The Influence of Nuclear Structure on the Hyperfine Structure of Heavy Elements

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The influence on the h.f.s. of the finite size of the nucleus is considered and the effect is calculated for simple models of the nuclear magnetism. It is pointed out that the distribution of magnetic dipole density over the nuclear volume may vary greatly from nucleus to nucleus depending on the relative contributions of spin and orbital magnetic moments to the total nuclear moment. On this basis an attempt is made to interpret the observed discrepancy between the h.f.s. ratio of the Rb isotopes and the ratio of the magnetic moments as determined by the magnetic resonance method. A study of such anomalies may give some information regarding the structure of nuclear moments, in particular, regarding the nuclear g_L-factor.

I. INTRODUCTION

A RECENT accurate determination[1] of the nuclear moments of the Rb isotopes by the magnetic resonance method has indicated that the ratio of the h.f.s. splittings in Rb^{85} and Rb^{87}, measured previously with great precision,[2] does not agree exactly with the value calculated from the ratio of the moments, if the nuclei are considered as point dipoles. The h.f.s. ratio is found to be larger by 0.33 percent, while the experimental uncertainty involved in the comparison is judged to be about 0.05 percent.

It has been pointed out by Bitter[3] that anomalies of this order may be expected if the nuclear magnetic moments are represented by some distribution of magnetism over the nuclear volume rather than by a point dipole. Since effects of this type might offer information regarding the structure of nuclear moments, we shall attempt a somewhat more detailed analysis of the problem.

II. QUALITATIVE CONSIDERATIONS

In the non-relativistic approximation, the h.f.s. of an s-state is proportional to the average electron density at the location of the nuclear magnetic moment. It is here assumed that the moment distribution is spherically symmetric, but even large angular asymmetries have only a minor effect. At a small distance R from a central charge Ze, the electron density is proportional to $1-2ZR/a_0$ where a_0 is the radius of the hydrogen atom. Inside the nucleus itself, however, the wave function decreases somewhat more slowly with distance. Thus, in a nucleus of uniform charge distribution, the electron density varies approximately as $1-ZR^2/a_0R_0$, where R_0 is the nuclear radius.

In a model in which the nuclear magnetic moment is considered as a smeared-out dipole distribution, the h.f.s. would thus be expected to differ from the value calculated for a point dipole at the nuclear center by a factor $1+\epsilon$, where

$$\epsilon \approx -(ZR_0/a_0)(R^2/R_0^2)_{\text{Av}}. \tag{1}$$

For heavy atoms, relativity becomes of importance and its main effect in the present connection is to increase the absolute magnitude of the electron density at the nucleus by a factor of about $(a_0/2ZR_0)^{2(1-\rho)}$, where $\rho=(1-Z^2\alpha^2)^{\frac{1}{2}}$ and α is the fine structure constant. The total h.f.s. is increased in corresponding measure and, thus,

$$\epsilon \approx -(ZR_0/a_0)(a_0/2ZR_0)^{2(1-\rho)}(R^2/R_0^2)_{\text{Av}}. \tag{2}$$

The assumption of a uniform distribution of the nuclear charge restricts our considerations to nuclei containing a large number of protons. For the lightest elements the dependence of the electronic wave function on the position of the individual protons must be taken into account.[4]

If the magnetic moment is uniformly distributed over the nucleus, we have $(R^2/R_0^2)_{\text{Av}}=\frac{3}{5}$ and, by putting $R_0=1.5\times10^{-13}$ cm $A^{\frac{1}{3}}$, values for ϵ are obtained which for light elements are of the order of 0.01 of a percent and which become quite appreciable, of the order of several percent, for the heaviest elements.

For $p, d\cdots$ states the h.f.s. anomaly is negligible in the non-relativistic approximation, in which the corresponding wave functions vanish at the center. In heavy elements, however, due to relativity effects, the value of ϵ may become appreciable for $p_{\frac{1}{2}}$-states. For these states the small components have the character of s-wave functions and determine the density at the center. The value of ϵ will therefore be of the order of

* On leave from Institute for Theoretical Physics, Copenhagen, Denmark.

[1] F. Bitter, Phys. Rev. 75, 1326 (1949).

[2] S. Millmann and P. Kusch, Phys. Rev. 58, 438 (1940).

[3] F. Bitter, Phys. Rev. 76, 150 (1949); see also H. Kopfermann, *Kernmomente* (Akademische Verlagsgesellschaft, Leipzig, 1940), p. 17.

[4] A. Bohr. Phys. Rev. 73, 1109 (1948).

$Z^2\alpha^2$ of that for an $S_{\frac{1}{2}}$-state. For an electron of higher total angular momentum than $\frac{1}{2}\hbar$, the influence on the h.f.s. of the finite size of the nucleus may be disregarded.

In the particular case of Rb, one obtains from (2) a value for ϵ of about 0.4 percent. Whereas effects of this magnitude are thus to be expected in the h.f.s. of each isotope, it might at first appear surprising that the ratio of the h.f.s. shows an anomaly of the same order. In fact, it becomes necessary to assume an essentially different distribution of magnetism in the two nuclei.

It appears that a natural explanation of such variations is possible if one considers the nuclear magnetic moment as composed of two intrinsically different parts, a spin moment and an orbital moment. In fact, the latter part, originating from currents in the nucleus, will in general be equivalent to a magnetic dipole distribution which increases toward the center, and should therefore produce smaller anomalies in the h.f.s. than the former, the spin part. For example, a rotating charged sphere is equivalent to a magnetization increasing toward the center and would give rise to a value of ϵ only about half that corresponding to a sphere of uniform magnetization. Depending on the proportion of spin magnetic moment and orbital magnetic moment in the nucleus, considerable variations in the h.f.s. anomalies may thus occur. Indeed, a nucleus in which the spin moment is directed oppositely to that of the total nuclear magnetic moment might even have a larger h.f.s. than that corresponding to a point dipole.

In the case of Rb, the values of the angular momenta and the magnetic moments actually indicate a very different alignment of spin and orbital momentum in the two isotopes, and therefore an essentially different distribution of magnetism. Indeed, while the magnetic moment of Rb^{87} ($I=\frac{3}{2}$, $\mu=2.75$) appears primarily due to the spin moment of the odd proton, the moment of Rb^{85} ($I=5/2$, $\mu=1.35$) would seem to be largely of orbital type with only a small, and perhaps even negative, spin contribution. On the basis of such considerations, it thus appears possible to understand the comparatively large decrease in the h.f.s. of Rb^{87} relative to that of Rb^{85}.

It is of interest that no effect of comparable magnitude has been found in the cases of Ga[5] or Tl[6] where the two isotopes have the same spins and comparable moments and, therefore, presumably, similar distributions of magnetism.

III. H.F.S. DUE TO SPIN MAGNETIC MOMENT

In the more quantitative considerations, we shall treat separately the cases of spin and orbital moment. In the former case we represent the nucleus by a distribution of magnetic moment, given by a density function $w(\mathbf{R})$, and having the direction of the spin angular momentum $\mathbf{s}$. Denoting the spin g-factor by g_s, the vector potential produced by the nucleus may be written

$$\mathbf{A}_s(\mathbf{r})=-\int d\tau_R w(\mathbf{R})g_s\left(\mathbf{s}\times\nabla_r\frac{1}{|\mathbf{r}-\mathbf{R}|}\right), \quad (3)$$

where

$$\int d\tau_R w(\mathbf{R})=1.$$

The magnetic interaction of the nucleus with an atomic electron is given by $e\boldsymbol{\alpha}\mathbf{A}(\mathbf{r})$ and, considering the interval rules, it is necessary only to evaluate the diagonal matrix element of this operator for the state in which the electronic, as well as the nuclear, angular momentum has its maximum component in the z direction. Denoting the electron part of the wave function by ψ, one finds that,

$$W\equiv\int d\tau_r\psi^* e\boldsymbol{\alpha}\mathbf{A}(\mathbf{r})\psi$$
$$=\pm 2e\int d\tau_r F(r)G(r)\frac{1}{r}(\mathbf{A}\times\mathbf{r}_z), \quad (4)$$

where F and G represent the two radial wave functions. The upper and lower sign refer to $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ states, respectively.

Introducing (3) into (4), it is convenient to write

$$W_s=\pm\frac{16\pi}{3}eg_s\int d\tau_R w(\mathbf{R})q_s. \quad (5)$$

If the distribution $w(\mathbf{R})$ is spherically symmetric, q_s takes the simple form

$$q_s=s_z\int_R^\infty FGdr. \quad (6)$$

In the case of angular asymmetries, one must add to (6)

$$\left\{\tfrac{1}{2}s_x\frac{3XZ}{R^2}+\tfrac{1}{2}s_y\frac{3YZ}{R^2}+\tfrac{1}{2}s_z\frac{3Z^2-R^2}{R^2}\right\}\int_0^R\frac{r^3}{R^3}FGdr, \quad (7)$$

but since the effect of such asymmetries for not too extreme models is of only minor influence, we shall in the following use the simple expression (6) for q_s.

Defining

$$\kappa_s\equiv\int_0^R FGdr\Big/\int_0^\infty FGdr, \quad (8)$$

it follows that W_s is decreased by the relative amount $(\kappa_s)_{\mathrm{Av}}$, as a consequence of the deviation of the nuclear magnetization from a point dipole.

IV. H.F.S. DUE TO ORBITAL MOMENT

It may first be noted that the magnetic field of a current density distribution $\mathbf{i}$, for which $\operatorname{div}\mathbf{i}=0$, is

[5] R. B. Pound, Phys. Rev. **73**, 1112 (1948).
[6] H. Poss, Phys. Rev. **75**, 600 (1949).

TABLE I. Values of b for $s_{\frac{1}{2}}$- and $p_{\frac{1}{2}}$-states.

Z	$b(s_{\frac{1}{2}})$	$b(p_{\frac{1}{2}})$
10	0.08 percent	
20	0.20	
30	0.38	0.02 percent
40	0.68	0.05
50	1.12	0.13
60	1.71	0.29
70	2.52	0.60
80	3.58	1.16
90	4.80	2.11

equivalent to that produced by a magnetic dipole density $\boldsymbol{\mu}$ given by $\mathbf{i}=-c\,\mathrm{rot}\boldsymbol{\mu}$, and thus the problem of an orbital moment is reducible to that considered in the preceding paragraph.

Still, in many cases, it is more convenient to consider directly the interaction of a moving nuclear particle with the atomic electron. Denoting by Ze and M the charge and mass of the particle, and its wave function by $\varphi(\mathbf{R})$, we may use the expression

$$\mathbf{A}_L(\mathbf{r})=\frac{Ze}{Mc}\int d\tau_R\varphi^*(\mathbf{R})\frac{1}{|\mathbf{r}-\mathbf{R}|}\mathbf{P}\varphi(\mathbf{R}) \qquad (9)$$

and, introducing in (4) one finds that W_L can be expressed in the form

$$W_L=\pm\frac{16\pi}{3}eg_L\int d\tau_R w(\mathbf{R})q_L, \qquad (10)$$

where $w(\mathbf{R})=|\varphi(\mathbf{R})|^2$ and $g_L=Ze/2Mc$. Furthermore,

$$q_L=L_Z\left\{\int_R^\infty FGdr+\int_0^R\frac{r^3}{R^3}FGdr\right\}, \qquad (11)$$

where $\mathbf{L}$ is the orbital angular momentum of the particle. Thus, if

$$\kappa_L\equiv\int_0^R\left(1-\frac{r^3}{R^3}\right)FGdr\Big/\int_0^\infty FGdr, \qquad (12)$$

the contribution to the h.f.s. of the orbital angular momentum is decreased by a relative amount $(\kappa_L)_{\mathrm{Av}}$.

V. EVALUATION OF ELECTRONIC WAVE FUNCTIONS

In order to calculate κ_s and κ_L, it is necessary to evaluate the electronic wave functions in the interior of the nucleus, in which region they deviate significantly from the wave functions corresponding to a point nucleus. Representing the nucleus by a homogeneously charged sphere of radius R_0, the potential in this region is given by

$$V=(\tfrac{3}{2}-\tfrac{1}{2}x^2)Ze/R_0 \quad x\equiv r/R_0<1. \qquad (13)$$

Solving the radial wave equations by an expansion in powers of x, one finds, for an $s_{\frac{1}{2}}$-state

$$G=k\left(1-\frac{3}{8}\gamma^2x^2+\frac{1}{10}\gamma^2x^4+\cdots\right) \quad \gamma\equiv Ze^2/\hbar c$$
$$F=\tfrac{1}{2}k\gamma x\left(1-\left(\frac{1}{5}+\frac{9}{40}\gamma^2\right)x^2+\frac{1}{10}\gamma^2x^4+\cdots\right) \qquad (14)$$

and, for a $p_{\frac{1}{2}}$-state

$$F=k\left(1-\frac{3}{8}\gamma^2x^2+\frac{1}{10}\gamma^2x^4+\cdots\right)$$
$$G=-\tfrac{1}{2}k\gamma x\left(1+\frac{4}{3}\frac{R_0mc}{\gamma\hbar}-\left(\frac{1}{5}+\frac{9}{40}\gamma^2\right)x^2+\frac{1}{10}\gamma^2x^4+\cdots\right), \qquad (15)$$

where k is a constant depending on the normalization of the entire wave function. These approximate expressions have an accuracy of about one percent, even for Z-values corresponding to the heaviest elements.

The integrals in the numerators of (8) and (12) may now be expressed in terms of powers of R. The main term is proportional to R^2 and since the higher terms never amount to more than ten percent of the leading term, one may conveniently replace these terms by appropriate multiples of R^2. If values are chosen intermediate between those corresponding to a uniform distribution $w(\mathbf{R})$ and those corresponding to a surface distribution, the error involved will, for the most plausible models, only amount to a few percent. In this manner, one obtains

$$\int_0^R FGdr=0.23k^2R_0\gamma(1-0.2\gamma^2)\times\frac{R^2}{R_0^2}\left[(-1)\left(1+1.44\frac{R_0mc}{\gamma\hbar}\right)\right] \qquad (16)$$
$$\int_0^R FG\left(1-\frac{r^3}{R^3}\right)dr=0.62\int_0^R FGdr,$$

where the factor in square brackets is to be included for $p_{\frac{1}{2}}$-states only.

Outside the nucleus the electron moves in a Coulomb field screened at larger distances by the other atomic electrons. However, we need consider only the unscreened part of the field, since the h.f.s. interaction takes place primarily in this central part of the atom.

The integral in the denominators of (8) and (12) may thus be expressed in terms of the well-known solutions to the wave equation for an unscreened Coulomb field, normalized relatively to the wave functions in the interior of the nucleus by the boundary conditions at the nuclear surface.[7] One finds approximately

$$\int_0^\infty FGdr = \tfrac{1}{4}k^2\frac{\hbar}{mc}\left(\frac{2R_0Z}{a_0}\right)^{2(1-\rho)}\frac{3}{\rho(4\rho^2-1)}\{(2\rho-1)!\}^2 \times\begin{cases}(1-\tfrac{1}{2}\gamma^2)^2 & (s_{\frac{1}{2}}) \\ (-1)\dfrac{4}{3\gamma^2}(1-0.72\gamma^2)^2 & (p_{\frac{1}{2}})\end{cases} \quad (17)$$

where $\rho=(1-\gamma^2)^{\frac{1}{2}}$. The use of the wave functions for an unscreened field in the evaluation of (17) is always well justified for s-states, and is also valid for $p_{\frac{1}{2}}$-states in heavy atoms.

It follows that the values of κ_s and κ_L are independent of k and, consequently, of the particular $s_{\frac{1}{2}}$- or $p_{\frac{1}{2}}$-state under consideration. One may conveniently write

$$\kappa_s = bR^2/R_0^2$$
$$\kappa_L = 0.62bR^2/R_0^2, \quad (18)$$

where the coefficients b depend only on Z and R_0. For $R_0 = 1.5\times10^{-13}$ cm$\times A^{\frac{1}{3}}$, the values of b for $s_{\frac{1}{2}}$- and $p_{\frac{1}{2}}$-states are given in Table I.

VI. COMBINED EFFECT OF SPIN AND ORBITAL MOMENT

If the total nuclear magnetic moment is composed of a spin part and an orbital part, the relative change in the total h.f.s. splitting may be written

$$\epsilon = -(\kappa_s)_{\text{Av}}\alpha_s - (\kappa_L)_{\text{Av}}\alpha_L, \quad (19)$$

where α_s and α_L represent the fractions of the nuclear moment due to spin moment and orbital moment, respectively. These quantities may be expressed in terms of the g-factors:

$$\alpha_s = \frac{g_s}{g_I}\frac{g_I-g_L}{g_s-g_L} \quad \alpha_L = 1-\alpha_s, \quad (20)$$

where g_I is the total nuclear g-factor.

The experiments determine most easily the difference of the ϵ-values for two isotopes. Of course, the values of $(\kappa_s)_{\text{Av}}$ and $(\kappa_L)_{\text{Av}}$ may vary somewhat from isotope to isotope, but, it appears that larger effects may be expected due to differences in α_s and α_L. If we neglect the fluctuations in $(\kappa_s)_{\text{Av}}$ and $(\kappa_L)_{\text{Av}}$ and if, moreover, the values of g_s and g_L are the same for the two isotopes, as seems a plausible assumption for isotopes differing by an even number of neutrons, one obtains the expression:

$$\Delta \equiv \epsilon(1)-\epsilon(2) = ((\kappa_s)_{\text{Av}}-(\kappa_L)_{\text{Av}})\frac{g_sg_L}{g_s-g_L}\left(\frac{1}{g_I(1)}-\frac{1}{g_I(2)}\right) \quad (21)$$

for the influence of the finite size of the nucleus on the h.f.s. ratio of isotopes 1 and 2.

It is, of course, difficult to estimate the mean values of $(R/R_0)^2$ entering into the expressions for $(\kappa_s)_{\text{Av}}$ and $(\kappa_L)_{\text{Av}}$. It may be noted that for a uniform distribution over the sphere, the mean value equals $\frac{3}{5}$, but one might well imagine a tendency of the unpaired particles which contribute to the nuclear moment to stay near the surface, in which case the mean value would be somewhat larger. Since, however, it cannot exceed unity, one may tentatively assume a value for $(R^2/R_0^2)_{\text{Av}}$ of about $\frac{4}{5}$. On this assumption one obtains, from (18) and (21),

$$\Delta \approx 0.3b\frac{g_sg_L}{g_s-g_L}\left(\frac{1}{g_I(1)}-\frac{1}{g_I(2)}\right), \quad (22)$$

but it need not be emphasized that the approximation may be rather crude.

VII. DISCUSSION AND COMPARISON WITH EXPERIMENTAL DATA

The expression (22) for Δ involves the spin and orbital g-factors, the values of which must be expected to depend on the type of nuclei. The large majority of nuclei having angular momenta contain an odd number of nucleons. In this case it is most often assumed that the spin g-factor equals that of the odd particle, i.e., $g_s = g$ (proton) for Z odd and $g_s = g$ (neutron) for Z even. The choice of g_L, however, is more uncertain and we shall consider two possible assumptions regarding the origin of the orbital momentum.

On the one hand, we may assume with Margenau and Wigner[8] that the nuclear matter as a whole is involved in the orbital momentum. This leads in a plausible manner to $g_L = Z/A$, in units of $e/2Mc$, M being the nucleon mass.

On the other hand, Schmidt[9] has tried to account for nuclear moments by ascribing the orbital momentum to the motion of the odd particle in the nucleus. This leads to $g_L = 1$ for Z odd and $g_L = 0$ for Z even. It may be added that these g-values do not necessarily imply a single particle model of the nuclear moment. Any model in which, for Z odd, only protons, and for Z even, only neutrons, contribute to the orbital momentum leads to the same g-values and is therefore equivalent for our purpose.

The two assumptions regarding g_L lead to appreciably different values for Δ, and the phenomenon in question might thus offer some evidence regarding the nature of the orbital momentum.

The value of Δ has as yet been measured only for the Rb isotopes ($Z=37$, $A=85$ and 87), for which has been found $\Delta = 0.33\pm0.05$ percent for the ground state, which is an $s_{\frac{1}{2}}$ term. Expression (22) gives $\Delta = 0.11$ percent for $g_L = Z/A = 0.43$ and $\Delta = 0.29$ percent for $g_L = 1$. The

[7] See G. Racah, Nuovo Cimento **8**, 178 (1931); also J. E. Rosenthal and G. Breit, Phys. Rev. **41**, 459 (1932).

[8] H. Margenau and E. Wigner, Phys. Rev. **58**, 103 (1940).

[9] Th. Schmidt, Zeits. f. Physik **106**, 358 (1937).

latter value is in good agreement with the empirical value while the estimate for $g_L = Z/A$ is too small by a factor three. Although the approximations involved in this estimate are somewhat crude, it seems difficult to obtain a sufficiently large value of Δ for $g_L = Z/A$, even under rather extreme assumptions regarding the values of $(\kappa_s)_{Av}$ and $(\kappa_L)_{Av}$ in the two isotopes.

In this connection it may be mentioned that the Schmidt model is also favored by other evidence regarding nuclear magnetic moments. In fact, as is well known, the g_I-values of all odd nuclei fall within the limits given by the Schmidt model, whereas the g_I-values of a number of nuclei with high spin fall outside the limits predicted by the model in which $g_L = Z/A$. Moreover, reference may be made to the recent successes achieved by the individual particle model of nuclear structure in accounting for the angular momenta of nuclei.

Measurements of the h.f.s. anomalies in other elements would be desirable. There exist a number of odd elements (e.g., Sb, Eu, Ir) having isotopes with widely different g_I-values and for which the value of Δ according to (22) should be appreciable. In the case of even elements with isotopes of odd atomic number (22) gives a vanishing Δ for the Schmidt model, whereas for $g_L = Z/A$, values of Δ of more than one percent would be expected in several cases (e.g., Yb and Hg).

A number of elements (e.g., Na and K) have isotopes of odd numbers of protons and neutrons whose spin and h.f.s. have been measured. These nuclei are of special interest for the problem of nuclear structure, and a study of their h.f.s. anomalies might give some indication regarding the composition of their moments.

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On the $\alpha-\beta$- and $\alpha-K$-Branching of the Heaviest Natural and Artificial Radioactive Substances*

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Based on simple empirical regularities in α-decay properties of the heavy elements, the following substances which show β-emission or K-capture should also be α-radioactive with α–β- or α–K-branching ratios of at least 10^{-10}: $MsTh_2$, $MsTh_1$, AcK; ${}_{88}Ra^{225}$, ${}_{91}Pa^{233}$, ${}_{93}Np^{234}$, ${}_{93}Np^{235}$, ${}_{93}Np^{236}$, ${}_{93}Np^{238}$, ${}_{93}Np^{239}$, ${}_{94}Pu^{241}$, ${}_{95}Am^{242}$.

I. INTRODUCTION

IN addition to the long known dual disintegration of the C-bodies of the three natural radioactive disintegration series, the existence of a number of other double disintegrations has been established. M. Perey[1] succeeded in proving that actinium emits a weak α-radiation. This disintegration creates element 87 (francium). B. Karlik's and T. Bernert's[2] proof of the dual disintegration of RaA made it possible to establish the occurrence in the natural disintegration series of element 85 (astasium) through the isotope ${}_{85}At^{218}$.** Prior to this, element 85 has already been artificially obtained and chemically investigated through reaction ${}_{83}Bi^{209}(\alpha,2n){}_{85}At^{211}$ by D. R. Corson, K. R. McKenzie, and E. Segre.[3] This element shows α–K-branching. In the newly established $4n+1$-series ${}_{83}Bi^{213}$ suffers an α–β-decay.[4]

II. EMPIRICAL REGULARITIES OF THE α-DISINTEGRATION OF THE HEAVIEST NUCLEI

In the range of α-instability of the heaviest nuclei all elements emitting β-rays or showing K-capture must also be α-radioactive. Based on simple empirical regularities, therefore, branching ratios for some of these radioactive elements which have not as yet been measured are stated below. However, only dual disintegrations with branching-ratios of 10^{-10} or higher will be mentioned.

If the disintegration energies of the natural α-radioactive elements are plotted in a diagram as function of their mass numbers[5] (Fig. 1), a family of curves is formed by lines connecting nuclei of the same atomic

* This paper was written in August, 1947.

[1] M. Perey, J. de phys. et rad. **10**, 435 (1939). M. Perey and Lecoin, J. de phys. et rad. **10**, 439 (1939).

[2] B. Karlik and T. Bernert, Zeits. f. Physik **123**, 51 (1944).

** In the case of ThA and AcA, also, long range α-particles had been observed.[2] The simplest explanation would be a dual decay of ThA and AcA. But applying energy-conservation rules it follows that there exist considerable difficulties in accepting this interpretation.

[3] Corson, McKenzie, and Segrè, Phys. Rev. **57**, 1087 (1940).

[4] Hagemann, Katzin, Studier, Ghiorso, and Seaborg, Phys. Rev. **72**, 252 (1947); English, Cranshaw, Demers, Harvey, Hincks, Jelley, and May, Phys. Rev. **72**, 253 (1947).

[5] J. Schintlmeister, Oesterreichische Chemiker Zeitung Nr. 17, 1938, Nr. 9/12, 1943. Regularities in Geiger Nuttal diagrams referring to the individual values of Z were pointed out by Berthelot in the J. de phys. et rad., serie VIII, **3**, 17 (1942) and by N. Feather, Proc. Roy. Soc. Edinburgh, **62**, 211 (1946).