in both cases, *below* the transition point by the formulas

$$p_3=p_3{}^0\cdot X_e, \quad (3)$$

$$p_4=p_4{}^0\cdot(1-X_e)^x\cdot\exp[G_4(T;x)-G_4(T;x_0)] \quad (4)$$

and *above* the transition point by the formulas

$$p_3=p_3{}^0X, \quad (5)$$

$$p_4=p_4{}^0\cdot(1-X)\cdot\exp[G_4(T;1)-G_4(T;x_0)]=p_4{}^{0\prime}\cdot(1-X). \quad (6)$$

In these formulas $p_3{}^0$ and $p_4{}^0$ are the vapor pressures of pure He^3 and He^4, and $p_4{}^{0\prime}$, for temperatures *below* the lambda-point of *pure* He^4, is the *extrapolated* vapor pressure of the phase *II*. When x is supposed to be independent of the concentration, X, one must substitute in (3) and (4) the relation $x=x_0(T)$, but when x is supposed to change with X one must substitute $x=x(T,X)$ as determined from the condition $dG/dx=0$ at constant values of T and X. For very small concentrations in both cases the vapor pressures become

$$p_3=p_3{}^0\cdot X/x, \quad p_4=p_4{}^0. \quad (7)$$

[1] J. de Boer, Phys. Rev. 72, 852 (1949).
[2] Taconis, Beenakker, Nier, and Aldrich, Physica 15, 733 (1949).
[3] Communication at the M.I.T. Conference (September 6–10, 1949).

Regularization as a Consequence of Higher Order Equations

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We consider any invariant differential equation in space-time of finite order $2n$ and write it in the factorized form:

$$\prod_{i=1}^{n}(\Box^2-m_i{}^2)\phi(x)=0. \quad (1)$$

For simplicity we assume $\phi(x)$ to be a scalar field. The Green function of (1) which satisfies

$$\prod_i(\Box^2-m_i{}^2)\bar{D}(x)=-\delta(x) \quad (2)$$

is given by:

$$\bar{D}(x)=\frac{(-1)^{n-1}}{(2\pi)^4}\int e^{ikx}\prod_i(k^2+m_i{}^2)^{-1}d^4k. \quad (3)$$

For quantization of ϕ we need the commutator function

$$D(x-x')=\frac{1}{i}[\phi(x),\phi^*(x')],$$

which is an odd invariant function obeying (1). This determines it to be

$$D(x)=-\frac{i(-1)^{n-1}}{(2\pi)^3}\int e^{ikx}\delta[\prod_i(k^2+m_i{}^2)]\epsilon(k)dk \quad (4)$$

(in Schwinger's notation), the numerical factor we get from the relation

$$\bar{D}(x)=-\tfrac{1}{2}D(x)\epsilon(x). \quad (5)$$

Developing the δ-function

$$\delta[\prod(k^2+m_i{}^2)]=\sum_i\frac{\delta(k^2+m_i{}^2)}{\prod_{j\neq i}(m_j{}^2-m_i{}^2)}$$

and the denominator

$$\prod_i(k^2+m_i{}^2)^{-1}=\sum(k^2+m_i{}^2)^{-1}\prod_{j\neq 1}(m_j{}^2-m_i{}^2)^{-1},$$

we obtain

$$D=c_iD_{m_i};\quad \bar{D}=c_i\bar{D}_{m_i};\quad c_i=\prod_{j\neq 1}(m_j{}^2-m_i{}^2)^{-1}(-1)^{n-1}. \quad (6)$$

D is a sum of ordinary D functions with different masses, the coefficients obey the following conditions:

$$\sum c_i=0,\quad \sum_i c_i\sum_{j\neq i}m_j{}^2=0\cdots\sum_i c_i\prod_{j\neq i}m_j{}^2=(-1)^{n-1}. \quad (7)$$

Using (6), (7), and the well-known property of $\bar{D}_m$, we can again prove (2); (1) for D is trivial after (6).

The first condition of (7) is Pauli's regularization condition which cancels the δ-singularity of the D function. For more than one mass there cannot be a δ-singularity in D, as (4) is then a convergent integral. Corresponding to the regularization method there is a fourth-order equation sufficient to make the self-energy of the nucleus finite, if we use ϕ as the meson field, as (using the corresponding Feynmann's function)

$$\int\frac{\gamma_5[\gamma_i(p_i-k_i)-m]\gamma_5d^4k}{[k^2+\mu_1{}^2][k^2+\mu_2{}^2][(p-k)^2+m^2]}$$

is a convergent integral. This shows that the equations of Podolsky, Bhabha, Born, and Green, etc., if quantized, lead automatically to regularization and give a simpler model of subtracting fields than the otherwise *ad hoc* assumed one.

The Photo-Disintegration of Nitrogen at Energies of 20 Mev to 100 Mev

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The photo-disintegration of several gaseous elements by x-rays from the 100-Mev betatron[1] is being studied in a cloud chamber, and the relative abundance of various modes of nuclear disintegration directly compared. The data obtained so far include about 5000 nuclear disintegrations in nitrogen. The energy dependence of some of these events is discussed in this note.

The cloud chamber[2] developed by the authors is suited for this type of investigation because it can be operated directly in a high energy x-ray beam of relatively strong intensity owing to its good clean-up characteristics,[3] and operation of the chamber at the rate of an expansion every five seconds permits the accumulation of adequate data in a relatively short time. The betatron is adjusted for an intensity level near its maximum output and is pulsed in synchronism with the cloud chamber. The x-ray beam, defined by thick sections of lead, is $\frac{1}{8}$ in.$\times\frac{3}{4}$ in. in cross section, and enters the cloud chamber through a thin Lucite window.

The disintegrations observed in the cloud chamber are classified according to the number of visible tracks as follows: (1) "Singles," heavily ionizing tracks of short range (<1 cm). (2) "Flags," consisting of two charged members, one of which is short and heavily ionizing. (3) "Stars," with three to six charged members; as the multiplicity of the star increases there is an increasing probability that one member will be too short to be visible. The total number of flags and stars, and their ratios, are given in Table I for peak betatron energies from 20 to 100 Mev.

The relative yield of flags at peak betatron energies of 50 and 100 Mev has been obtained by normalizing some of the data as follows: The 10-minute beta-activity induced in thin copper samples placed in the x-ray beam is taken as a monitor of the beam intensity in the energy range of the betatron spectrum near 25 Mev.[4] Since it is difficult to build up this activity with x-ray pulses coming only once in five seconds, a Victoreen r-thimble is used to measure the relative integrated intensity for each cloud-chamber run. A separate run at the same energy, with the betatron operating at 60 pulses per second, provides excitation for the copper sample, the integrated intensity again being measured with an r-thimble. The Cu^{62} activity corresponding to the integrated x-ray intensity for the cloud-chamber run is derived from these readings. The ratio of flags to copper activity at 50 and 100 Mev gives the relative yield at these peak energies. The results are shown in Table II; the estimated accuracy of each ratio is ten percent.

The ratio of flags to singles has been measured for a portion of the data taken at 100 Mev, for gas pressures of one-half and one