in both cases, *below* the transition point by the formulas

$$p_3 = p_3^0 \cdot X_e, \quad (3)$$

$$p_4 = p_4^0 \cdot (1 - X_e)^x \cdot \exp[G_4(T; x) - G_4(T; x_0)] \quad (4)$$

and *above* the transition point by the formulas

$$p_3 = p_3^0 X, \quad (5)$$

$$p_4 = p_4^0 \cdot (1 - X) \cdot \exp[G_4(T; 1) - G_4(T; x_0)] = p_4^{0'} \cdot (1 - X). \quad (6)$$

In these formulas p_3^0 and p_4^0 are the vapor pressures of pure He^3 and He^4, and $p_4^{0'}$, for temperatures *below* the lambda-point of *pure* He^4, is the *extrapolated* vapor pressure of the phase *II*. When x is supposed to be independent of the concentration, X, one must substitute in (3) and (4) the relation $x = x_0(T)$, but when x is supposed to change with X one must substitute $x = x(T, X)$ as determined from the condition $dG/dx = 0$ at constant values of T and X. For very small concentrations in both cases the vapor pressures become

$$p_3 = p_3^0 \cdot X/x, \quad p_4 = p_4^0. \quad (7)$$

[1] J. de Boer, Phys. Rev. 72, 852 (1949).
[2] Taconis, Beenakker, Nier, and Aldrich, Physica 15, 733 (1949).
[3] Communication at the M.I.T. Conference (September 6–10, 1949).

Regularization as a Consequence of Higher Order Equations

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December 19, 1949

WE consider any invariant differential equation in space-time of finite order $2n$ and write it in the factorized form:

$$\prod_{i=1}^{n}(\Box^2 - m_i^2)\phi(x) = 0. \quad (1)$$

For simplicity we assume $\phi(x)$ to be a scalar field. The Green function of (1) which satisfies

$$\prod_i(\Box^2 - m_i^2)\bar{D}(x) = -\delta(x) \quad (2)$$

is given by:

$$\bar{D}(x) = \frac{(-1)^{n-1}}{(2\pi)^4}\int e^{ikx}\prod_i(k^2 + m_i^2)^{-1}d^4k. \quad (3)$$

For quantization of ϕ we need the commutator function

$$D(x - x') = \frac{1}{i}[\phi(x), \phi^*(x')],$$

which is an odd invariant function obeying (1). This determines it to be

$$D(x) = -\frac{i(-1)^{n-1}}{(2\pi)^3}\int e^{ikx}\delta[\prod_i(k^2 + m_i^2)]\epsilon(k)dk \quad (4)$$

(in Schwinger's notation), the numerical factor we get from the relation

$$\bar{D}(x) = -\tfrac{1}{2}D(x)\epsilon(x). \quad (5)$$

Developing the δ-function

$$\delta[\prod(k^2 + m_i^2)] = \sum_i \frac{\delta(k^2 + m_i^2)}{\prod_{j \neq i}(m_j^2 - m_i^2)}$$

and the denominator

$$\prod_i(k^2 + m_i^2)^{-1} = \sum(k^2 + m_i^2)^{-1}\prod_{j \neq 1}(m_j^2 - m_i^2)^{-1},$$

we obtain

$$D = c_i D m_i; \quad \bar{D} = c_i \bar{D} m_i; \quad c_i = \prod_{j \neq 1}(m_j^2 - m_i^2)^{-1}(-1)^{n-1}. \quad (6)$$

D is a sum of ordinary D functions with different masses, the coefficients obey the following conditions:

$$\sum c_i = 0, \quad \sum_i c_i \sum_{j \neq i} m_j^2 = 0 \cdots \sum_i c_i \prod_{j \neq i} m_j^2 = (-1)^{n-1}. \quad (7)$$

Using (6), (7), and the well-known property of $\bar{D}_m$, we can again prove (2); (1) for D is trivial after (6).

The first condition of (7) is Pauli's regularization condition which cancels the δ-singularity of the D function. For more than one mass there cannot be a δ-singularity in D, as (4) is then a convergent integral. Corresponding to the regularization method there is a fourth-order equation sufficient to make the self-energy of the nucleus finite, if we use ϕ as the meson field, as (using the corresponding Feynmann's function)

$$\int \frac{\gamma_5[\gamma_i(p_i - k_i) - m]\gamma_5 d^4k}{[k^2 + \mu_1^2][k^2 + \mu_2^2][(p - k)^2 + m^2]}$$

is a convergent integral. This shows that the equations of Podolsky, Bhabha, Born, and Green, etc., if quantized, lead automatically to regularization and give a simpler model of subtracting fields than the otherwise *ad hoc* assumed one.

The Photo-Disintegration of Nitrogen at Energies of 20 Mev to 100 Mev

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December 30, 1949

THE photo-disintegration of several gaseous elements by x-rays from the 100-Mev betatron[1] is being studied in a cloud chamber, and the relative abundance of various modes of nuclear disintegration directly compared. The data obtained so far include about 5000 nuclear disintegrations in nitrogen. The energy dependence of some of these events is discussed in this note.

The cloud chamber[2] developed by the authors is suited for this type of investigation because it can be operated directly in a high energy x-ray beam of relatively strong intensity owing to its good clean-up characteristics,[3] and operation of the chamber at the rate of an expansion every five seconds permits the accumulation of adequate data in a relatively short time. The betatron is adjusted for an intensity level near its maximum output and is pulsed in synchronism with the cloud chamber. The x-ray beam, defined by thick sections of lead, is $\frac{1}{8}$ in.$\times\frac{3}{4}$ in. in cross section, and enters the cloud chamber through a thin Lucite window.

The disintegrations observed in the cloud chamber are classified according to the number of visible tracks as follows: (1) "Singles," heavily ionizing tracks of short range (<1 cm). (2) "Flags," consisting of two charged members, one of which is short and heavily ionizing. (3) "Stars," with three to six charged members; as the multiplicity of the star increases there is an increasing probability that one member will be too short to be visible. The total number of flags and stars, and their ratios, are given in Table I for peak betatron energies from 20 to 100 Mev.

The relative yield of flags at peak betatron energies of 50 and 100 Mev has been obtained by normalizing some of the data as follows: The 10-minute beta-activity induced in thin copper samples placed in the x-ray beam is taken as a monitor of the beam intensity in the energy range of the betatron spectrum near 25 Mev.[4] Since it is difficult to build up this activity with x-ray pulses coming only once in five seconds, a Victoreen r-thimble is used to measure the relative integrated intensity for each cloud-chamber run. A separate run at the same energy, with the betatron operating at 60 pulses per second, provides excitation for the copper sample, the integrated intensity again being measured with an r-thimble. The Cu^{62} activity corresponding to the integrated x-ray intensity for the cloud-chamber run is derived from these readings. The ratio of flags to copper activity at 50 and 100 Mev gives the relative yield at these peak energies. The results are shown in Table II; the estimated accuracy of each ratio is ten percent.

The ratio of flags to singles has been measured for a portion of the data taken at 100 Mev, for gas pressures of one-half and one

TABLE I. Nuclear disintegrations observed in nitrogen at various peak x-ray energies. These data show the relative abundance of the various types of stars observed at different energies, but are not normalized to yield excitation curves for any one type.

Peak energy	Number of observed tracks per disintegration 2	3	4	5	Stars/Flags
20 Mev	6	0	0	0	0
25	40	0	1	0	0.02
30	125	4	1	0	0.04
40	493	57	20	0	0.16
50	517	79	40	0	0.23
60	147	29	24	0	0.36
70	197	44	22	0	0.34
100	2346	469	296	17	0.33

TABLE II. Yield of flags in nitrogen at peak energies of 50 and 100 Mev relative to the beta-activity (Cu^{62}) induced in Cu samples irradiated by x-rays of the corresponding peak energy and of the same relative intensity.

Peak energy	No. of flags	No. of flags / Copper activity
50 Mev	191	1.00
100	212	0.89

atmosphere. The lower pressure is used to check the possible loss of the shortest singles. The ratio is 4.0, based on a total count of 442 singles and 1623 flags; the same ratio is obtained for both pressures. This result includes a correction for some singles caused by neutron induced reactions,[5] particularly $N^{14}(n, p)C^{14}$.

From the data of Table I and Table II we note the following: (1) The most numerous events are the flags, which according to a preliminary analysis appear to be mostly (γ, pn) disintegrations. The singles, representing (γ, n) disintegrations, comprise only about 16 percent of the events at 100-Mev peak energy. (2) There is no appreciable production of flags by x-rays with energy between 50 and 100 Mev. An approximate normalization of the data taken with a peak x-ray energy of 20 Mev indicates the yield below 20 Mev to be about 5 percent of that with 50-Mev peak energy. (3) There is a significant increase in the ratio of three and four prong stars to flags up to 60 Mev.

We wish to express our appreciation to the betatron operators, Mr. E. L. Martin and Mr. J. McNamara. We are indebted to Dr. G. C. Baldwin and Mr. C. E. Pickert for the use of counting equipment and assistance in the counting of the Cu^{62} activities used to normalize our data at various energies. The cloud-chamber equipment used in this investigation was developed under Contract N7 onr 332 with the ONR.

[1] Some disintegrations caused by radiation from the 100-Mev betatron have been observed in a cloud chamber by G. C. Baldwin and G. S. Klaiber, Phys. Rev. **70**, 259 (1946).
[2] E. R. Gaerttner and M. L. Yeater, Rev. Sci. Inst. **20**, 588 (1949).
[3] Additional discrimination against the electron background is obtained through the use of a short growth time for the tracks.
[4] The $Cu^{63}(\gamma, n)Cu^{62}$ cross section reaches its maximum value at 24 Mev. See G. C. Baldwin and G. S. Klaiber, Phys. Rev. **73**, 1156 (1948).
[5] A correction has been made for these on the basis of their distribution in and out of the beam in the cloud chamber and the intensity of the neutron background as measured in a study of proton recoils observed when the cloud chamber was filled with hydrogen. The number of flags and stars produced by neutrons is negligible.

Erratum: Microwave Spectrum of CH_2CFCl

[Phys. Rev. **77**, 148 (1950)]

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FIGURE 1 of this Letter to the Editor was unfortunately misplaced, and appears on page 152 (January 1, 1950) of this volume.

Erratum: Non-Linear I-V Characteristic of Ge at Very Low Temperatures

[Phys. Rev. **77**, 152 (1950)]

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FIGURE 1 of this Letter to the Editor was unfortunately misplaced, and appears on page 148 (January 1, 1950) of this volume.