Nuclear Shell Models*,†

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The principal features of the several shell models now under discussions are described briefly.

The magnetic moments of odd nuclei have been interpreted as (a) supporting an extreme single particle model (Schmidt limits) and (b) as generally consistent with a uniform model (Margenau-Wigner limits). The evidence is consistent with a composite interpretation based on (a) the approximate validity of the extreme single particle model for nuclei lacking one particle to make a closed shell or possessing one particle in excess of the number making up a closed shell, and (b) the approximate validity of the uniform model for all other odd nuclei.

Remarks on quadrupole moments, isomerism and beta-decay supplement an earlier discussion.

CONTEMPORARY nuclear theory bears some resemblance to the periodic system of the elements and theories of chemical valence and structure. The sources of the parallelism are known: the exclusion principle and the isotropy of space, the latter in the form of the quantum theory of the parity and angular momentum operators.

The number of neutrons N and of protons Z are the basic parameters for the characterization and interpretation of nuclear structure. An imposing body of evidence supports the conclusion that $N=50$, 82, and 126 and $Z=50$ and 82 are associated with particularly stable and abundant nuclear species.[1,2] Other numbers may be added to this list, among them N or $Z=2$, 8, 10, and 20.[3–5]

The interpretation of the "magic" numbers as occupation numbers of single-particle levels in a suitably proportioned potential well was discussed by Elsasser in the same paper in which the special properties of the numbers 50, 82, and 126 were first recognized. Other closely related schemes have been proposed recently. Figure 1 summarizes the various schemes at present under discussion. Column 1 shows the levels and occupation numbers of an isotropic harmonic oscillator. The modifications introduced by distorting the potential into the shape of a rectangular well appear in column 2. These potentials have in common the occupation numbers 2, 8, and 20.[6]

Column 3 exhibits the level order and occupation numbers in the scheme devised by Nordheim.[7] The guiding principle here is the production of the magic numbers with a minimal departure from the level order in the rectangular potential well.

The j–j coupling scheme[8,9] appears in column 4. All levels with orbital angular momentum $l>0$ are split into two, a lower level with total angular momentum $i=l+1/2$ and an upper with $i=l-1/2$. This is essentially the Breit-Inglis[10–12] rule for the effect of spin-orbit coupling. What is new is the assumption that the spin-orbit coupling is large and particularly so for levels with $l\geqq 4$.

The remaining columns 5–9 show the Feenberg-Hammack[13] scheme based on Elsasser's suggestion of a central elevation in the potential well.[1] A physical basis for the central elevation is now provided by the effect of the Coulomb repulsion between protons on the variation of particle density within the nucleus. The repulsion causes the density to vary from a minimum value at the center of the nucleus to a maximum near the boundary.[14] The optimum single-particle potential well is pictured as a distorted mirror image of the particle density, being deepest where the density attains a maximum value and shallowest at the center where the minimum density occurs (Fig. 2B). It is postulated that levels with large particle density in the central region of the nucleus ($2s$, $2p$, and $2d$) are pushed up by the developing central elevation with sufficient rapidity to permit closed shells at 50, 82, and 126. Other levels are displaced relatively little because the wave functions are extremely small in the region of the elevation.

In light nuclei the particle density may be pictured as a bell-shaped curve. The corresponding optimum single-particle potential then resembles an inverted bell and may be approximated roughly by a rectangular well with a central depression (Fig. 2A). A sufficiently deep central depression pulls the $2s$ level below $1d$, thus permitting a closed shell at N or $Z=10$ and the closing of the $1d$ shell at N or $Z=20$. This explanation serves all the proposed schemes.

* Assisted by the Joint Program of the ONR and AEC.

† Based on two lectures at the symposium on modern physics sponsored by the Oak Ridge National Laboratory and the Oak Ridge Institute of Nuclear Studies, August, 1949.

[1] W. Elsasser, J. de phys. et rad. **5**, 625 (1934).

[2] M. G. Mayer, Phys. Rev. **74**, 235 (1948).

[3] W. D. Harkins, Phys. Rev. **76**, 1538 (1949).

[4] E. P. Wigner, Phys. Rev. **51**, 947 (1937).

[5] W. H. Barkas, Phys. Rev. **55**, 691 (1939).

[6] The notation is that proposed by M. G. Mayer. np, for example, denotes a state with one unit of orbital angular momentum and a radial wave function possessing $n-1$ nodes at finite values of the radial coordinate (excluding one at the origin).

[7] L. W. Nordheim, Phys. Rev. **75**, 1894 (1949).

[8] M. G. Mayer, Phys. Rev. **75**, 1969 (1949).

[9] Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949). For obscure reasons these authors prefer the oscillator potential.

[10] D. R. Inglis, Phys. Rev. **50**, 783 (1936).

[11] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948).

[12] E. Feenberg, Phys. Rev. **76**, 1275 (1949).

[13] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).

[14] E. Feenberg, Phys. Rev. **59**, 593 (1941).

The existence of alternative structures for the closed shells at N or $Z=50$, 82, and 126 may be a clue to the exceptional stability of these shells. In molecular theory the existence of alternative qualitative descriptions often provides an additional element of stability through the operation of the quantum-mechanical resonance effect. Similarly, in the nuclear problem, it is not unlikely that the optimum representation of the closed shell structure may be a linear combination of the various simple possibilities provided by the different schemes rather than exclusively one or the other.

In this connection it is interesting to consider the possibility that the dependence of level position on the variation of particle density within the nucleus may cause the order of levels to depend on whether certain orbits are occupied or empty. For example, eight particles in $2s$ and $2p$ orbits contribute materially to the particle density in the inner region of the nucleus, thus opposing the tendency toward the formation of a semihollow nucleus and the correlated development of a central elevation in the potential well. On the other hand, the transfer of eight particles from $2s$ and $2p$ into $1g$ orbits should materially reduce the central particle density and consequently accelerate the development of the central elevation in the potential well with the result that the unoccupied $2s$ and $2p$ levels are raised above the partially occupied $1g$ level. In the former situation, a closed shell may form at N or $Z=50$, utilizing only $G_{9/2}$ orbits in accordance with the postulates of the Mayer scheme. The latter situation produces a closed shell at N or $Z=50$ with $1g$ fully occupied. Similar conditions may prevail near N or $Z=82$ yielding two possible structures for a closed shell at 82, one with $2s$, $3s$, and $2p$ orbits fully occupied and only the $H_{11/2}$ subshell of $1h$ filled and a second with $2s$ and $2p$ empty and $1h$ completely filled.

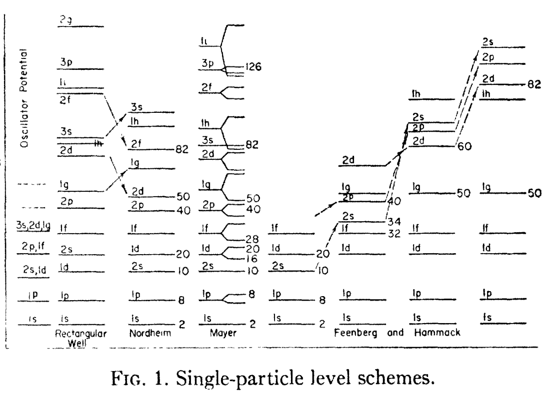

FIG. 1. Single-particle level schemes.

MAGNETIC MOMENTS

The magnetic moments[15] of odd nuclei are plotted in Fig. 3 (odd Z) and Fig. 4 (odd N). Large circles denote nuclei with closed ($\pm$) shells (one particle lacking to make a closed shell or one particle in excess of the number required for a closed shell). The theoretical interpretation of these diagrams is based on the study of simple limiting situations. Results of great value have been derived from the assumption that the nucleus possesses a definite orbital angular momentum and a total angular momentum $I=l+1/2$ or $I=l-1/2$ (Russel-Saunders coupling). The magnetic moment is given by the formula (in nuclear magneton units)

$$\mu=\tfrac{1}{2}\cdot I\left[g_l+g_s+(g_l-g_s)\frac{l(l+1)-\frac{3}{4}}{I(I+1)}\right]. \tag{1}$$

Here

$$g_s=5.58 \text{ (odd proton)}$$
$$=-3.82 \text{ (odd neutron)}, \tag{2}$$

while g_l depends on special properties of the model. Two models have been employed in this connection.

(a) Extreme single-particle model. The odd nucleon moves in the spherically symmetric field of a core containing all the other particles. Even parity and zero angular momentum are assigned to the core in harmony with the empirical rule that even-even nuclei have always zero spin in the ground state. All angular momentum and parity properties are associated with the odd nucleon. Thus,

$$g_l=1 \text{ (odd proton)}$$
$$=0 \text{ (odd neutron)}. \tag{3}$$

The dashed curves in Figs. 3 and 4 are the Schmidt limits[16] computed from Eqs. (1)–(3). It is not difficult to see that the experimental points on each diagram group themselves into two broad bands between the theoretical limits and paralleling them. This relation between theory and experiment has been interpreted as evidence for the substantial validity of the extreme single particle model and for the assumption that the orbital angular momentum l is a good quantum number (allowing, of course, notable exceptions). Inglis[17] presents theoretical arguments for this point of view. Paradoxically, the recognition of the magic numbers and the associated closed ($\pm$) shell systems weakens the case for the general validity of the extreme single-particle model. One expects somewhat closer conformity to the extreme single-particle model from closed ($\pm$) shell system than from other odd nuclei. Actually such

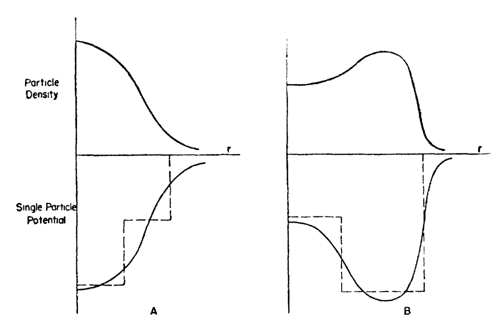

FIG. 2. (A) Particle density and optimum single particle potential function in a light nucleus. (B) Particle density and optimum single particle potential function in a heavy nucleus.

[15] (a) H. H. Goldsmith and D. R. Inglis, *The Properties of Atomic Nuclei I* (Information and Publications Division, Brookhaven National Laboratory, Upton, New York, October 1, 1948). (b) Livingston, Gilliam, and Gordy, Phys. Rev. **76**, 443 (1949). (c) L. Davis, Phys. Rev. **76**, 435 (1949).

[16] T. Schmidt, Zeits. f. Physik **106**, 358 (1937).

[17] D. R. Inglis, Phys. Rev. **53**, 470 (1938); **60**, 837 (1941).

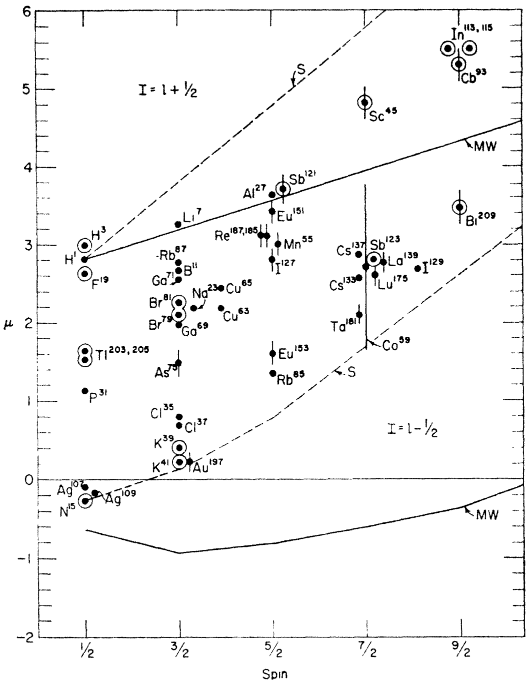

FIG. 3. Magnetic moments of the odd proton nuclei. S—Schmidt limits, MW—Margenau-Wigner limits. Circles ⊙ label nuclei containing closed (±) shells: one particle lacking to make a closed shell or one particle in addition to a closed shell.

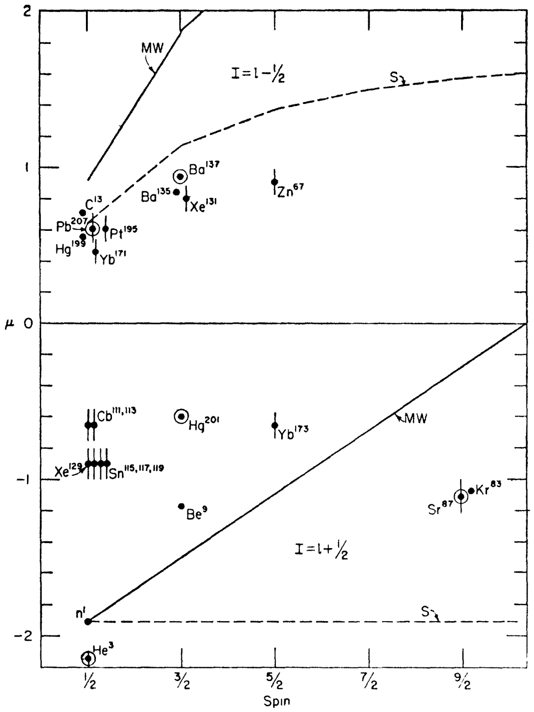

FIG. 4. Magnetic moments of the odd neutron nuclei. S—Schmidt limits, MW—Margenau-Wigner limits. Circles ⊙ label nuclei containing closed (±) shells: one particle lacking to make a closed shell or one particle in addition to a closed shell.

a distinction is not apparent relative to the Schmidt limits. The absence of a distinction puts in question the relevancy of the Schmidt limits for odd nuclei not of the closed (±) shell type.

Deviations from the Schmidt limits suggest a generalization of the single-particle model in which nuclear states are linear combinations of states with $l=I-1/2$ and $l=I+1/2$. The opposite parity of single-particle wave functions differing by one unit in l requires a corresponding ambivalence in the parity of the core.[7] In this form the model lacks plausibility. It seems preferable to admit that a single-particle model is not competent to deal with deviations from the Schmidt limits.[18]

The fact that all experimental magnetic moments lie close to or between the Schmidt limits supports the assumption of a predominantly doublet character for the low states of odd nuclei. It must be considered unlikely that a widespread occurrence of strong quartet components would never result in a breach of the limits.[18] Thus, for example, an odd proton nucleus in a state containing equal parts of $^4P_{5/2}$ and $^2D_{5/2}$ might have $\mu \sim 7.1$.

(b) Uniform model. This model, developed by Margenau and Wigner[18] from a suggestion of K. Way, distributes the orbital angular momentum uniformly over all the particles in the nucleus with the result

$$g_l \sim Z/A > 0.4 \quad (4)$$

[18] H. Margenau and E. P. Wigner, Phys. Rev. **58**, 103 (1939).

for both odd N and odd Z. The solid lines in Figs. 3 and 4 are the Margenau-Wigner limits computed from Eqs. (1), (2), and (4). Moments falling between the MW limits are interpreted as evidence that the orbital angular momentum is not a constant of motion. States with $l=I-1/2$ and $l=I+1/2$ may have the same parity in a many particle model and, consequently, may combine to produce intermediate values of the moments.

TABLE I. Possible doublet states for partially filled shells.[a] Number of linearly independent states.

Configuration	^2S	^2P	^2D	^2F	^2G	^2H	^2I	^2K	^2L	^2M	^2N	^2O
$(np)^{1,5}$		1										
$(np)^3$		1	1									
$(nd)^{1,9}$			1									
$(nd)^{3,7}$		1	2	1	1	1						
$(nd)^5$	1	1	3	2	2	1	1					
$(nf)^{1,13}$				1								
$(nf)^{3,11}$		1	2	2	2	2	1	1	1			
$(nf)^{5,9}$		4	5	7	6	7	5	5	3	2	1	1
$(nf)^7$	1	5	7	10	10	9	9	7	5	4	2	1
$(ng)^{1,17}$					1							
$(ng)^{3,15,\,b}$		1	2	2	3	3	2	2	2	1	1	1

[a] Gibbs, Wilbur, and White, Phys. Rev. **29**, 790 (1927).
[b] Computed by the author.
The analysis of the g^n configurations has been completed by Mr. K. C. Hammack. In odd nuclei maximum degeneracy occurs always for ^2G states. Table I should be supplemented by a similar table for the degeneracy of singlet states in configurations containing an even number of equivalent orbits. The most significant property of the singlet table is a relatively high degeneracy of ^1S states. This circumstance is helpful in understanding the empirical rule that even-even nuclei have always zero spin in the ground state.

TABLE II. Three particles in *nd* orbits (central elevation level scheme).

Nucleus	Spin	Magnetic moment	Uniform model interpretation
$_{12}Mg^{25}$	5/2[a]	−0.96	$D_{5/2}$
$_{13}Al^{27}$	5/2	3.64	$D_{5/2}$
$_{42}Sr^{91}$	5/2[b]	——	——
$_{42}Mo^{95}$	5/2[b]	——	——
$_{53}I^{127}$	5/2	2.8	$D_{5/2}\gg F_{5/2}$
$_{75}Re^{185}$	5/2	3.3	$D_{5/2}\gg F_{5/2}$
$_{75}Re^{187}$	5/2	3.3	$D_{5/2}\gg F_{5/2}$
$_{54}Xe^{129}$	1/2	−0.9	$S_{1/2}\sim P_{1/2}$
$_{53}I^{129}$	7/2	2.74	$F_{7/2}\gg G_{7/2}$

[a] Crawford, Kelly, Shawlow, and Grey, Phys. Rev. **76**, 1423 (1949).
[b] Derived from the analysis of radioactivity and isomerism; reference 13 of this paper. *Added in proof:* Confirmed for Mo^{95} by hyperfine structure measurements (private communication from Prof. J. E. Mack).

Eight intermediate and heavy nuclei have moments falling outside of the MW limits.‡ It is noteworthy that six of the eight contain closed (±) shells for which the single-particle model might be expected to yield better results than the uniform model. Thus the exceptional cases occur where failures are not unexpected. The evidence is consistent with a composite interpretation based on (a) the approximate validity of the extreme single-particle model for nuclei containing closed (±) shells and (b) the approximate validity of the uniform model for all other odd nuclei. Under (a) l is generally a fairly good quantum number while under (b) admixtures of $l=I-1/2$ and $l=I+1/2$ in varying proportions are common.

No explanation has yet been advanced for the peculiar fact visible in both Figs. 3 and 4 that practically all moments (excluding the exceptional closed (±) shell systems) fall between the MW limit for $I=l+1/2$ and the Schmidt limit for $I=l-1/2$.

The question of the admixture of higher values of S (quartet components) already referred to becomes less pressing with recognition of an adequate interpetation, requiring no quartet component,[18] for the moments falling outside of the MW limits.

In summary the single-particle model, while not at all plausible on the basis of present notions concerning nuclear forces, receives support from the apparent grouping of the moments into irregular broad bands paralleling the Schmidt limits. No obvious distinction appears between closed (±) shell nuclei and other types. At the opposite extreme the uniform model is consistent with most of the data provided that l is not generally a good quantum number. Moreover, the failures are confined (with two exceptions) to the class of closed (±) shell nuclei for which the single-particle model is most likely to possess a useful degree of validity.

For odd nuclei not of the closed (±) shell-type configuration analysis provides a working compromise between the oversimplification of the extreme single-particle model and the intractability of the uniform model. The spherically symmetric core (of even parity) contains the even group of particles and the largest number of particles in the odd group forming a closed shell. The remaining particles or holes of the odd group are placed in appropriate single particle orbits. For example, $_{53}I^{127}$ is represented by a $(2d)^3$ configuration (three protons in $2d$ orbits) on a core containing 50 protons and 74 neutrons while $_{49}In^{115}$ requires a $1g$ orbit occupied by a proton hole on a core containing 50 protons and 66 neutrons.[19] In special cases a complete configuration for all particles outside of closed shells appears most plausible; $_{11}Na^{23}$ is an example. However, the $(1d)^1(1d)^2$ configuration contains no symmetrical ^2P state and fails, just as does the single-particle model, to account for the $P_{3/2}$ character of the ground state.

TABLE III. Three holes in *nd* orbits (central elevation level scheme).

Nucleus	Spin	Magnetic moment	Uniform model interpretation
$_{17}Cl^{35}$	3/2	0.82	$D_{3/2}>P_{3/2}$
$_{17}Cl^{37}$	3/2	0.68	$D_{3/2}>P_{3/2}$
$_{16}S^{33}$	3/2†	——	——
$_{56}Ba^{135}$	3/2	0.84	$D_{3/2}>P_{3/2}$
$_{79}Au^{197}$	3/2	0.195	$D_{3/2}>P_{3/2}$
$_{57}La^{139}$	7/2	2.76	$F_{7/2}>G_{7/2}$
$_{44}Ru^{101}$	——	——	——

† *Added in proof:* Confirmed by C. K. Jen, Bull. Am. Phys. Soc. **25**, No. 1, 35 (1950).

TABLE IV. Three particles or holes in $2p$ orbits (central elevation level scheme).

Nucleus	Spin	Magnetic moment	Uniform model interpretation
$_{37}Rb^{87}$	3/2	2.75	$P_{3/2}$
$_{37}Rb^{85}$	5/2	1.35	$D_{5/2}\sim F_{5/2}$
$_{30}Zn^{67}$	5/2	0.9	$F_{5/2}>D_{5/2}$
$_{63}Eu^{151}$	5/2	3.4	$D_{5/2}$
$_{63}Eu^{153}$	5/2	1.5	$D_{5/2}\sim F_{5/2}$

‡ $_{13}Al^{27}$ is not included among the exceptions because a more detailed treatment using exact values of Z/A in estimating g_l would place the magnetic moment of this nucleus between the accurate MW limits.

TABLE V. Shell structure of nuclei involved in beta-transitions showing the unique first-forbidden type of energy distribution.

Nucleus	Configuration	Parity	Spin
$_{17}Cl^{38}$	$(1d)^{-3}(1f)^1$	odd	..2...
$_{18}A^{38}$	$(1d)^{-2}$	even	0
$_{19}K^{42}$	$(1d)^{-1}(1f)^3$	odd	..2...
$_{20}Ca^{42}$	$(1f)^2$	even	0
$_{38}Sr_{89}$	$(1d)^1$	even	5/2
$_{39}Y_{89}$	$(2p)^{-1}$	odd	1/2
$_{38}Sr^{90}$	$(2p)^{-2}(2d)^2$	even	0
$_{39}Y^{90}$	$(2p)^{-1}(2d)^1$	odd	..2...
$_{40}Zr^{90}$	closed shells	even	0
$_{38}Sr^{91}$	$(2d)^3$	even	..5/2...
$_{39}Y^{91}$	$(2p)^{-1}$	odd	1/2
$_{40}Zr^{91}$	$(2d)^1$	even	5/2
$_{55}Cs^{137}$	$(2d)^5$	even	7/2[a]
$_{56}Ba^{137}$	$(1h)^{-1}$	odd	11/2
$_{56}Ba^{137}$	$(2d)^{-1}$	even	3/2[a]

[a] Experimental value.

[19] W. Heisenberg, Ann. d. Physik **10**, 888 (1931). The theory of holes is developed in this paper.

Table I contains the number of linearly independent doublet states generated by configurations of equivalent orbits. A certain tendency to simulate the results of the extreme single-particle model may be inferred from the fact that the smallest value of orbital angular momentum associated with maximum degeneracy is identical with that of a single orbit. Thus, for example, the $(1f)^7$ configuration with ten linearly independent ^2F states might be expected to favor $l=3$ for the ground state in agreement with the spin and magnetic moment of $_{27}Co^{59}$ (referred to the MW limits).

The threefold occurrence of *nd* orbits in the central elevation level scheme suggests a search for regularities based on repetitions of $(nd)^{3,5,7}$ configurations. Results, shown in Tables II and III, conform well to expectations based on the double degeneracy of ^2D states in the $(nd)^{\pm 3}$ configuration and the Breit-Inglis rule. In Mayer's level scheme $_{56}Ba^{135}$ and $_{79}Au^{197}$ are assigned $2d_{3/2}$ orbits.

No regularity is apparent in the $(nd)^5$ series. $_{15}P^{31}$ with $I=1/2$ and $\mu=1.13$ requires the interpretation $S_{1/2}+P_{1/2}$ (with approximately equal weights). Mayer assigns a $2s$ orbit to the odd proton, bracketing the $2s$ level between $1d_{5/2}$ and $1d_{3/2}$. This procedure requires both a splitting of the $1d$ level in the sense of the Breit-Inglis rule and an upward displacement of $2s$ relative to $1d$, thus combining features of the j–j coupling and central elevation level schemes. The occurrence of a large $P_{1/2}$ component is not explained. The partial configuration interpretation applied to the central elevation or Nordheim level schemes (identical in this region) provides two closely spaced configurations, $(2s)^2(1d)^5$ and $(2s)^1(1d)^6$ from which two ^2S and one ^2P states are derived.

The group of nuclei with $I=7/2$ in Fig. 3 all require the interpretation $G_{7/2} \gg F_{7/2}$ if the extreme single-particle model is favored.* However, only one nucleus in this group, $_{51}Sb^{123}$, contains a closed $(\pm)$ shell. For the others, the uniform model seems preferable, requiring the reversed inequality $F_{7/2} \gg G_{7/2}$.

The difficulties created by the uncritical application of the extreme single particle model are well illustrated by $_{27}Co^{59}$. All level schemes place the odd proton in a $1f$ orbit in contradiction with the $G_{7/2}$ interpretation derived from the Schmidt limits. On the other hand, the partial configuration $(1f)^7$ yields a wide range of possibilities with a predominant $F_{7/2}$ *a priori* most likely.

Nuclei containing half-filled $2p$ shells (according to the FH level scheme) provide material for Table IV. With theoretical possibilities limited to ^2P and ^2D (odd parity), the strong representation of $F_{5/2}$ components in three out of five examples is a serious difficulty for the level scheme, although less so now than earlier when the theoretical interpretation was based on the Schmidt limits. It is satisfactory that $_{37}Rb^{87}$ with a closed neutron shell $(N=50)$ conforms to the theoretical possibilities of the $(2p)^3$ configuration. Mayer's scheme provides an $F_{5/2}$ component at N or $Z=37$, but no $D_{5/2}$, while at $Z=63$, the reverse is true. The two level schemes disagree regarding the parity of the europium isotopes, central elevation requiring odd and j–j coupling even parity.

* The inequalities here and in Tables II–IV express relations between the statistical weights of the two Russel-Saunders components required for a complete description.

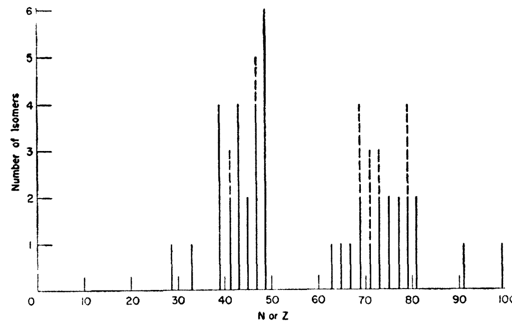

FIG. 5. Distribution of isomerism in odd nuclei. N or Z denotes the odd member of the N, Z pair.

The copper and gadolinium isotopes $_{29}Cu^{63,65}$ and $_{31}Ga^{69,71}$ form an interesting group with $P_{3/2} \gg D_{3/2}$ according to the MW limits. Mayer interpolates the $2p_{3/2}$ level between $1f_{7/2}$ and $1f_{5/2}$ to account for the properties of this group in terms of the extreme single-particle model. However, it is then necessary to assign the odd proton at $Z=35$ to an $f_{5/2}$ orbit in disagreement with the spins and moments of the bromine isotopes. A somewhat weaker interpretation is supplied by the partial configurations $(2s)^2(1f)^9$ for copper and $(2s)^2(1f)^{11}$ for gadolinium. The first with four linearly independent ^2P states is indeed quite plausible. Excited configurations of the types $(2s)^2(1f)^{2n}2p$ and $(1f)^{2n+1}$ may assist in stabilizing the $2P$ state. The bromine isotopes then mark the beginning of the $2p$ shell. The evidence is features of the j–j coupling and central elevation schemes: splitting of the $1f$ level and a rise of the $2p$ level with increasing Z (starting with $2p$ between $1f_{7/2}$ and $1f_{5/2}$ at $Z=29$).

A puzzling feature of the magnetic moment distribution is the large number of odd neutron nuclei with $I=1/2$ and magnetic moments requiring the interpretation $S_{1/2} > P_{1/2}$. The group contains $N=63$, 65, 67, 69, and 75,[20] while Z ranges from 48 to 54. It is evident from Table I that S states generated by configurations of equivalent orbits are extremely rare. Possibly configurations containing a singly occupied s orbit are required; however, it is not clear why the state of minimum angular momentum and even parity should be favored in so many cases.

The fact that the series begins at $Z=48$ suggests an interpretation consistent with the postulates of the central elevation scheme. In this scheme the $2s$ proton level crosses $1g$ and $2d$ just before $Z=48$. The simultaneous complete absence of neutrons from the $2s$

[20] $_{52}Te^{123}(N=71)$ and $_{52}Te^{125}(N=73)$ with $I=1/2$ are likely candidates for places on this list (G. R. Fowles, Phys. Rev. **76**, 571 (1949) and J. E. Mack and O. H. Arroe, Phys. Rev. **76**, 1002 (1949)).

neutron orbit may carry the system beyond the energy minimum resulting from the readjustment of particle density under the action of the Coulomb repulsion between protons. The experimental results permit the interpretation that the optimum distribution of nucleons when the 2*s* proton orbit is empty occurs with one neutron occupying a 2*s* orbit. One pictures the singly occupied 2*s* orbit constrained to lie just below the nearest level. When doubly occupied or completely empty it rises above the latter.

QUADRUPOLE MOMENTS

Nuclei with $I>1/2$ are expected to possess an electric quadrupole moment Q.[21] In the extreme single-particle model Q vanishes for odd N and takes negative values for odd Z (charge distribution flattened along axis of spin). Empirically quadrupole moments are more often positive (charge distribution elongated along axis of spin) than negative. The partial configuration method offers possibilities for obtaining positive values of Q through the operation of the hole mechanism since a hole in a proton shell behaves like a negatively charged particle.

One may hope to obtain a significant correlation between theory and experiment, at least as regards sign, for the closed ($\pm$) shell nuclei adjoining the magic numbers. Small negative Q's for ${}_{51}Sb^{121}$, ${}_{51}Sb^{123}$, and ${}_{83}Bi^{209}$ and a positive Q for ${}_{49}In^{115}$ are consistent with theory.[15a, 22]

The bromine isotopes with positive values of Q are not in accord with theoretical expectations for a singly occupied 2*p* orbit. However, there is scarcely ground for a theoretical prediction where the shell structure does not make itself apparent in increased stability of the closed shell. Large positive values of Q, requiring a considerable elongation of the charge distribution along the axis of spin, occur in the region where the j–j coupling and central elevation level schemes place the filling of the 1*h* orbits. Several authors[23–25] have pointed out indications of a general correlation between quadrupole moments and shell structure. On this, judgment must be deferred until more experimental evidence is available. Caution is required in attempting to relate the large quadrupole moments to special schemes of shell structure. Generally a good wave function for the ground state can only be obtained as a sum of linearly independent elementary functions. The quadrupole moment is then a double sum involving both diagonal and non-diagonal matrix elements with respect to the linearly independent basis. Large values of the moment require suitable phase relations between the elementary functions making up the wave function and a sufficient number of large non-diagonal matrix elements.

ISOMERISM AND BETA-DECAY

Nuclear isomerism denotes long-lived (half-lives ranging from 10^{-7} second to 1 year) excited states decaying by the emission of electromagnetic radiation, the production of internal conversion electrons, pair production, or radioactive transitions.[26] Figure 5 shows the distribution of odd nuclei possessing isomeric states as a function of the odd member of the N, Z number pair. The existence of well-defined "islands" of isomerism points to a close connection with shell structure. Indeed, a plausible condition for isomerism is the occurrence of closely spaced single-particle levels, differing by several units in l, at the top of the filled level distribution. Possibilities for satisfying this condition occur in all three level schemes although the schemes differ in regard to the possible parity changes. In particular the j–j coupling scheme allows only isomeric transitions with change in parity when the spin change exceeds two units. The crossing of levels in the central elevation scheme provides a number of pairs of closely spaced configurations with the same and with opposite parity. Isomeric transitions with and without change in parity occur in each large island of isomerism indicating need for all the theoretical possibilities of the central elevation scheme.

The careful theoretical analysis by Axel and Dancoff[26] confirms the earlier deductions from a Wiedenbeck-type chart.[13] There is no change in parity at N or $Z=43$ (two examples), 45 (two examples), 47 (five examples), 63, 73, 79. A parity change occurs at N or $Z=39$ (three examples), 41 (two examples), 43, 49 (three examples), 69 (two examples), 71, 73, 75, 77 (two examples), 79, and 81 (two examples).

In relation to the theory of beta-decay, shell models assist in determining the order of allowed and forbidden transitions between ground states by fixing the parities and in some cases the spins of parent and daughter nuclei. The information derived from the models is particularly useful in an extended radioactive series involving two or more beta-transitions in cascade.

The recent discovery of the unique first forbidden distribution in the beta-decay ${}_{17}Cl^{38}$, ${}_{19}K^{42}$, ${}_{38}Sr^{89}$, ${}_{38}Sr^{90}$, ${}_{39}Y^{90}$, ${}_{39}Y^{91}$, and ${}_{55}Cs^{137}$ has verified a number of deductions based on shell structure, selection rules and observed half-lives and maximum energies.[13, 27–34] These deductions from the central elevation level scheme are summarized in Table V; the results are also all compatible with the j–j coupling level scheme.

The high binding energies of two silicon isotopes,[35] inferred from radioactive decay energies, suggests the formation of a particularly stable structure at $Z=14$. The j–j coupling level scheme permits a closed subshell containing six like particles in $1d_{5/2}$ orbits; the reversal of the 2*s*, $1d_{5/2}$ level order encountered earlier in the discussion of the ${}_{15}P^{31}$ spin places the closed structure at $Z=14$.

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