$\times 10^{-4}$, where $g_J(^2P_{\frac{1}{2}})=0.66599$. For indium F/R is 0.94930. Insertion of the numerical quantities in Eq. (6) gives $Q^{115}=1.161\times 10^{-24}$ cm^2.

It is to be noted that the diamagnetic correction customarily applied to observed nuclear g values has not been applied here. It is not at all clear from the present state of the theory that this correction should be made. It seems doubtful that the theory is sufficiently good to enable one to calculate quadrupole moments with a precision limited only by uncertainties in observed quantities. It should also be noted that in several instances the measured ratio of the h.f.s. separations of the same states of two isotopes is not simply derived from the measured ratio of the nuclear magnetic dipole moments.[14] The presence of such an effect in indium would cause the calculated quadrupole moment to be in error. For atoms with the same spin, the magnitude of the effect presumably is small compared to the error arising from uncertainties in the application of the diamagnetic correction.

[14] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948); F. Bitter, Phys. Rev. **76**, 150 (1949); P. Kusch and A. K. Mann, Phys. Rev. **76**, 707 (1949).

To a very high order of approximation

$$Q^{115}/Q^{113}=b^{115}/b^{113}=1.0146$$

which leads to the value $Q^{113}=1.144\times 10^{-24}$ cm^2, where the diamagnetic correction has not been made.

CONCLUSIONS

The h.f.s. of the $^2P_{3/2}$ state of indium can be described exactly in terms of two interaction constants. No experimental evidence has been found to indicate the existence of a nuclear moment of higher order than the quadrupole moment. This is the heaviest atom in which a critical search has been made for higher order nuclear moments.

The addition of two neutrons to In^{113} produces an isotope which differs only very slightly from In^{113} in certain measurable nuclear properties. The spins are identical, the magnetic dipole moments differ by 0.22 percent, and the electric quadrupole moments differ by 1.46 percent. This is the greatest degree of similarity observed for any pair of isotopes of odd mass differing in mass by two units.

PHYSICAL REVIEW VOLUME 77, NUMBER 4 FEBRUARY 15, 1950

Further Data on the Spin Gyromagnetic Ratio of the Electron*,†

A. K. MANN‡ AND P. KUSCH
Columbia University, New York, New York

(Received October 26, 1949)

The atomic beam magnetic resonance method has been used to compare the g_J values of In in the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ states in order to obtain a value of the spin gyromagnetic ratio of the electron. The determination of the g_J ratio involved only the measurement of frequencies of lines in the h.f.s. spectra of both states at constant magnetic field. Lines with widely different frequency dependence on field were selected to avoid possible systematic errors arising from inhomogeneities in the field. The result of these measurements is $g_J(^2P_{3/2})/g_J(^2P_{\frac{1}{2}})=2(1.00200\pm 0.00006)$. Assuming Russell-Saunders coupling, and $g_L=1$, the spin gyromagnetic ratio of the electron may be calculated. The result is $g_S=2(1.00133\pm 0.00004)$, which is to be compared with the value $g_S=2(1.00119\pm 0.00005)$ obtained by Kusch and Foley. The discrepancy is greater than the sum of the experimental errors. The $^2P_{\frac{1}{2}}$ state of In is believed to be free of significant perturbations which might affect the total electronic g value. However, in view of possible perturbations of the $^2P_{3/2}$ state, the agreement must be considered as very good. The present result confirms the conclusions of the previous experiment, both as to the existence of the intrinsic magnetic moment of the electron, and as to its approximate magnitude.

INTRODUCTION

In a recent experiment, Kusch and Foley[1] have measured the ratio of the electron spin g value (g_S) to the orbital g value (g_L) by comparing the total electronic g values of gallium in the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states, indium in the $^2P_{\frac{1}{2}}$ state, and sodium in the $^2S_{\frac{1}{2}}$ state. Three independent intercomparisons gave three values for the ratio g_S/g_L which are in agreement to within 7 parts in 10^5. In addition, the experimental result is in good agreement with the result obtained by Schwinger[2] from a first-order theoretical investigation.

The method employed by Kusch and Foley avoids the difficulty of producing a magnetic field which is known to a high order of precision in terms of absolute standards, and which is required for a precision measurement of the absolute value of a gyromagnetic ratio. However, the validity of any individual determination of the ratio g_S/g_L by the method of Kusch and Foley may be limited by deviations of the properties of the atomic systems from the simple description implicit in

* A brief account of the material in this paper was given at the April, 1949 meeting of the American Physical Society.
† This research has been supported in part by the ONR.
‡ Now at the University of Pennsylvania, Philadelphia, Pennsylvania.

[1] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).

[2] J. Schwinger, Phys. Rev. **73**, 416 (1948).

TABLE I. Observations of lines in the spectra of the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states of indium, and a comparison of the apparent values of H'.

State	Line	Frequency mc/sec.	H'	Deviation
$^2P_{\frac{1}{2}}$	$5,5\leftrightarrow 5,4$	23.056	355.582	-0.018
	$5,4\leftrightarrow 5,3$	23.152	355.596	-0.004
	$5,3\leftrightarrow 5,2$	23.247	355.617	$+0.016$
	$5,-1\leftrightarrow 5,-2$	23.639	355.606	$+0.006$
		$H'(^2P_{\frac{1}{2}})$ =	355.600 ±	0.011
$^2P_{3/2}$	$6,-4\leftrightarrow 6,-3$	134.043	356.308	-0.001
	$6,-5\leftrightarrow 6,-4$	139.916	356.287	-0.022
	$5,-5\leftrightarrow 5,-4$	142.941	356.297	-0.012
	$6,-6\leftrightarrow 6,-5$	146.075	356.345	$+0.036$
		$H'(^2P_{3/2})$ =	356.309 ±	0.018
		$\Delta H'/H'=0.00199$		

the theory underlying the experiment. In general, such deviations cannot be calculated with precision, and the validity of an individual measurement of g_S/g_L is determined by its consistency with values of the ratio obtained from other atomic systems. The fact that three independent sets of experimental data yield substantially identical values indicates that perturbations which affect the total electronic g values of the atomic energy states are small. It is, however, desirable to investigate various atomic systems in an effort to improve the precision of the experimental value of g_S/g_L. In the experiment to be described in this paper, a value of g_S/g_L is obtained from a comparison of the g_J values of indium in the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states. This measurement is possible because the interaction constants of indium in the $^2P_{3/2}$ state have recently been measured[3] with sufficient precision to allow the use of that state in the present experiment.

THEORY OF THE EXPERIMENT

The g_J value of any atomic energy state may be expressed as a linear combination of g_S and g_L. In particular, if Russell-Saunders coupling is assumed,

$$\begin{aligned} g_J(^2P_{3/2}) &= g_S/3+2g_L/3 \\ g_J(^2P_{\frac{1}{2}}) &= -g_S/3+4g_L/3. \end{aligned} \tag{1}$$

It is evident that a measurement of the ratio $g_J(^2P_{3/2})/g_J(^2P_{\frac{1}{2}})$ will yield a value of the ratio g_S/g_L. For reasons discussed previously,[1] it is convenient to take $g_L=1$, and, on the basis of this assumption, to calculate g_S directly.

If the interaction constants which determine the h.f.s. levels of the electronic states in question are known, it is possible to find the ratio of the g_J values of those states by observing chosen lines in the h.f.s. spectrum of each state in a fixed magnetic field. The expressions for the frequencies of the h.f.s. lines always contain the quantity $H'=(\mu_0H/h)\times 10^{-6}$, in mc/sec., which occurs in the products g_JH' and g_IH'. Accordingly, if lines are observed, at the same value of the magnetic field, in both the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states, the product g_JH' can be found for each of the states, and the ratio of the g_J values can be determined. For convenience in the actual calculation, we have assumed that $g_J{}^0(^2P_{\frac{1}{2}})=2/3$ and that $g_J{}^0(^2P_{3/2})=4/3$. Unless the ratio of the g_J values is exactly as assumed, the calculated values of H' will not be identical and the true ratio of the g_J values may be found from the calculated values of H'. If $\Delta H'=H'(^2P_{3/2})-H'(^2P_{\frac{1}{2}})$, then

$$\begin{aligned} R &= g_J(^2P_{3/2})/g_J(^2P_{\frac{1}{2}}) \\ &= [g_J{}^0(^2P_{3/2})/g_J{}^0(^2P_{\frac{1}{2}})][1+\Delta H'/H'(^2P_{\frac{1}{2}})]. \end{aligned} \tag{2}$$

From Eqs. (1), it follows at once that

$$g_S/g_L=2(2R-1)/(R+1). \tag{3}$$

If g_S differs from the value predicted by the Dirac theory by a small quantity, δ_S, and if g_L is assumed equal to one, we may write

$$g_S=2(1+\delta_S)$$

where

$$\delta_S=(R-2)/(R+1).$$

For small $\Delta H'$, $\delta_S=2\Delta H'/3H'$, where it is not necessary within the present precision to distinguish between $H'(^2P_{3/2})$ and $H'(^2P_{\frac{1}{2}})$.

To find the value of H' which corresponds to any observed line frequency in the h.f.s. spectrum of the $^2P_{\frac{1}{2}}$ state, it is necessary to know, in addition to the spin, the h.f.s. separation at zero field ($\Delta\nu$) and the ratio $g_I/g_J(^2P_{\frac{1}{2}})$. These quantities have been measured with high precision,[4] and application of the well-known equations[1] which describe the energies of the h.f.s. levels of an atom for which $J=\frac{1}{2}$, readily yields H'. To find the value of H' which corresponds to any observed line frequency in the spectrum of the $^2P_{3/2}$ state, it is necessary to know the interaction constants a and b, and the ratio $g_I/g_J(^2P_{3/2})$. The values of a and b have been reported in the preceding paper.[3] The ratio $g_I/g_J(^2P_{3/2})$ can be found to sufficient accuracy, since g_I is very much smaller than g_J, by using the value of $g_I/g_J(^2P_{\frac{1}{2}})$ and assuming that $g_J(^2P_{3/2})/g_J(^2P_{\frac{1}{2}})$ is the same for indium as for gallium.[1] In the most general case, it is then necessary to find a value of H' such that the difference between roots of the appropriate secular equations has a value corresponding to the observed line frequency. The secular equations are of considerable complexity and will not be reproduced here.

PROCEDURE

The general experimental procedures employed in this experiment are identical to those previously described[1] for the determination of the ratio g_S/g_L from a comparison of the g_J values of gallium in the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states. The experimental difficulties are, however,

[3] A. K. Mann and P. Kusch, Phys. Rev. 77, 427 (1950).

[4] H. Taub and P. Kusch, Phys. Rev. 75, 1481 (1949).

considerably greater in the present case. This arises from the fact that the spin of indium is 9/2 as compared to a spin of 3/2 for gallium, and that the population of indium in the $^2P_{3/2}$ state is very much smaller than that of gallium in the same state. The high multiplicity of states corresponding to the high spin value and the low population of the upper state result in lines of small intensity which are difficult to observe.

Of the lines in the $^2P_{3/2}$ state which are designated in the weak field notation by $F,\ m\leftrightarrow F,\ m\pm1$ those lines which become $m_J,\ m_I\leftrightarrow m_J\pm1,\ m_I$ at high fields are the most satisfactory for determining the value of g_J, because at weak fields the frequencies of those lines are $\mu_0 g_F H/h$, and at high fields the frequencies are, except for constant terms, $\mu_0 g_J H/h$. At all magnetic fields the line frequencies have a first-order dependence on both g_J and H. This is not true of the lines $F,\ m\leftrightarrow F,\ m\pm1$ which become the lines $m_J,\ m_I\leftrightarrow m_J,\ m_I\pm1$ at high fields because these lines are dependent on the field at high field only through the term $\mu_0 g_I H/h$. The use of the lines $F,\ m\leftrightarrow F\pm1,\ m\pm1$ would increase the difficulty of the experiment and its interpretation since the expressions for these lines contain, in addition to the field dependent terms $\mu_0 g_F H/h$, large constant terms. In the apparatus employed in the present experiment, it was possible to observe a number of the most favorable set of lines, namely, those lines $F,\ m\leftrightarrow F,\ m\pm1$ for which the energy levels are shown in Fig. 1 of the preceding paper. The remainder of the lines $F,\ m\leftrightarrow F,\ m\pm1$ were not observable because at high fields they become the lines $m_J,\ m_I\leftrightarrow m_J,\ m_I\pm1$, and the deflection properties of the apparatus prevent such lines from being observed. The fact that a large number of lines are unobservable in this apparatus simplifies the interpretation of the data since identification of the observed lines can be made without extensive calculation, and inappropriate lines will not be observed.

In the $^2P_{\frac{1}{2}}$ state, transitions of the type $F,\ m\leftrightarrow F,\ m\pm1$ were observed. These are again, for the reasons given above, the most satisfactory transitions for determining g_J. It should be noted that a high field condition for indium in the $^2P_{3/2}$ state occurs at a very much lower absolute value of the field than the high field condition for the $^2P_{\frac{1}{2}}$ state of indium. Accordingly, a field strength at which high field conditions prevail in the deflecting fields for the $^2P_{3/2}$ state represents low intermediate field conditions for the $^2P_{\frac{1}{2}}$ state. This circumstance permits the observation of all transitions $F,\ m\leftrightarrow F,\ m\pm1$ in the $^2P_{\frac{1}{2}}$ state, even though these would become $m_J,\ m_I\leftrightarrow m_J,\ m_I\pm1$ transitions at high field. The intensity of these lines is low because the moment change involved in the transitions is small. However, under experimental conditions for which it is possible to observe the low intensity lines in the $^2P_{3/2}$ state, it is also possible to observe these lines in the $^2P_{\frac{1}{2}}$ state.

The value of the magnetic field employed in this experiment was about 250 gauss. At this field the width of the observed lines was substantially determined by the transit time width and did not appear to be affected by inhomogeneities in the magnetic field. At larger values of the magnetic field, the increase in field inhomogeneity in our magnet resulted in increased width of the more field dependent lines in the spectrum of the $^2P_{3/2}$ state. The lower limit of the magnetic field is, of course, determined by the requirement that the lines $F,\ m\leftrightarrow F,\ m\pm1$ in the $^2P_{\frac{1}{2}}$ state, which are all coincident at weak magnetic field, should be completely resolved. At the field strengths at which observations were actually made, the frequencies of the lines arising in the $^2P_{3/2}$ state are markedly different in their field dependence. The fact that the results are identical, within experimental error, for all lines in the $^2P_{3/2}$ state, indicates that effects due to field inhomogeneity were negligible.

Table II. A summary of all data taken in the present experiment.

Run	$\Delta H'/H'$	Deviation
1	0.00207	+0.00007
2	0.00186	−0.00014
3	0.00209	+0.00009
4	0.00199	−0.00001
5	0.00200	0.0
	Mean 0.00200±0.00006	

The particular apparatus used in this experiment was the same as that used in the measurement of the interaction constants of indium 115, and was described in detail in the preceding paper. However, at the field strengths employed in this experiment, the field was not entirely constant. In order that a measurement of the ratio of the g_J values may be made, transitions resulting in lines whose frequencies are to be compared must occur in the same magnetic field, although no further knowledge of the field is required. It was therefore necessary to correct for variations in the magnetic field. The procedure for making this correction has been discussed at length in previous references.[1,4] In this experiment, the principle limitation on the precision of measurement was imposed by the line widths and not by the stability of the field.

The rarer of the two isotopes of indium (In^{113}), with a relative abundance of about 5 percent, has no effect on the present observations. The lines in the $^2P_{\frac{1}{2}}$ state are observed under very weak field conditions. The frequencies are given almost entirely by $\mu_0 g_F H/h$; the lines are resolved because of a small differential quadratic term containing the factor $1/\Delta\nu$. Since the $\Delta\nu$'s of the two isotopes are equal to within 1 part in 500, the lines of both isotopes are substantially coincident at the magnetic fields in question. The actual shift of the lines due to the isotope effect is very much less than that calculated from the $\Delta\nu$'s because of the low abundance of In^{113}. In the $^2P_{3/2}$ state, the lines of the two isotopes are entirely resolved, and only a fortuitous coincidence of two lines at a given field strength is

TABLE III. Observed ratios of atomic g values and the corresponding values of δ_S obtained in this experiment, and the experiment of Kusch and Foley.

	Experimental ratio	δ_S
I	$\frac{g_J(\mathrm{Ga}\,{}^2P_{3/2})}{g_J(\mathrm{Ga}\,{}^2P_{\frac{1}{2}})}=2(1.00172\pm0.00006)$	0.00114 ± 0.00004
II	$\frac{g_J(\mathrm{Na}\,{}^2S_{\frac{1}{2}})}{g_J(\mathrm{Ga}\,{}^2P_{\frac{1}{2}})}=3(1.00242\pm0.00006)$	0.00121 ± 0.00003
III	$\frac{g_J(\mathrm{Na}\,{}^2S_{\frac{1}{2}})}{g_J(\mathrm{In}\,{}^2P_{\frac{1}{2}})}=3(1.00243\pm0.00010)$	0.00121 ± 0.00005
IV	$\frac{g_J(\mathrm{In}\,{}^2P_{3/2})}{g_J(\mathrm{In}\,{}^2P_{\frac{1}{2}})}=2(1.00200\pm0.00006)$	0.00133 ± 0.00004

possible. Such a coincidence would still introduce an error of magnitude appreciably less than our experimental error.

EXPERIMENTAL RESULTS

The results of one series of observations are shown in Table I in order to indicate the degree of consistency of individual measurements. In the first column of the table the atomic state is given. The second column lists the transitions denoted by the weak field quantum numbers. The third column shows the observed line frequencies. Each of the values in this column is the mean of several observations all reduced to a common value of the magnetic field. In the fourth column are given the values of H' computed from the equations for the line frequencies. The fifth column in Table I lists the deviations of the individual values of H' from the mean. It should be noted that the difference between the values of $H'({}^2P_{3/2})$ and $H'({}^2P_{\frac{1}{2}})$ is approximately thirty-nine times as great as the largest of the mean deviations.

A summary of the results of all the data taken in the present experiment is shown in Table II. Each of the values listed in this table represents a series of observations similar to that given in Table I. The experimental conditions under which these observations were made were purposely varied between successive series. A very large number of lines in the spectrum of the ${}^2P_{\frac{1}{2}}$ state were observable, and the particular lines which were observed were chosen at random. In all cases, however, enough lines were observed, with reduced precision, to make the identification of the lines positive. Six lines were observable in the spectrum of the ${}^2P_{3/2}$ state, and the particular lines chosen for observation were varied between series. In addition, the magnitude of the magnetic field strength was varied between series, and the direction of the magnetic field was changed from time to time. It is therefore to be expected that the errors in the determination of $\Delta H'/H'$ are primarily statistical in character.

DISCUSSION

The values of the ratio $g_J(\mathrm{In}\,{}^2P_{3/2})/g_J(\mathrm{In}\,{}^2P_{\frac{1}{2}})$, and of δ_S, obtained in the present experiment are presented in Table III, which also gives the results of the experiment of Kusch and Foley. It is seen that the last value of δ_S differs from the average of the first three values by more than the sum of the experimental errors.

It appears likely that this discrepancy is the result of effects similar to those mentioned in an earlier section of this paper. It is evident that perturbations of the atomic states in question could in principle bring about deviations from the g_J values computed on the basis of the Russell-Saunders coupling scheme. Such deviations have been found, for example, by Kusch and Taub[5] in the g_J values of rubidium and caesium, which are greater than the g_J of the other alkalis by 5 and 13 parts in 10^5, respectively. It will be noted that these effects are of the magnitude of the discrepancy between the values in Table III. The possible origin of such perturbations has been discussed previously.[1] At the present time, no quantitative theory of the effect is available.

From the agreement of the data in rows II and III of Table III, it may be inferred that perturbations which affect the g_J values of the ${}^2P_{\frac{1}{2}}$ states of indium and gallium are small, and that therefore the g_J values are nearly those characteristic of a pure ${}^2P_{\frac{1}{2}}$ state. It is, consequently, reasonable to assume that it is the ${}^2P_{3/2}$ state of indium which is perturbed sufficiently to account for the discrepancy in Table III. In spite of this discrepancy, however, the present result must be considered as confirming the conclusions of the previous experiment,[1] both as to the existence of the intrinsic magnetic moment of the electron, and as to its approximate magnitude.

[5] P. Kusch and H. Taub, Phys. Rev. 75, 1477 (1949).