Nevertheless Roberts was fully justified in obtaining this as a logical deduction from the only developed theory then available for his purpose.

The derivation in detail of the relativistic equation given above will be published later.

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## Calculation of Decay Times from Coincidence Experiments

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USING delayed coincidence circuits for the measurement of short radioactive half-lives one has to measure the genuine coincidence rate $N_g$ as a function of an artificial delay time $T$ deliberately put in either channel. The mean life $\Theta$ has to be calculated from the delay curve $N_g(T)$ of the experiment.

Such calculations[1,2] are generally based on some assumptions or knowledge concerning the time delays in the equipment and concerning the resolving time $\tau$. The result of the calculations is a theoretical delay curve, which can be compared with the measured delay curve.

It is easy to show that $\Theta$ can be calculated in a simple way without knowing the time delays within the equipment, without knowing $\tau$, and without using any theoretical delay curves.

Our method is based on the calculation of the moments of delay curves. It will be proved in another paper[3] that all the moments of the delay curve $N_g(T)$ can be calculated from the moments of two curves:

1. A delay curve $\nu_g(T)$ which is measured under the condition that the corresponding event (which give counts in the individual channels) have no time delay between them (e.g., $\gamma$–$\gamma$-coincidences with negligible time delay or a single particle which traverses both counters). $\nu_g(T)$ is thus the characteristic delay curve of the equipment.

2. The curve giving the probability $p(t)dt$ of the time delay between $t$ and $t+dt$ of the two events in the actual decay-experiment (e.g., in a decay experiment, where the parent gives counts in one channel and the daughter with the mean life $\Theta$ gives counts in the other channel, $p(t)=(1/\Theta)\exp(-t/\Theta)$.

Measuring $N_g(T)$ and $\nu_g(T)$ we normalize both to the same area, i.e.,

$$\int_{-\infty}^{+\infty} N_g(T)dT=\int_{-\infty}^{+\infty}\nu_g(T)dT=I. \tag{1}$$

Then,

$$\begin{aligned}M^{(n)}[N_g(T)]=&M^{(n)}[\nu_g(T)]M^{(0)}[p(t)]\\&+\binom{n}{1}M^{(n-1)}[\nu_g(T)]M^{(1)}[p(t)]+\cdots\\&+\binom{n}{n-1}M^{(1)}[\nu_g(T)]M^{(n-1)}[p(t)]\\&+M^{(0)}[\nu_g(T)]M^{(n)}[p(t)],\end{aligned} \tag{2}$$

where we use the well-known definition:

$$M^{(n)}[f(x)]=\int_{-\infty}^{+\infty}x^n f(x)dx. \tag{3}$$

Equations (2) are valid under the most general circumstances,[3] e.g., when equal or unequal time delays, due to different decay times, occur in both counting channels. In such cases $p(t)$ is composed of the different decay functions.

The use of (2) permits the determination of all the moments of $p(t)$ and hence $p(t)$ is completely determined.

To show the use of (2) in the simple case given above (parent gives counts in one channel, daughter with a mean life $\Theta$ in the other), we take the first moment:

$$M^{(1)}[N_g(T)]=M^{(1)}[\nu_g(T)]+I\Theta.$$

Thus:

$$\Theta=\frac{M^{(1)}[N_g(T)]}{I}-\frac{M^{(1)}[\nu_g(T)]}{I}, \tag{4}$$

which states that the "center of gravity" of the delay curve in a decay experiment will be displaced by the mean life of the daughter substance. If the delay curve of the equipment in symmetric around $T=0$, then $\Theta$ is simply the "center of gravity" of the delay curve of the decay experiment. It is obvious that $\Theta$ can be determined accurately in this way from measurable quantities, even if the $N_g(T)$ and $\nu_g(T)$ curves overlap appreciably, in which case one usually says that $\Theta$ is smaller than the resolving time $\tau$ of the equipment.

When the counts from the same daughter substance appear in both channels (e.g., $\gamma$–$\gamma$-experiments), we get symmetric delay curves and use the second moment of the delay curves for the determination of $\Theta$. In experiments, where two or more decay processes occur, we take more moments for the calculation of the decay constants.

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## On the Energy Spectrum of Heavy Nuclei in Primary Cosmic Radiation

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BRADT and Peters[1] have obtained recently the intensity of heavy nuclei in primary cosmic radiation reaching the earth at 55° and 30° north geomagnetic latitudes. According to their figures, the ratio $I/I'$ of the intensities at these two latitudes is 4 for helium, 3.4 for carbon, nitrogen, and oxygen, and 3.5 for nuclei of atomic number greater than 10. The difference between these ratios is not significant and lies within the experimental error.

If one assumes that most of the heavy nuclei in the experiment of Bradt and Peters reach the point of observation at an angle not greater than 30° from the zenith, one can attempt to make an estimate of the energy spectrum of the heavy nuclei, assumed to be of the form $K/E^\gamma$, using for this purpose the geomagnetic latitude effect for particles arriving essentially in the vertical direction.

The principle of this analysis has already been outlined.[2] To begin with, the geomagnetic effects depend only on the value of the energy of the particle measured in Störmers. Therefore, the energy in Störmers is first converted into energy in more usual units, say in Bev, for different values of $Z$ from $Z=2$ to $Z=30$. Knowing these, one can plot curves giving the geomagnetic cut-off at the two latitudes as a function of the atomic number $Z$. The results of our calculation are contained in Table I.

TABLE I. Cut-off energy.

| $Z$ | 2 | 6 | 8 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|
| Cut-off at 55° (Bev) | 1.4 | 4.2 | 5.6 | 7.0 | 10.5 | 14.0 |
| Cut-off at 30° (Bev) | 14 | 42 | 56 | 70 | 105 | 140 |
| $\gamma$ | 1.60 | 1.53 | 1.53 | 1.53 | 1.56 | 1.56 |