of those particles for which ϕ is real in $U(\phi)=H$ will, for a sinusoidal field at least, approach asymptotically a linear function of distance, as may be seen in the following:

$$E^2\gg 1:\quad E-(E^2-1)^{\frac{1}{2}}\rightarrow 1/2E.$$

Thus, for high energy,

$$\begin{pmatrix}\beta_w=1\\E^2\gg 1\end{pmatrix}:\quad \frac{dE}{dx}\rightarrow k[\epsilon^2-\{(1/2E)-H\}^2]^{\frac{1}{2}},$$

$$\begin{bmatrix}\beta_w=1\\E^2\gg 1\\E\gg 1/2H\end{bmatrix}:\quad \frac{dE}{dx}\rightarrow k[\epsilon^2-H^2]^{\frac{1}{2}}=\text{constant}.$$

A more general solution of the dynamical motion, which is equivalent to Eq. (6) for a sinusoidal field, is that obtained by finding $\phi(x)$. This solution is:

$$\left.\begin{aligned}\beta_w\neq 1:\quad &(1-\beta_w^2)^{-1}\int_{\phi_0}^{\phi(x)}d\phi\left\{\frac{\pm\beta_w\chi(\phi)}{[\chi^2(\phi)-(1-\beta_w^2)]^{\frac{1}{2}}}-1\right\}=kx,\\ \beta_w=1:\quad &\int_{\phi_0}^{\phi(x)}\frac{d\phi}{\chi^2(\phi)}-[\phi(x)-\phi_0]=2kx,\end{aligned}\right\}\quad (7)$$

where the $\pm$ sign depends on which branch of the $E(\phi)$ curve in Eq. (4) the particle is orbiting. Here there are no restrictions on the shape function $f(\phi)$ of the field, from which $\chi(\phi)$ is obtained. When Eq. (7) is solved for $\phi(x)$, we can then use Eq. (3) to find $U[\phi(x)]$ and finally Eq. (2) to get $E(x)$. The expressions in Eq. (7) might be called the equations of bunching.

It is a general result of the foregoing solutions that the energy spectrum of a beam of particles from a traveling wave linear accelerator is a strong function of the initial phase spread of the beam so that pre-bunching is a necessity for any appreciable final energy homogeneity of the beam.

Detailed considerations of all the above questions will be taken up in a forthcoming paper.

* This work has been supported in part by the Air Materiel Command, the Army Signal Corps, and the ONR.

[1] See J. Schwinger, Phys. Rev. **75**, 1912 (1949).
[2] J. C. Slater, Rev. Mod. Phys. **20**, 505, 508 (1948).

Note on Nuclear Models

HOFF LU
University of Chekiang, Hangchow, China
November 21, 1949

NUCLEAR models in which the protons are concentrated on or near, or uniformly distributed over, the surface of a sphere have been recently proposed by several investigators.[1-3] For the same value of Coulomb repulsion energy, a given nucleus represented by such a neutron-core model will be smaller than that by the usually assumed model in which the protons are uniformly distributed over the volume of a sphere. In other words, the new model will yield a smaller value of r_0 in the formula $r=r_0A^{\frac{1}{3}}$ for nuclear radii than the customary model, when it is computed either from the semi-empirical binding-energy equation[4] or from a comparison of the binding energies of contiguous isobars.[5]

Assume the protonic charge be uniformly distributed over the surface. Then, the constant coefficient of the Coulomb energy term in the semi-empirical equation[4] is $a_3=\frac{1}{2}(e^2/r_0)$ instead of $a_3\approx(3/5)(e^2/r_0)$ so that with the value of a_3 given in Lapp and Andrews' book, we find, for this neutron-core model,

$$r_0=1.23_+\times 10^{-13}\text{ cm},$$

and, for the customary model, $r_0=1.48\times 10^{-13}$ cm. Also, the difference in Coulomb energy of two contiguous isobars due to the additional repulsion of the extra proton in the Coulomb field of the isobar with the lower Z is now $C=(Z+\frac{1}{2})e^2/r$ instead of $C=(6/5)Ze^2/r$ so that, with Bethe's value of C for $_7N^{13}-{}_6C^{13}$, we have, for this neutron-core model,

$$r_0=1.34_-\times 10^{-13}\text{ cm},$$

instead of 1.47×10^{-13} cm obtained by Bethe for the customary model. The two results from the new model do not appear to agree as well as those from the customary model. This, however, is not entirely unexpected, since for a nucleus as light as C^{13} our new model with charge spread over a surface layer cannot be anything but a very crude model, and we should expect that, for the isobar N^{13} having the extra proton, a model with a proton on the surface of the C^{13} model should be closer to the true situation. Thus, by considering the charge of the extra proton to remain unspread, we have $C=Ze^2/r$, giving $r_0=1.23_-\times 10^{-13}$ cm, which brings the two results in agreement again.

Comparing the above results with the value $r_0\approx 1.5\times 10^{-13}$ cm for most α-emitters as given by Gamow's theory of α-decay, one may, at first sight, stick to the customary model. However, on our proton-shell model, one should expect that the radius, R, of a nucleus of mass number A as defined in Gamow's theory should be equal to our model radius, r, for a nucleus of mass number $(A-4)$ plus an effective radius, r_α, of α-particle. Since r_α cannot be well defined, and since our formula

$$r=1.23\times 10^{-13}A^{\frac{1}{3}}\text{ cm}\qquad (1)$$

is not expected to hold for very light nuclei, we may calculate r_α from the values of R and r. A more accurate calculation of the nuclear radii of α-emitters have been recently made by Preston.[6] His values of R give r_0 that range from 1.28 (ThC–ThC″) to 1.62×10^{-13} cm (Th$-$MsTh$_1$) with a great majority of cases having r_0 around 1.5×10^{-13} cm. Using these values of R, we find r_α to range from 0.28 to 2.4×10^{-13} cm with a great majority of cases within 1.7 to 1.8×10^{-13} cm. These latter values are somewhat less than 1.96×10^{-13}, the value given by (1). This is reasonable since $_2He^4$ is very tightly bound as is well known (it is the only element hitherto unaffected by neutron bombardment).

Recently, Fernbach *et al.*,[7] using data of scattering of fast neutrons by nuclei from Li to U, have shown that $r_0=1.37\times 10^{-13}$ cm. Weisskopf and Ewing[8] have also obtained the best fit of data on the anomalous scattering of protons by medium weight nuclei by assuming $r_0=1.3\times 10^{-13}$ cm. These values are in better agreement with ours than what the customary model gives.

Therefore, it appears likely that the proton-shell neutron-core model is closer to the true nuclear structure than the customary model.

[1] J. G. Winans, Phys. Rev. **71**, 379 (1947); **72**, 435 (1947).
[2] B. Davison and W. H. Watson, Phys. Rev. **71**, 742 (1947).
[3] I. N. Sneddon and B. F. Tonschek, Proc. Camb. Phil. Soc. **44**, 402 (1948).
[4] R. E. Lapp and H. L. Andrews, *Nuclear Radiation Physics* (Prentice-Hall, Inc., 1948) p. 146, Eqs. (7) and (8).
[5] H. A. Bethe, Phys. Rev. **54**, 436 (1938).
[6] M. A. Preston, Phys. Rev. **69**, 535 (1946).
[7] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1354 (1949).
[8] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).

Measurement of Some Weak γ-Ray Intensities

G. R. BISHOP, RICHARD WILSON, AND H. HALBAN
Clarendon Laboratory, Oxford, England
December 19, 1949

WE have measured the intensities of some weak high energy γ-rays by counting the photo-protons produced by the disintegration of deuterium in an ionization chamber counter.[1] The 2.62 Mev γ-ray from Rd Th was used for energy calibration. The intensities have been measured relative to other γ-ray lines