

The difference of  $\gamma$  between  $Z=2$  and  $Z=20$  does not seem to be significant. The average value of  $\gamma$  is 1.64. Thus their spectrum is not significantly different from that of the primary protons which seem to constitute the majority of the primary cosmic radiation.

It is obvious that the result reported here is only a first approximation and is subject to revision. For clearly, the photographic plate is a very poor directional instrument and, in addition, there are at present only two measured points on the curve of the latitude effect of the heavy nuclei.

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<sup>1</sup> H. E. Bradt and B. Peters (private communication). I am indebted to the authors for informing me of their results before publication. These figures seem to agree with the results of the University of Minnesota's group reported at the Chicago meeting of the American Physical Society, November, 1949.

<sup>2</sup> Vallarta, Perusquia, and de Oyarzábal, *Phys. Rev.* **71**, 393 (1947).

## Longitudinal Photons in Quantum Electrodynamics

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IN the covariant electrodynamics of Tomonaga and Schwinger<sup>1</sup> considerable complications are introduced whenever the effects of longitudinal and transverse photons are considered separately. Schwinger has pointed out that such a separation is in practice unnecessary, but he has been obliged to start from a treatment with separation in order to demonstrate that a treatment without separation is permissible.<sup>2</sup> Another discussion of the question was given by Hu,<sup>3</sup> but again an elimination of the longitudinal field had to be performed in order finally to show that such an elimination was unnecessary. In the radiation theory of Feynman<sup>4</sup> the longitudinal and transverse photons are never separated; but the argument justifying this procedure<sup>5a</sup> is not immediately applicable to the Schwinger formalism. The present author's formal proof<sup>5</sup> of equivalence of the Schwinger and Feynman theories takes for granted the validity of the unseparated treatment of the radiation field. A simple proof of the correctness of the unseparated treatment of the field, within the framework of the Schwinger theory, is therefore still lacking. The purpose of this letter is to sketch such a proof.

Let  $a_\lambda$ ,  $a_\lambda^*$  be absorption and emission operators for photons, so that

$$A_\mu(x) = \sum_{k,\lambda} (a_\lambda e^{ik \cdot x} + a_\lambda^* e^{-ik \cdot x}) e_{\lambda\mu} g(k), \quad (1)$$

where  $e_\lambda$  is the polarization vector of the photon  $\lambda$ , and  $g(k)$  is a certain function of  $k$ . Corresponding to a single momentum  $k$ , we may choose the  $e_\lambda$ ,  $\lambda=1, 2, 3, 4$ , so that

$$e_\lambda \cdot e_\mu = \delta_{\lambda\mu}, \quad (e_3 + ie_4)_\mu = f(k) k_\mu. \quad (2)$$

Then  $\lambda=1, 2$  are transverse photons,  $\lambda=3, 4$  longitudinal and scalar. This choice of  $e_\lambda$ , together with the reality condition for the  $A_\mu$ , implies that  $a_\lambda^*$  is the Hermitian conjugate of  $a_\lambda$  for  $\lambda=1, 2, 3$ , and of  $(-a_\lambda)$  for  $\lambda=4$ . The vacuum state  $\Psi_0$  satisfies the conditions

$$a_1 \Psi_0 = a_2 \Psi_0 = 0 \quad (\text{definition of vacuum}), \quad (3)$$

$$(a_3 + ia_4) \Psi_0 = 0 \quad (\text{supplementary condition}). \quad (4)$$

Also the  $a_\lambda$  satisfy commutation relations

$$[a_\lambda, a_\mu] = [a_\lambda^*, a_\mu^*] = 0, \quad [a_\lambda, a_\mu^*] = \delta_{\lambda\mu}. \quad (5)$$

From (2)-(5) one deduces the vacuum expectation values of

products of two  $a_\lambda$  as follows. Let a function  $\phi(k)$  be defined by

$$\langle a_3^* a_3 \rangle_0 = (e_3 \cdot k)^2 \phi(k).$$

Then by (2) and (4)

$$\langle a_3^* a_4 \rangle_0 = (e_3 \cdot k)(e_4 \cdot k) \phi(k)$$

and similarly

$$\begin{aligned} \langle a_\lambda^* a_\mu \rangle_0 &= \phi(k)(e_\lambda \cdot k)(e_\mu \cdot k), \quad \lambda, \mu = 1, 2, 3, 4, \\ \langle a_\lambda a_\mu^* \rangle_0 &= \delta_{\lambda\mu} + \phi(k)(e_\lambda \cdot k)(e_\mu \cdot k). \end{aligned} \quad (6)$$

These relations yield

$$\begin{aligned} \langle A_\mu(x) A_\nu(x') + A_\nu(x') A_\mu(x) \rangle_0 \\ = \hbar c \delta_{\mu\nu} D^{(0)}(x-x') + (\partial^2 / \partial x_\mu \partial x_\nu) \Phi(x-x'), \end{aligned} \quad (7)$$

where  $\Phi$  is a function left undetermined by the conditions (2)-(5). Schwinger<sup>6</sup> gives as his prescription that we are to take  $\Phi=0$  in (7). Within the Schwinger theory, this prescription is the only point at which the correctness of the treatment of longitudinal photons comes in question.

In consequence of the gauge-invariance of electrodynamics, and in accordance with the argument of Feynman cited above, (7) will always be used in the evaluation of matrix elements of operators of the form

$$\int K_{\mu\nu}(x, x') A_\mu(x) A_\nu(x') dx dx', \quad (8)$$

where  $K_{\mu\nu}$  is a tensor satisfying the conservation laws

$$\partial K_{\mu\nu} / \partial x_\mu = 0, \quad \partial K_{\mu\nu} / \partial x_\nu = 0. \quad (9)$$

The second term on the right of (7) contributes zero to all matrix elements of (8). Therefore all results of the theory are independent of  $\Phi$ , and will be given correctly by taking  $\Phi=0$  in (7). This fact justifies the unseparated treatment of the transverse and longitudinal fields.

<sup>1</sup> S. Tomonaga *et al.*, *J. Phys. Soc. Japan* **2**, 172, 199 (1947); *Prog. Theor. Phys.* **2**, 198 (1947). J. Schwinger, *Phys. Rev.* **74**, 1439 (1948); **75**, 651 (1949); **76**, 790 (1949).

<sup>2</sup> See the discussion leading to Eq. (3.40), J. Schwinger, *Phys. Rev.* **75**, 668 (1949).

<sup>3</sup> Ning Hu, *Phys. Rev.* **76**, 391 (1949).

<sup>4</sup> R. P. Feynman, *Phys. Rev.* **74**, 1430 (1948); **76**, 769 (1949).

<sup>5a</sup> R. P. Feynman, *Phys. Rev.* **76**, 780 (1949), Section 8.

<sup>5</sup> F. J. Dyson, *Phys. Rev.* **75**, 486 (1949).

<sup>6</sup> J. Schwinger, *Phys. Rev.* **75**, 668 (1949), Eqs. (3.40) and (3.41).

## The Microwave Spectrum of Bromine Monofluoride\*

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SIX lines have been observed in the region 20.9 to 21.5 thousand megacycles per second when  $\text{BrF}_3$  or  $\text{BrF}_5$  was admitted to the absorption cell. These six lines become considerably more intense when  $\text{F}_2$  and  $\text{Br}_2$  are admitted to the cell at approximately equal pressures, and six additional weak lines become observable. The line frequencies and the Stark shifts can be readily interpreted as the  $J=0 \leftrightarrow 1$  transition for  $\text{BrF}$ . The existence of this spectrum constitutes the first direct evidence for the chemically stable existence of this particular diatomic halide.<sup>1</sup> The earlier observations of the emission spectrum<sup>2</sup> in addition to verifying the physical stability of  $\text{BrF}$ , suggest that this compound should be chemically stable.

The line frequencies, with their assignments, are given in Table I. In Table II, the constants derived from the spectrum assuming a  $\text{BrF}$  model are presented. The large values of the nuclear quadrupole coupling constant and of the dipole moment to be expected