The difference of $\gamma$ between $Z=2$ and $Z=20$ does not seem to be significant. The average value of $\gamma$ is 1.64. Thus their spectrum is not significantly different from that of the primary protons which seem to constitute the majority of the primary cosmic radiation.

It is obvious that the result reported here is only a first approximation and is subject to revision. For clearly, the photographic plate is a very poor directional instrument and, in addition, there are at present only two measured points on the curve of the latitude effect of the heavy nuclei.

It is a pleasure to thank Professor J. R. Oppenheimer for his kind hospitality at the Institute for Advanced Study.

[1] H. E. Bradt and B. Peters (private communication). I am indebted to the authors for informing me of their results before publication. These figures seem to agree with the results of the University of Minnesota's group reported at the Chicago meeting of the American Physical Society, November, 1949.

[2] Vallarta, Perusquía, and de Oyarzábal, Phys. Rev. 71, 393 (1947).

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# Longitudinal Photons in Quantum Electrodynamics

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December 12, 1949

IN the covariant electrodynamics of Tomonaga and Schwinger[1] considerable complications are introduced whenever the effects of longitudinal and transverse photons are considered separately. Schwinger has pointed out that such a separation is in practice unnecessary, but he has been obliged to start from a treatment with separation in order to demonstrate that a treatment without separation is permissible.[2] Another discussion of the question was given by Hu,[3] but again an elimination of the longitudinal field had to be performed in order finally to show that such an elimination was unnecessary. In the radiation theory of Feynman[4] the longitudinal and transverse photons are never separated; but the argument justifying this procedure[4a] is not immediately applicable to the Schwinger formalism. The present author's formal proof[5] of equivalence of the Schwinger and Feynman theories takes for granted the validity of the unseparated treatment of the radiation field. A simple proof of the correctness of the unseparated treatment of the field, within the framework of the Schwinger theory, is therefore still lacking. The purpose of this letter is to sketch such a proof.

Let $a_\lambda$, $a_\lambda^*$ be absorption and emission operators for photons, so that

$$A_\mu(x)=\sum_{k,\lambda}(a_\lambda e^{ik\cdot x}+a_\lambda^* e^{-ik\cdot x})e_{\lambda\mu}g(k), \tag{1}$$

where $e_\lambda$ is the polarization vector of the photon $\lambda$, and $g(k)$ is a certain function of $k$. Corresponding to a single momentum $k$, we may choose the $e_\lambda$, $\lambda=1, 2, 3, 4$, so that

$$e_\lambda\cdot e_\mu=\delta_{\lambda\mu},\quad (e_3+ie_4)_\mu=f(k)k_\mu. \tag{2}$$

Then $\lambda=1, 2$ are transverse photons, $\lambda=3, 4$ longitudinal and scalar. This choice of $e_\lambda$, together with the reality condition for the $A_\mu$, implies that $a_\lambda^*$ is the Hermitian conjugate of $a_\lambda$ for $\lambda=1, 2, 3$, and of $(-a_\lambda)$ for $\lambda=4$. The vacuum state $\Psi_0$ satisfies the conditions

$$a_1\Psi_0=a_2\Psi_0=0\quad\text{(definition of vacuum)}, \tag{3}$$

$$(a_3+ia_4)\Psi_0=0\quad\text{(supplementary condition)}. \tag{4}$$

Also the $a_\lambda$ satisfy commutation relations

$$[a_\lambda, a_\mu]=[a_\lambda^*, a_\mu^*]=0,\quad [a_\lambda, a_\mu^*]=\delta_{\lambda\mu}. \tag{5}$$

From (2)–(5) one deduces the vacuum expectation values of products of two $a_\lambda$ as follows. Let a function $\phi(k)$ be defined by

$$\langle a_3^*\ a_3\rangle_0=(e_3\cdot k)^2\phi(k).$$

Then by (2) and (4)

$$\langle a_3^*\ a_4\rangle_0=(e_3\cdot k)(e_4\cdot k)\phi(k)$$

and similarly

$$\begin{aligned}\langle a_\lambda^*\ \ a_\mu\rangle_0&=\phi(k)(e_\lambda\cdot k)(e_\mu\cdot k),\quad \lambda, \mu=1, 2, 3, 4,\\ \langle a_\lambda\ \ a_\mu^*\rangle_0&=\delta_{\lambda\mu}+\phi(k)(e_\lambda\cdot k)(e_\mu\cdot k).\end{aligned} \tag{6}$$

These relations yield

$$\begin{aligned}\langle A_\mu(x)A_\nu(x')+A_\nu(x')A_\mu(x)\rangle_0\\ =\hbar c\delta_{\mu\nu}D^{(1)}(x-x')+(\partial^2/\partial x_\mu\partial x_\nu)\Phi(x-x'),\end{aligned} \tag{7}$$

where $\Phi$ is a function left undetermined by the conditions (2)–(5). Schwinger[6] gives as his prescription that we are to take $\Phi=0$ in (7). Within the Schwinger theory, this prescription is the only point at which the correctness of the treatment of longitudinal photons comes in question.

In consequence of the gauge-invariance of electrodynamics, and in accordance with the argument of Feynman cited above, (7) will always be used in the evaluation of matrix elements of operators of the form

$$\int K_{\mu\nu}(x, x')A_\mu(x)A_\nu(x')dxdx', \tag{8}$$

where $K_{\mu\nu}$ is a tensor satisfying the conservation laws

$$\partial K_{\mu\nu}/\partial x_\mu=0,\quad \partial K_{\mu\nu}/\partial x_\nu'=0. \tag{9}$$

The second term on the right of (7) contributes zero to all matrix elements of (8). Therefore all results of the theory are independent of $\Phi$, and will be given correctly by taking $\Phi=0$ in (7). This fact justifies the unseparated treatment of the transverse and longitudinal fields.

[1] S. Tomonaga *et al.*, J. Phys. Soc. Japan **2**, 172, 199 (1947); Prog. Theor. Phys. **2**, 198 (1947). J. Schwinger, Phys. Rev. **74**, 1439 (1948); **75**, 651 (1949); **76**, 790 (1949).

[2] See the discussion leading to Eq. (3.40), J. Schwinger, Phys. Rev. **75**, 668 (1949).

[3] Ning Hu, Phys. Rev. **76**, 391 (1949).

[4] R. P. Feynman, Phys. Rev. **74**, 1430 (1948); **76**, 769 (1949).

[4a] R. P. Feynman, Phys. Rev. **76**, 780 (1949), Section 8.

[5] F. J. Dyson, Phys. Rev. **75**, 486 (1949).

[6] J. Schwinger, Phys. Rev. **75**, 668 (1949), Eqs. (3.40) and (3.41).

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# The Microwave Spectrum of Bromine Monofluoride*

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December 21, 1949

SIX lines have been observed in the region 20.9 to 21.5 thousand megacycles per second when $BrF_3$ or $BrF_5$ was admitted to the absorption cell. These six lines become considerably more intense when $F_2$ and $Br_2$ are admitted to the cell at approximately equal pressures, and six additional weak lines become observable. The line frequencies and the Stark shifts can be readily interpreted as the $J=0\leftrightarrow1$ transition for BrF. The existence of this spectrum constitutes the first direct evidence for the chemically stable existence of this particular diatomic halide.[1] The earlier observations of the emission spectrum[2] in addition to verifying the physical stability of BrF, suggest that this compound should be chemically stable.

The line frequencies, with their assignments, are given in Table I. In Table II, the constants derived from the spectrum assuming a BrF model are presented. The large values of the nuclear quadrupole coupling constant and of the dipole moment to be expected

TABLE I. Observed transitions and frequencies $J=0\leftrightarrow 1$.

| $v=0$ | $\mathrm{Br}^{79}\mathrm{F}$ | $\mathrm{Br}^{81}\mathrm{F}$ |
|---|---|---|
| F 3/2–1/2 | 20,985.5 mc/sec. | 20,928.4 mc/sec. |
| 3/2–5/2 | 21,202.6 | 21,110.4 |
| 3/2–3/2 | 21,475.4 | 21,337.5 |
| $v=1$ | | |
| F 3/2–1/2 | 20,828.9 mc/sec. | 20,772.3 mc/sec. |
| 3/2–5/2 | 21,045.6 | 20,954.6 |
| 3/2–3/2 | 21,319.4 | 21,181.7 |

TABLE II. Molecular constants of BrF.

| | $\mathrm{Br}^{79}\mathrm{F}$ | $\mathrm{Br}^{81}\mathrm{F}$ |
|---|---|---|
| $B_e$ | $10,706.9_5$ mc/sec. | 10,655.7 |
| $I_e$ | $78.355(10)^{-40}$ g cm$^2$ | $78.658(10)^{-40}$ |
| $\alpha$ | 156.3 mc/sec. | 155.8 mc/sec. |
| $r_e$ | $1.759(10)^{-8}$ cm | |
| $\mu$ | 1.29 debye units | |
| $eqQ$ | +1089.0 mc/sec. | +909.2 |

in this dihalide are to be noted. The most convincing evidence that this spectrum is due to BrF is obtained when the ratio of $B_e$ values derived from the spectrum 0.995213, is compared with that computed from the mass measurements of the bromine isotopes[3] 0.995207.

While the fit of the triplets to the conventional quadrupole coupling expression, including second-order corrections, was not entirely satisfactory, the precision of the frequency measurements did not appear to justify the evaluation of the magnetic coupling factor, as was done for the dihalide FCl by Gilbert, Roberts, and Griswold.[4]

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* This document is based on work performed under contract for the AEC by Carbide and Carbon Chemicals Corporation, at Oak Ridge, Tennessee.

[1] H. S. Booth and J. T. Pinkston, Chem. Rev. **41**, 421 (1947).

[2] VanPeter Heinz Broderson and Hans Joachim Schumacher, Zeits. f. Naturforschg. **2a**, 358 (1947).

[3] K. Ogata, Phys. Rev. **75**, 200 (1949).

[4] Gilbert, Roberts, and Griswold, Phys. Rev. **76**, 1723 (1949).

## Intensity Distribution of the Second Positive ($C^3\Pi\rightarrow B^3\Pi$) Band System of Molecular Nitrogen

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December 12, 1949

RECENT interest shown in the intensity distributions of the vibrational band-systems of molecular nitrogen radiated from auroral displays, and from the night sky[1] prompts this brief account of some calculations of the intensity distributions of the second positive band system, made in connection with an experimental investigation being undertaken here of the kinetics of excitation of the energy levels of molecular nitrogen in gaseous electrical discharge.

The integrated intensity of a molecular band may be written:

$$I(v',v'')=CN(v')E^4(v',v'')p(v',v''), \tag{1}$$

where $N(v')$ is the population of molecules in the $v'$ vibrational level of the upper electronic state concerned in the transition, $E(v',v'')$ is the energy radiated in the transition between $v'$ and $v''$, $v''$ is the vibrational level in the lower electronic state upon which the transition ends, and

$$p(v',v'')=\left|\int_{-\infty}^{\infty}\psi(v')M\psi(v'')dx\right|^2,$$

where $\psi(v')$ and $\psi(v'')$ are the respective wave functions of the levels $v'$ and $v''$, $x$ is the internuclear separation, and $M$ is the dipole moment of the molecule. $p$ is a fractional transition probability as $\Sigma_{v''}\,p(v',v'')=1$ for each $v'$. $C$ is a constant of proportionality.

Following a method suggested and used first by Pearse and Gaydon,[2] and later used by Miss Pillow,[3] we have calculated the values of $p(v',v'')$ by numerical integration of the wave-function product, for all the observed bands of the second positive system of nitrogen under the same conditions of simplifying approximation as are used in references 2 and 3. Briefly, these are (a) treating $M$ as constant, and (b) linearly distorting the Hermite polynomial forms of $\psi(v')$ and $\psi(v'')$ expected for a harmonic oscillator, to fit the empirical, yet experimentally justified Morse curves for the $C^3\Pi$ and $B^3\Pi$ states of nitrogen.

These values of $p(v',v'')$ are shown in Table I. Table II shows the values of $E^4(v',v'')p(v',v'')$ adjusted to a value of 100 at the (0, 0) band. It will be evident from Eq. (1) that the values of Table II, when compared with measured intensities will yield information relating to the population $N(v')$ of the upper level concerned in the transition, and therefore to the probable major mechanism responsible for the excitation of the level.

TABLE I. $p(v',v'')$.

| $v' \backslash v''$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.53 | 0.27 | 0.12 | 0.05 | 0.02 | 0.01 | | | | | |
| 1 | 0.43 | 0.04 | 0.21 | 0.12 | 0.08 | 0.07 | 0.05 | 0.00 | | | |
| 2 | 0.21 | 0.39 | 0.01 | 0.10 | 0.08 | 0.07 | 0.08 | 0.05 | $0.00_3$ | | |
| 3 | 0.01 | 0.37 | 0.26 | 0.02 | 0.03 | 0.06 | 0.07 | 0.10 | 0.08 | 0.00 | |
| 4 | — | 0.04 | 0.45 | 0.16 | 0.08 | 0.01 | 0.04 | 0.04 | 0.08 | 0.09 | 0.01 |

TABLE II. $p(v',v'')E^4(v',v'')$.

| $v' \backslash v''$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 100 | 40 | 14 | 4.8 | 1.3 | 0.39 | | | | | |
| 1 | 110 | 8.9 | 32 | 14 | 7.7 | 5.3 | 2.7 | 0.01 | | | |
| 2 | 67 | 99 | 1.6 | 17 | 11 | 7.2 | 6.2 | 2.9 | 0.15 | | |
| 3 | 3.1 | 120 | 67 | 4.5 | 5.9 | 7.4 | 7.6 | 8.4 | 5.1 | $0.00_5$ | |
| 4 | — | 15 | 150 | 43 | 17 | 1.0 | 5.6 | 4.6 | 6.8 | 6.0 | 0.64 |

Similar calculations are being made for the other molecular nitrogen band systems and for the NO$\beta$- and $\gamma$-systems in connection with the experimental work referred to above. The possible geometry of a three-dimensional representation of the Tables I and II is also being worked upon.

While these calculations were being made an interesting paper by Bates[1] appeared, in which some similar work was reported. However, as data for far fewer bands of the second positive system were reported there, it is thought worth while to present the above calculations.

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[1] *The Emission Spectra of The Night Sky and Aurorae* (The Physical Society, London, 1948); D. R. Bates, Proc. Roy. Soc. **A196**, 217 (1949).

[2] R. W. B. Pearse and A. G. Gaydon, Proc. Roy. Soc. **A173**, 37 (1939).

[3] M. E. Pillow, Proc. Phys. Soc. London **62**, 237 (1949).

## New Types of Microwave Transitions Involving $l$-Type Doubling in OCS and HCN*

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December 12, 1949

MICROWAVE measurements of the vibration-rotation interaction called $l$-type doubling are available for a number of linear $XYZ$ molecules.[1] Two new types of transitions involving this interaction have been observed which confirm the expected