

On the Secondary Particles of Local Penetrating Showers*

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An analysis of the secondary particles of the local penetrating showers has been done by means of absorption experiments and delayed coincidence measurements. The large majority of those particles, filtered with about 12 in. of lead, appear to be mesons, which have been identified as π -mesons in our preceding paper. The average number of those mesons per shower is about four. The analysis appears to give a lower limit of 1200 g/cm² for the mean free path for nuclear absorption of π -mesons having energies of some hundred Mev. This result is discussed in connection with other experiments on local penetrating showers.

THE results reported in our previous paper¹ show that very likely the mesons produced in a hard shower event (HS) are π -mesons, but they give no indication about the fraction of the secondary particles which is composed of mesons. In what follows we will describe further experiments (I, II, III, IV) which we performed at 11,300 ft., in order to make an analysis of the secondary particles as well as to obtain evidence for nuclear interactions of the π -mesons.

I

The first experiment, I, was made with the same apparatus as described in our previous paper. We measured the absorption in lead and iron of the secondary particles of a HS and of the normal penetrating component (NP). Since only the measurement of the absorption in lead was originally planned, the experiment was made in two different series, as Table I shows. The absorbers were inserted just above Tray C (Fig. 1 of first paper), and a plate of sulfur five inches thick was constantly kept under the same tray. The absorbing powers could then be determined as the

TABLE I. Absorption of HS and NP particles in lead and in iron (Experiment I).

$ABC \times h^{-1}$	$A_2B \times h^{-1}$	$A_2BC \times h^{-1}$	Absorber
5580	108.2 ± 1.2	39.9 ± 0.7	90 g/cm ² Pb + $\frac{1}{4}$ in. brass + $\frac{1}{4}$ in. Al
5920	107.9 ± 1.3	51.5 ± 0.9	no Pb + $\frac{1}{4}$ in. brass + $\frac{1}{4}$ in. Al
5405	104 ± 1	40.5 ± 0.6	83 g/cm ² Fe
5755	103.7 ± 1	51.5 ± 0.6	None

Absorption powers (g⁻¹/cm²)

$$(A_{Pb})_{NP} = \frac{340}{5750 \times 90} = 6.57 \times 10^{-4}$$

$$(A_{Pb})_{HS} = \frac{11.6}{45.7 \times 90} = (2.82 \pm 0.3) \times 10^{-3}$$

$$(A_{Fe})_{NP} = \frac{350}{5580 \times 83} = 7.56 \times 10^{-4}$$

$$(A_{Fe})_{HS} = \frac{11}{46 \times 83} = (2.88 \pm 0.25) \times 10^{-3}$$

ratios:

$$\frac{[(ABC)_{no\ abs} - (ABC)_{abs}]}{[(ABC)_{no\ abs} + (ABC)_{abs}]}$$

We will refer to such ratios as $(A_{Fe})_{NP}$, $(A_{Pb})_{NP}$. The analogous ratios for A_2BC coincidences, which represent HS events, will be referred to as $(A_{Fe})_{HS}$ and $(A_{Pb})_{HS}$. The identification of the number of secondary particles absorbed, with the decrease in the rate A_2BC , is certainly incorrect because when two counters of C were discharged, the apparatus, during this experiment I, only registered them as one. However, the comparison between lead and iron is not affected appreciably, especially since it turns out that $(A_{Pb})_{HS}$ is about equal to $(A_{Fe})_{HS}$. An error of a few percent, due to the use of a mechanical register not preceded by a scaling circuit, must be supposed to affect both $(A_{Fe})_{NP}$ and $(A_{Pb})_{NP}$ about in the same amount, thus affecting only slightly the ratio $(A_{Fe}/A_{Pb})_{NP}$. The obtained value in g⁻¹cm², were

$$\begin{aligned} (A_{Pb})_{NP} &= 6.57 \times 10^{-4}; & (A_{Fe})_{NP} &= 7.56 \times 10^{-4}; \\ (A_{Pb})_{HS} &= (2.82 \pm 0.3) \times 10^{-3}; \\ (A_{Fe})_{HS} &= (2.88 \pm 0.25) \times 10^{-3} \end{aligned}$$

from which

$$(A_{Fe}/A_{Pb})_{NP} = 1.15, \quad (A_{Fe}/A_{Pb})_{HS} = 1.02 \pm 0.15.$$

Our value of 1.15 for $(A_{Fe}/A_{Pb})_{NP}$ compares well with the one obtained by Koenig² who measured the absorption of the hard component at sea level. $(A_{Fe}/A_{Pb})_{HS}$ is practically equal to $(A_{Fe}/A_{Pb})_{NP}$. This result, which shows that among the secondary particles emerging downward from the lead block, the proportion of electrons is unimportant, also gives a first indication that π -mesons are not removed by nuclear interactions with an absorption cross section as large as the geometrical cross section of iron nuclei.

For in the case of such a large cross section, we would obtain the values $(A_{Pb})_{HS} = 6.25 \times 10^{-3}$ and $(A_{Fe})_{HS} = 9.65 \times 10^{-3}$, which are much higher than the obtained ones, even allowing for the multiplicity mentioned before. But even more remarkable, the ratio $(A_{Fe})_{HS}/(A_{Pb})_{HS}$ should be 1.55, while it is only 1.02 ± 0.15 .

* Reported at the Echo Lake Conference of Cosmic Rays (June 22-28, 1949).

¹ O. Piccioni, Phys. Rev. 77, 1 (1950).

² H. P. Koenig, Phys. Rev. 69, 590 (1946).

II

In Experiment II, again using the apparatus described in our previous paper, we measured the rates of the delayed coincidences when the absorber above Tray C was, in periodical alternation, 1, 3, and 5 in. of iron. Below Tray C, as we have said, there was always 5 in. of sulfur. The three rates of the delayed coincidences are obviously proportional to the three values of the differential range spectrum corresponding to the three values of the range 1, 3, and 5 in. of iron. Table II shows that such a spectrum is flat in the rather narrow region considered. Unfortunately, practical reasons prevented from extending such a measurement to a wider interval of ranges. Nevertheless, we think that the obtained values are of some significance.

Let $N(E)dE$ be the energy spectrum of the mesons emerging from the lead block, and E_1, E_3, E_5 the energies corresponding to the three ranges of 1, 3, 5 in. of iron.* The ratio $N(E_1)/N(E_5)$ would be equal to the ratio of the rates of the delayed coincidences D_1/D_5 if the mesons were affected only by ionization losses. If the mesons were strongly interacting with the nuclei, some of the mesons which emerge from the lead with energy E_5 would be prevented by such nuclear interactions from crossing all the iron. Thus they could not give all the "due" delayed coincidences with the 5-in. iron absorber, with the consequence that $N(E_1)/N(E_5)$ would be smaller than D_1/D_5 . Now the experimental value of D_1/D_5 is 0.95 ± 0.1 and if $N(E_1)/N(E_5)$ were to be less than that, we should conclude that the energy spectrum of the locally produced mesons, as they emerge from 15 in. of lead, increases with their energy. Other data on the spectrum of mesons produced by nucleons of average energy of the order of 10 Bev, such as those which supposedly cause our HS³ are not yet available.

However, even though it is obvious that the determination of the range spectrum and of the nuclear interaction interfere with each other to some extent, it does not seem likely that, in the narrow interval of ranges which is of interest to us, the shape of the energy spectrum can have a very important role. If we assume that the energy spectrum is flat in the interval E_1 to E_5 , then we can say that the probability for a π -meson to undergo a nuclear absorption along its path does not increase by more than 0.1, in absolute value, when the path itself increases by 4 in. of iron. This determines a mean free path of more than 800 g/cm^{-2} , which must be considered as an average over the whole path of the meson, except the last one inch of iron. As to the corresponding interval of energies, we can say that the low limit is 60 Mev and the high limit is the initial energy

* The decay electrons may be produced either above or below Tray C, hence the range of the mesons entering the absorber and decaying in the vicinity of Tray C is not well defined. The total thickness of the absorber seems to be the best average of the range. This point has however but little bearing on the whole discussion.

³ T. G. Walsh and O. Piccioni, Echo Lake Conference on Cosmic Rays (June 22-28, 1949).

TABLE II. Delayed coincidences as function of the thickness of the absorber.

Time (hr.)	ABC_{del}/h^{-1} (total delays in 4 channels less casuals)	$A \times B \times h^{-1}$	$A_2BC_{\text{del}}/h^{-1}$ (total delays in 4 channels less casuals)	Absorber
210.98	14.5 ± 0.42	104.0	0.86 ± 0.06	1 in. iron + $\frac{1}{4}$ in. brass
188.10	15.5 ± 0.45	105.2	0.98 ± 0.07	3 in. iron + $\frac{1}{4}$ in. brass
227.20	15.3 ± 0.4	104.2	0.90 ± 0.06	5 in. iron + $\frac{1}{4}$ in. brass

E_0 of the meson at its production. To determine the value of E_0 we should know where the mesons are actually produced. Again, the flatness of the obtained range spectrum suggests that the place of production is well above Tray C, at least for most the mesons. It is then reasonable to believe that most mesons giving delayed coincidences are produced above Tray A with an energy E_0 of about 0.5 Bev. Of course, the mesons which do not stop in the absorber are produced with higher energy.

III

Experiment III was made in order to determine the fraction of HS delayed coincidences in which no ionizing secondary emerged from the lead, entering the iron absorber. Such a case would correspond for instance to the production in the iron of a low energy meson by a secondary neutron. The possibility that a large fraction of the delayed coincidences is due to mesons generated with a low initial energy by secondary nucleons in the part of the absorber which is nearest to Tray C, can already be ruled out by the dependence of the delayed coincidence rate with the thickness of the iron, as determined in Experiment II. The 4 in. of iron would indeed absorb considerably the secondary nucleons, while the observed flat curve gives no indication of a strong absorption. However, a fraction like 20 percent for the delayed coincidences due to a meson production in such low energy events, is not in contrast with the experimental errors of Experiment II.

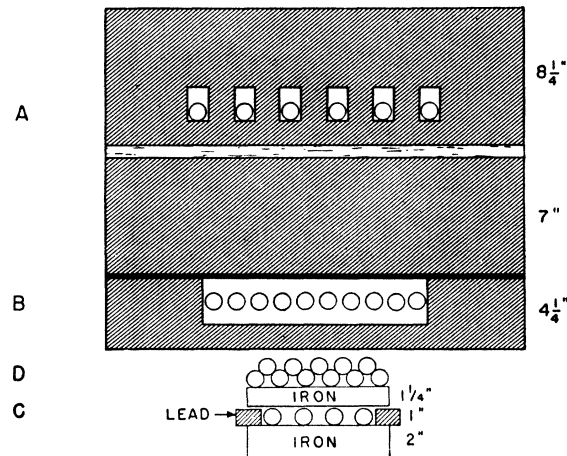


FIG. 1. Arrangement of apparatus.

TABLE III. HS delayed coincidences not accompanied by an ionizing secondary (Experiment III, see also Fig. 1).

Time (hr.)	Total counts				A_2BC_{del} not accompanied by a discharge in D	A_2BC_{del} accompanied by a discharge in D
	A_2BC	A_2BD	A_2BCD			
131.8	2853	6795	2674		11	49

The apparatus was arranged as shown in Fig. 1 of this paper. The coincidences A_2BC , A_2BD , A_2BDC , A_2BC_{del} , and A_2BDC_{del} were registered.

The result of this experiment (Table III) shows that very likely a fraction perhaps more than 10 percent of the delayed coincidence are not accompanied by an incoming ionizing secondary particle. If we are to interpret such a result on the basis of the hypothesis sketched above, we would say that for each registered HS, neutrons of energy of some hundred Mev enter the iron in number of the order of one. This does not appear to be much in contrast with any available experimental datum.

Whether the above explanation is true or not, it appears that the rate of the HS delayed coincidences should be corrected on the basis of Experiment III, if we want to associate a delayed coincidence with an ionizing particle reaching the end of its range in the absorber. This conclusion is of interest for the interpretation of the following experiment.

IV

Experiment IV was performed in order to determine the number of secondary particles when the absorber was, alternatively, 1 and 5 in. of iron. The experiment was made with the same apparatus of Fig. 1 of our first paper, except for Tray C , which was substituted by Tray C_1 of nine counters, 1 in. diameter having $2\frac{1}{2}$ in. center-to-center distance. Tray C_1 was also 1 in. lower than the Tray C of Fig. 1, first paper. The absorber below Tray C_1 was always 5 in. sulfur. The electronic circuit connected with Tray C_1 registered separately

the coincidences A_2BC_1 , A_2BC_2 , A_2BC_3 , A_2BC_4 , where the subscript in C_n means that *more* than n counters were discharged among the nine of the tray. From the progressive decrease of the rates $A_2BC_1 \cdots A_2BC_4$ (Table IV), it is felt that the neglect of the cases where more than four counters are discharged does not constitute an error more than a very few percent. The same argument, together with the fact that the angle subtended by a counter of C_1 , in a plane perpendicular to the axis of the counter itself, with respect to a point at the level of Tray A is only about two degrees, suggests that only seldom must one counter of C_1 be crossed by two secondary particles.^b Similarly, the practical constancy of the ratio of $(ABC_{\text{del}})/(A_2BC_{\text{del}})$ measured with three different combinations for Tray C and the absorber above^c shows that the efficiency of the same tray, as detector of delayed electrons, is not too much affected by the multiplicity of the HS.

The data of Table IV allow us to make a comparison between the number $(HS)_F$ of secondary particles which fail to cross four inches of iron and the number $(HS)_{\pi\text{-ion}}$ of secondary particles which are mesons and reach the end of their range because of ionization losses, therefore producing a decay electron approximately one-half of the times. We know that the number of HS delayed coincidences, $(HS)_{\text{del}}$, is proportional to $(HS)_{\pi\text{-ion}}$. The coefficient of proportionality can be written GE , where G represents the coefficient of proportionality which should be used if all the mesons had a positive charge. E is the ratio of the number of positive mesons to the total number of mesons, and is equal to $(1+\epsilon)/(2+\epsilon)$, ϵ being the positive excess. G obviously depends on the geometry and on the thickness of the absorber, as well as on the energy spectrum of the delayed electrons and on the limits of delay within which the delayed coincidences are registered. Rather than compute G , we note that the product GE is about equal to the ratio of $(NP)_{\text{del}}/(NP)_F$, where $(NP)_F$, the decrease in the ABC coincidences when four inches of iron are inserted, is to be identified with $(NP)_{\mu\text{-ion}}$, namely, the number of μ -mesons stopped in the four

TABLE IV. Comparison between the absorption power and the decay-electron emitting power.

1	2	3	4	5	6	7	8	9	10*	Absorber above Tray C
Time (hr.)	$ABC \times h^{-1**}$	$ABC_{\text{del}} \times h^{-1}$ (total of the delays in 3 channels, less casuals)	$A_2B \times h^{-1}$	$A_2BC_{\text{del}} \times h^{-1}$ (total of the delays in 3 channels, less casuals)	$A_2BC_1 \times h^{-1}$	$A_2BC_2 \times h^{-1}$	$A_2BC_3 \times h^{-1}$	$A_2BC_4 \times h^{-1}$	Total of columns 6, 7, 8, 9 (h^{-1})	
165.33	5890	8.45 ± 0.3	108.8	0.56 ± 0.06	44.8	17.8	9.68	5.53	77.8 ± 0.7	1 in. of iron
156.50	5550	8.95 ± 0.3 8.7 ± 0.2	108.7	0.45 ± 0.06 0.505 ± 0.04	35.5	14.5	7.73	4.35	62.1 ± 0.6	5 in. of iron
	340								15.7 ± 0.1	Averages between 1 and 5 in. Differences between 1 and 5 in.

* A_2BC_n means a HS which discharged *more* than n counters in C . For instance a HS discharging 3 counters, gives one count A_2BC_1 , one A_2BC_2 , and one A_2BC_3 . Column 10 gives thus the total number of counters discharged, with the errors that multiplicities higher than four are taken into account as four.

** Measured with a scaling circuit.

^b See, for instance, the data on angular divergence, obtained with a cloud chamber by W. Fretter, Phys. Rev. 76, 511 (1949).

^c See Table I of the first paper and Tables II and III of the present one.

inches of iron by ionization losses. Indeed, as we have said in Section II of the first paper, within few percent $(NP)_F$ represent nothing else but μ -mesons stopped by ionization in the four inches iron, namely, the analog of $(HS)_{\pi\text{-ion}}$ for NP. Concerning the factor E , however, we should take into account the possibility for the positive excess ϵ_{HS} of the locally produced mesons, to be different from the positive excess ϵ_{NP} of the normal mesons. We note that if ϵ is changed by a factor two, namely, from 0.2 to 0.4, E only changes by a factor 1.07. From the considerations made in Section IV of the first paper, we are led to believe that neglecting such a possible difference between ϵ_{HS} and ϵ_{NP} we make an error not greater than 10 percent. We will take into account this 10 percent indeterminacy in the discussion. The factor G is the same for NP and HS, because the geometry, the circuit, the energy spectrum of the decay electrons, and the mean life for their production are the same in both cases. Our conclusion¹ that the HS mesons are π 's, obviously does not imply any practical difference in the value of G to be accepted for HS.

The number $(HS)_{\pi\text{-ion}}$ can then be obtained from the expression:

$$(HS)_{\pi\text{-ion}} = (NP)_F \frac{A_2BC_{\text{del}}}{ABC_{\text{del}}}$$

and, from Table IV, $(HS)_{\pi\text{-ion}} = 19.8 \pm 1.8 \text{ hr.}^{-1}$.

On the other hand, $(HS)_F$ is supposed to be equal to $(HS)_{\pi\text{-ion}}$, the number of mesons stopped by ionization, plus $(HS)_{\pi\text{-nucl}}$, the number of mesons absorbed by nuclear interaction, plus $(HS)_F$ the number of other particles than mesons, stopped or scattered away by the four inches iron absorber.

Now $(HS)_F$ is only 15.7 ± 1 , namely, less than $(HS)_{\pi\text{-ion}}$, which suggests that we should apply some correction either to $(HS)_F$ or to $(HS)_{\pi\text{-ion}}$ which is deduced from A_2BC_{del} . We already mentioned two corrections which might be needed. First, we concluded that from the uncertainty in the value of the positive excess of HS mesons may arise an uncertainty of ~ 10 percent in the computation of $(HS)_{\pi\text{-ion}}$. Second, and more important, in Experiment III we actually found evidence that a fraction of the A_2BC_{del} were due to non-ionizing particles. We estimated that such a fraction is not likely to be more than 20 percent, because of Experiment III. Introducing those two corrections, we can reduce the value of $(HS)_{\pi\text{-ion}}$ to about 14. But one would say that the difference $(HS)_F - (HS)_{\pi\text{-ion}}$ cannot be as large as 5, without introducing some rather artificial assumption. Taking such a number as an upper limit, we conclude that no more than five out of 77 particles are absorbed by the nuclei in the four inches of iron. That is to say that, the absorption mean free path of the π -mesons in iron is more than 1200 g/cm^{-2} . As to the average energy of the mesons to which such datum is referred, we can give an estimate extrapolating the iron absorption curve. The obtained average, is about 400 Mev.

The result of this experiment also show that $(HS)_X$, the number of particles different from mesons and stopping in the absorber, is unimportant. The proton component is practically extinguished by the lead above Tray C.

V. DISCUSSION

We see that, in progressive order of importance, the comparison of the absorption of lead and iron, the curve of the delayed coincidence rate-*versus*-thickness of absorber, the comparison of the absorption in iron with the rate of delayed coincidences, all point toward the conclusion that Λ_π , the absorption mean free path of π -mesons of energy of the order of few hundred Mev is much larger than the value corresponding to the geometrical size of iron nuclei ($\sim 100 \text{ g/cm}^{-2}$).

We should notice that in our apparatus a nuclear non-Coulombian scattering would have the same effect as a nuclear absorption only if the angle of scattering would be as large as 90 degrees. According to Marshak,⁴ if one uses the pseudoscalar theory of meson forces, the scattering cross sections, per unit solid angle, in the forward, transverse and backward directions are, respectively, proportional to the numbers 7; 2.5; 1. Referring to those values the ratio of the scattering in the backward hemisphere to the total scattering would then be as low as $\frac{1}{4}$, therefore we could not learn much about the nuclear scattering of π -mesons from our value of Λ_π . Experiments already reported by Fretter⁵ and by Lovati, Mura, Salvini, and Tagliaferri⁶ where secondary particles of penetrating showers were observed in a cloud chamber, gave very little evidence, if any, for the scattering of those particles. A number of secondary nuclear events were found in both of those experiments, from which a collision mean free path, in lead, of about 370 g/cm^{-2} according to the Italian group, and of less than 750 g/cm^{-2} according to Fretter could be derived. The last number is a result of a much larger number of events than was obtained by the Italian group, but is not corrected for the probability that a nuclear event would escape observation. We believe that, as the authors pointed out, the question of how many protons are among the particles observed in those experiments, is still an open one.

If the fraction of protons would be reasonably small, our result would appear to be in disagreement at least with the value reported by the Italian group. It is perhaps worth notice that the thickness of lead to be crossed by the secondary particles is much larger in our apparatus than it is in both the previously mentioned cloud-chamber experiments. That makes it impossible to assume that the fractional number of protons is the same in our experiment and in the cloud-chamber ones.

⁴ R. E. Marshak, Echo Lake Conference on Cosmic Rays (June 22-28, 1949).

⁵ W. B. Fretter, Echo Lake Conference on Cosmic Rays (June 22-28, 1949).

⁶ Lovati, Mura, Salvini, and Tagliaferri, Nuovo Cimento VI, 207 (1949).

On the other hand, it may be noted that Rochester, Butler, Mitra, and Rosser,⁷ working with a cloud chamber covered with 16.8 cm or less, of lead, found among the secondary particles of hard showers, 14 positive lightly ionizing tracks and two negatives, which, as noted by the authors, suggests that such particles were mainly protons. And it is significant that of the slow particles for which the British authors were able to recognize the mass, 23 were protons and 5 mesons.

We thus think that no real discrepancy can be found between our result on the nuclear absorption of π -mesons and any of the experimental evidence now available.

Our lower limit of 1200 g/cm⁻² for Λ_π seems therefore to be significant.

Finally, we would like to underline that from Experiment III we conclude that the locally produced

⁷ Rochester, Butler, Mitra, and Rosser, reported by G. D. Rochester, Symposium on Cosmic Rays, California Institute of Technology (June 21-23, 1948), *Rev. Mod. Phys.* 21, 20 (1949).

mesons detected with the delayed coincidence method are mainly generated not in low energy processes by secondary nucleons, but in high energy ones, and most likely in the very first nuclear collision of the arriving nucleon. This supports the standpoint which we have taken in the previous paper.

The average number of particles arriving on Tray C from the lead block for every HS (A_2BC) turns out to be about four, from the average number of counters discharged in Tray C_1 (1.7) and the fraction of the surface covered by the counters (1/2.25).

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The Effect of Screening on Beta-Ray Spectra and Internal Conversion*

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The Dirac equation for an electron in the field of a Thomas-Fermi-Dirac atom is solved numerically on the ENIAC for a large number of cases. The resulting wave functions are used to calculate the effect of screening (by the atomic electrons) on allowed beta-spectra and on the internal conversion coefficients of gamma-rays. It is seen that the negatron spectra are essentially unaffected by screening; positron spectra are affected appreciably in a direction such as to increase the number of low energy positrons. Where a comparison can be made between the present calculations and previous ones in which the screening has been neglected, it is seen that the effect of screening is to increase the conversion coefficients slightly. Most of the present calculations, however, are for the soft gamma-ray region, for which only approximate formulas for the conversion coefficients exist. Thus the results of this paper are used to test the accuracy of these formulas. An attempt is also made to classify a number of experimentally observed gamma-rays.

I. INTRODUCTION

THE purpose of the work described in this paper is to obtain solutions of the relativistic motion of an electron in an atomic potential, this potential being supplied by the statistical model of the atom, and to use the wave functions obtained to calculate internal conversion coefficients and beta-ray spectra. These results, when compared with those calculated using Coulomb wave functions, give the so-called "screening correction" due to atomic electrons.

Whereas the Coulomb correction to the shape of the allowed beta-ray spectrum (obtained by using Coulomb

eigenfunctions instead of free particle functions) is quite large, giving a factor of about 100 for elements of high nuclear charge and negatrons of low energy, the screening correction is generally assumed to be small. Approximate calculations of the screening correction by Rose¹ and Longmire and Brown² have verified this assumption, although the correction for low energy positrons was found to be appreciable. More accurate calculations of the screening correction seemed desirable since the deviations from the theoretical spectra found experimentally in Cu⁶⁴ and S³⁵ (both of the allowed type) have been interpreted^{3,4} as a failure of the Fermi

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¹ M. E. Rose, *Phys. Rev.* 49, 727 (1936).

² C. Longmire and H. Brown, *Phys. Rev.* 75, 1102, 264 (1949).

³ C. S. Cook and L. M. Langer, *Phys. Rev.* 73, 601 (1948).

⁴ Cook, Langer, and Price, *Phys. Rev.* 74, 548 (1948).