\gamma o}N_\gamma(5\ \mathrm{Mev})\cong(\Gamma_\gamma/L)N_\gamma(5\ \mathrm{Mev}). \quad (73)$$

The ratio of scattered photons to neutron emission is

$$\frac{\sigma_{(\gamma,\gamma)}}{\sigma_{(\gamma,n)}}\cong\frac{(\Gamma_\gamma/L)N_\gamma(5\ \mathrm{Mev})}{(D/2\pi)N_n(W-B)}=\frac{\Gamma_\gamma}{\Gamma}2\pi\frac{N_\gamma(5\ \mathrm{Mev})}{N_n(W-B)}, \quad (74)$$

where we have used the relation $LD=\Gamma=$ observed width of "resonance curve." The ratio Γ_γ/Γ is the ratio used for the Goldhaber-Teller theory. For Cu^{63} the energy $W-B\cong22-11=11$ Mev; so the ratio of levels reached by photon emission to those reached by neutron emission $N_\gamma(5\ \mathrm{Mev})/N_n(W-B)$ will be less than unity, and perhaps of order of magnitude 0.01; so this calculation gives a result that the ratio (scattered photons/emitted neutrons) would be roughly one-tenth that predicted from the Goldhaber-Teller theory.

Several reports have appeared very recently which confirm the experimental work discussed in this section. Strauch[32] has measured the transition curves in lead for photons from the Berkeley synchrotron causing the (γ,n) reaction in C^{12} or Cu^{63}. He finds the mean energy for photons causing these reactions to be 30 and 20 Mev, respectively, in agreement with Baldwin and Klaiber's measurements by a different method. Helmholz and

[31] This calculation of (photon emission/neutron emission) was done by H. A. Bethe and H. Hurwitz, of the Knolls Atomic Power Laboratory, General Electric Company, Schenectady, New York.

[32] K. Strauch, Phys. Rev. **78**, 84 (1950).

Strauch[33] have measured the yields for several photonuclear reactions with the Berkeley synchrotron, and find a preliminary value of 1 Mev-barn for the integrated $Cu^{63}(\gamma,n)$ cross section, in agreement with the work of Lawson and Perlman. Dressel, Goldhaber, and Hanson,[34] have measured resonance scattering of photons, using the 22-Mev Illinois betatron. They used the activation of Pr^{141} by the (γ,n) reaction to detect scattered photons of high energy. The cross section for resonance scattering of photons for lead nuclei is roughly 1 mb per steradian, and is considerably less than this for copper and antimony nuclei. Their upper limit for scattering by copper nuclei is about one-tenth that found by Gaertner and Yeater. The result for lead is consistent with the Goldhaber-Teller theory for a resonance width of 4 Mev, and is also consistent with the theory of this paper.*

Our calculations from sum rules do not give a definite prediction as to the shape of the $\sigma(W)$ curve; but only the integrated cross section and the mean energy for photon absorption. The curve could be broad, or could have a sharp resonance. If we take the alpha-particle model of the nucleus as a first approximation, we might expect a strong resonance at a photon energy of roughly 20 Mev, as photons of this energy are emitted with a high probability in the $T^3(p,\gamma)He^4$ reaction.[18] However, "alpha-particles" in the nucleus will certainly behave differently from free alpha-particles, and the resonance might be greatly broadened.

The alpha-particle model gives qualitatively satisfactory results concerning the ratio $\sigma_{(\gamma,p)}/\sigma_{(\gamma,n)}$ for heavy nuclei. Hirzel and Wäffler[35] found this ratio for a photon energy of 17 Mev to be many times greater than that predicted by the statistical theory of nuclei. Perlman and Friedlander[22] find even larger (γ,p) yields [roughly 10 percent of (γ,n) yields] using the G.E. betatron at 100 Mev, or 50 Mev electron energy. The only two isotopes which were measured by both groups are Si^{30} and Mo^{98}. The (γ,p) yield, relative to $Cu^{63}(\gamma,n)$ was increased 14 times for the former, and three times for the latter, by use of the higher energy photons from the betatron rather than the 17-Mev gammas.

Courant[36] assumed a surface photoelectric effect for the whole nucleus for proton emission and found too low a proton yield to account for Hirzel and Wäffler's results. Schiff[37] assumed that only nuclear levels of special properties could be excited by photons, and obtained rough agreement with Hirzel and Wäffler's results. However neither author found that the (γ,p) yield should increase rapidly with increasing photon energy.

We propose the following tentative picture which is similar to that suggested in a letter by Courant: The main process of nuclear absorption is by absorption of energy by a single proton in a nuclear alpha-particle. Usually this proton interacts with other nucleons before getting out of the nucleus; the excitation energy then becomes shared among many nucleons, and a neutron of energy corresponding to the nuclear temperature will in general be emitted, in accord with the statistical theory. Occasionally the proton escapes without making any collisions. (This will occur for a proton emitted from a surface alpha-particle in the right direction.) Since the mean free path for protons in a nucleus increases with increasing proton energy, the probability of proton escape increases with proton energy.

We want to make clear that these calculations from the alpha-particle model are tentative, and of a preliminary nature only.

In conclusion, we have shown by sum rules that there *are* dipole transitions, and that quadrupole transitions can account for only a small fraction of the observed integrated photonuclear cross sections. Goldhaber and Teller assume a specific model for dipole transitions; i.e., that there is a dipole vibration of the entire nucleus. From this model they obtain the integrated cross section, predict a sharp resonance curve for photon absorption, estimate the resonance energy and predict elastic nuclear scattering of photons. Making no *ad hoc* assumption of dipole vibration of the entire nucleus, we have found an integrated cross section somewhat larger than that of Goldhaber and Teller, due to exchange forces. Either integrated cross section agrees with present experiments. We find a reasonable mean energy for photon absorption, if a Yukawa well is used with not too high a fraction of exchange force. Experimental evidence on the shape of the $\sigma(W)$ curve is not yet decisive; we could obtain qualitative agreement with a resonance peak by consideration of the alpha-particle model. We predict a somewhat lower elastic scattering of photons than do Goldhaber and Teller. Either result is in agreement with present experiments. We can qualitatively understand the ratio of proton to neutron emission, using the alpha-

[33] A. C. Helmholz and K. Strauch, Phys. Rev. **78**, 86 (1950).

[34] Dressel, Goldhaber, and Hanson, Phys. Rev. **77**, 754 (1950).

* *Note added in proof:* Gaerttner and Yeater, Phys. Rev. **77**, 714 (1950) have recently measured the cross sections for photo-nuclear reactions in nitrogen and oxygen. Cloud-chamber observations of reactions produced by x-rays from the G.E. betatron showed "flags," singles, and stars. The flags are interpreted as (γ, pn) reactions, and the singles as (γ, n) processes. Gaerttner and Yeater found that flag production was the predominant process, for both oxygen and nitrogen. They calculate the total integrated photonuclear cross section as about 0.6 Mev-barn, for both nuclei. These measurements confirm the increase of the integrated photo-nuclear cross section due to exchange forces. Using this data, together with our Eq. (24), we find that the fraction of exchange force $x=2\mp 1$, where we have assumed a 30 percent uncertainty in their experimental result. (Equation (24) was calculated for heavy nuclei; surface effects might change the numbers for light nuclei such as oxygen and nitrogen.) Gaerttner and Yeater's measurements also show the importance of competition among different photonuclear reactions.

[35] Hirzel and Wäffler, Helv. Phys. Acta **20**, 373 (1947).

[36] E. D. Courant, Phys. Rev. **74**, 1226(A) (1948) and private communication.

[37] L. I. Schiff, Phys. Rev. **73**, 1311 (1948).

particle model, while Goldhaber and Teller's theory does not consider this question. We believe that many of the features of the nuclear photo-effect can be understood merely from the fact that there are dipole transitions, without a special model.

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The Fission Yield of Xe^{133} and Fine Structure in the Mass Yield Curve

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A mass spectrometer investigation of the xenon isotopes formed from U^{235} fission has been made and the fission yield and half-life of Xe^{133} determined. The fission gases were extracted, purified, and the mass spectrometer abundance data obtained within two weeks of the end of the irradiation period. The fission yield of Xe^{133} was found to be 6.29±0.01 percent, which is 20 percent higher than that expected from the mass yield curve. This is further evidence of fine structure in the mass fission yield curve. The half-life value of Xe^{133} was found to be 5.270±0.002 days.

THODE and Graham[1] first determined mass spectrometrically the relative abundances of stable isotopes of Xe and Kr resulting from the decay of fission product chains in U^{235} fission. In this way the relative fission yields of eight mass chains—83, 84, 85, 86, 131, 132, 134 and 136, were determined with considerable accuracy. When these yields are normalized to the mass-yield curve at a value of 2.8 percent for mass 131, the values fit the curve nicely with the exception of Xe^{134} which is about 35 percent above the normal fission yield curve. Glendenin[2] has pointed out that the value obtained by Thode and Graham[1] of Kr^{84} is also above the experimental mass yield curve by about 35 percent. These results indicated for the first time "fine structures" in the mass yield curve. Recently, Inghram, Hess, and Reynolds[3] reported isotope abundance data for fission product cesium, which also indicate anomalies in the mass yield curve.

If it is assumed that the yields should fall on a smooth curve, then the abnormal yields of Xe^{134} and Kr^{84} might be explained by delayed or prompt neutron emission. If this fission chain branching does occur, then the fission yield of adjacent chains will be affected. It seemed important, therefore, to determine accurately the fission yields of the 133 and 135 mass chains. In the original mass spectrometer investigations of the fission gases, Xe^{133} and Xe^{135} did not occur because of their short half-lives, and because it was not possible to get samples immediately after irradiation. However, with the Chalk River facilities of the National Research Council available, it has now been possible to extract Xe gas from irradiated uranium disks without a long "cooling" period and thus permit the investigation of 5.3-day Xe^{133}. In order to calculate the fission yield of Xe^{133} from abundance data, (after a definite irradiation and cooling time), it is necessary to know its half-life with considerable accuracy. Since the previous values obtained by radio-chemical methods,[4–12] were only good to 1 or 2 percent, a new and more accurate value was determined mass spectrometrically. This determination of the half-life of Xe^{133} and the determination of the fission yield for the 133 mass chain are reported in this paper.

THEORY

Half-Life Determination

By comparing the abundance of a radio-active isotope with that of a stable one with a mass spectrometer over a period of time, it is possible to follow its decay rate and thereby determine its half-life. By this method, very accurate half-life determinations are possible for isotopes with half-lives ranging from about one day to ten years. The fundamental decay equation is

$$n/n_0 = e^{-\lambda t} = \exp(-0.6932t/t_{\frac{1}{2}}),$$

where n is the concentration at time t and n_0 is the concentration at zero time.

By substituting for example the 133/131+132 ratio obtained with the mass spectrometer at different time

[1] H. G. Thode and R. L. Graham, Can. J. Research, **A25**, 1–14 (1947). Technical Report No. 35.
[2] G. L. Glendenin, Ph.D. thesis, Department of Chemistry, M.I.T. (August 1949).
[3] Inghram, Hess, and Reynolds, Phys. Rev. **76**, 1717 (1949).
[4] A. Langsdorf, Jr., Phys. Rev. **56**, 205 (1939).
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