TABLE I. Implied excitation energy of Be^7.

Run	E_p	E_n	R	E_n'	E
1	3.02	1.25	0.63	0.78_5	0.45
2	3.02	1.25	0.63	0.78_5	0.45
3	3.12	1.35	0.57	0.76	0.54
4	3.12	1.35	0.64	0.87	0.45
5	3.38	1.6	0.70	1.13	0.44

(Three of these curves are shown in reference 1.) E_p is the energy in Mev of the Van de Graaff protons, E_n the energy of the main neutron group, R the ratio of recoil energy at the anomalous peak to maximum recoil energy, E_n' the energy of the postulated slower neutron group, and E the excitation energy of Be^7 which would cause such a group.

Table I shows that only one run was inconsistent with the excited state hypothesis, the location of the distorted peak was in fact exceptionally vague in this one case. While the anomalous peak cannot be localized sufficiently to give a precise measure of the excitation energy, the other four runs put it between 420 and 480 kev.

Recent precision measurements indicate that such a Be^7 state exists.[2,3] However, the excitation of this state by 3 Mev protons striking lithium has escaped previous observation.[4] The reason perhaps is that the Li(p,n) yield curve, which was studied by Freier, Lampi, and Williams, does not offer as sensitive a test as the helium recoil data, in which the slower neutrons at optimum energies produce resonant forward-scattered recoils which are superposed, in the distribution curves, upon non-resonant faster neutron recoils of unfavorable scattering angle. If the implications drawn from this helium data are correct, the traces of the slower neutron group should be observable in many neutron resonance studies, especially by comparing resonance data taken with both Li(p,n) and D(d,n) neutron sources.

As to helium scattering itself, in reference 1 the smallness of the ratio of maximum to minimum differential scattering cross sections in all of the curves was taken to indicate a split resonance level, but this apparently small ratio may also be due really to a slower neutron group.

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Fast Protons from the Absorption of π^--Mesons by Nuclei*

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BY use of a very simple model for the production of stars resulting from the absorption of π^--mesons by nuclei it is possible to estimate (a) the number of "fast" protons (energy >30 Mev) in the stars, and (b) the average excitation energy. We assume that the meson is absorbed by a single proton in the nucleus producing a neutron which moves at high speed through a "gas" of nucleons, while momentum is conserved by the recoil of one or more neighboring nucleons. The fast nucleons may escape from the nucleus without collision or may undergo one or more collisions, thereby heating up the nucleus. Thus star fragments from π^--absorptions fall into two categories: high energy nucleons arising directly from the absorption process, and evaporation fragments whose energies are low and determined by the energy loss of the initial nucleon in traversing the nucleus. If the mean free path of the fast nucleons is of the order of or larger than the nuclear diameter, we would expect large numbers of fast nucleons as well as total excitation energies of the evaporation stars low compared to the meson rest mass of 146 Mev.

If, in addition to the collision cross section, the average energy loss per collision is known, one can estimate the probability that a nucleon makes a specified number of collisions before leaving the nucleus and the excitation energy of the residual nucleus. The energies involved are sufficiently high that, to a first approximation, the binding of the nucleons can be ignored. The total $n-p$ scattering $n-p$ scattering cross section is taken as $6.8/E$ barns where E is measured in Mev,[1] and we assume $\sigma_{nm}=\sigma_{p-p}=\frac{1}{4}\sigma_{n-p}$.[2] Since the nucleus is treated as a Fermi gas, the exclusion principle discriminates against collisions with small momentum transfer, thereby increasing the effective cross section by a factor of about 1.45.[3] The energy loss per collision has been estimated by Serber[4] as 25 Mev for energies of the order of 100 Mev. The calculation is greatly simplified by assuming forward scattering. The error involved is difficult to estimate, but is surely not very large and probably leads to a slight overestimate for the number of fast protons.

Two different models were used for the calculations.[5] I. The recoil momentum is taken up by a single nucleon so that the absorption results in two nucleons moving in opposite directions, each with half the meson rest energy. The recoil particle can be a neutron or a proton, and on the basis of an α-particle model the ratio of neutrons to protons is 2:1. Use of the ratio obtained by counting all neutrons and protons in the nucleus gives almost identical results. II. The recoil is a triton, the residual part of the α-particle of which the absorbing proton is taken to be a member. Using a triton binding energy of 8 Mev,[6] we find that the neutron carries away 95 Mev while the recoil triton has 31 Mev. The entire energy of the trition goes into heating up the nucleus.

With model I one calculates the probabilities that both nucleons make zero collisions, one makes one and the other zero, etc. After more than one collision the nucleon energy is degenerated below 30 Mev and is considered to be "lost" in the evaporation star. With model II the single nucleon is "lost" after more than two collisions. In this way one obtains an estimate of the number of absorptions which produce no stars (by star we mean evaporation star) and the number which yield fast protons. In all these calculations it is assumed that a charge exchange occurs in half the neutron-proton collisions.

The calculations were carried out for π^--absorptions in nitrogen and in silver so as to make possible a comparison with the observations in nuclear emulsions. The important results are given separately in Table I for nitrogen (taken as typical of the C, N, O group) and for silver (representing Ag, Br) since it should be possible experimentally to distinguish between the π^--stars produced in the light and heavy elements of the emulsion.

Evaporation stars of energy less than 40 Mev are classified separately since an excitation of that magnitude will produce stars consisting almost entirely of neutrons in nuclei as heavy as silver[7] and are therefore not observed. It is worth noting that model II

TABLE I.

	Model	Nitrogen	Silver
Number of fast protons (E>30 Mev) per 100 π^--mesons absorbed	I II	48 12	24 13
Average excitation energy of evaporation star	I II	31 Mev 55 Mev	78 Mev 70 Mev
Number of π^--absorptions giving no evaporation star (per 100)	I II	28 0	8 0
Number of evaporation stars with excitation <40 Mev per 100 π^--absorptions	I II	31 64†	12 43†

† The escaping fast nucleon is always a neutron.

predicts a large number of π^--absorptions in silver which produce no observable prongs (except possibly recoil nuclei) while model I predicts very few. Both models lead to a substantial number of fast protons too energetic to be accounted for by an evaporation process.

Meson absorptions giving rise to fast protons have been observed in photographic plates by Perkins[8] and by Cheston and Goldfarb[9] at Rochester. The data appear to fit reasonably well with our model II but the results are still inconclusive.

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Upper and Lower Bounds of Eigenvalues

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The Ritz variational method gives only upper bounds of eigenvalues of Hermitian operators. Many authors[1] attempted to generalize the method and derived various formulas to estimate lower bounds of eigenvalues. But it seems that as yet the mutual relations and relative merits of these formulas have not been fully discussed from a systematic point of view.

We carefully examined these formulas and reached the conclusion that the formula of Temple[1] is the most precise one among them notwithstanding that it is the oldest as well as the simplest one.

Moreover, we could generalize the Temple formula to the case of higher eigenvalues of operators which are not necessarily bounded below. Let λ be a non-degenerate eigenvalue of a Hermitian operator H and let $\alpha<\beta$ be two numbers such that the interval (α, β) contains λ but no other points of the spectrum of H. Let w be an approximate eigenfunction and calculate

$$\eta=(w, Hw)/\|w\|^2, \quad \epsilon=\|(H-\eta)\|/\|w\|.$$

Then we can show that

$$\eta-\frac{\epsilon^2}{\beta-\eta}\leqq\lambda\leqq\eta+\frac{\epsilon^2}{\eta-\alpha}, \tag{1}$$

provided $\epsilon^2<(\eta-\alpha)(\beta-\eta)$.

This formula is symmetric with respect to upper and lower bounds, as it should be. If in particular λ is the lowest eigenvalue, we can put $\alpha=-\infty$ and (1) reduces to the Ritz-Temple formula. It should be noted that (1) gives λ within the error of the order ϵ^2, which is very small if ϵ is small, i.e. w is a good approximate eigenfunction. Also we can show that (1) is in precision not behind any of the formulas cited above,[1] so long as the latter is not incorrect. In fact, it can even be shown that (1) is the best possible estimate if η, ϵ, α and β are the only available data.

Another advantage of (1) is that it can be applied to higher eigenvalues without the preliminary procedure of orthogonalization which is necessary in the Ritz method.

We can also estimate the error of the approximate eigenfunction w. Further (1) can be generalized to the case of degenerate eigenvalues.

These formulas proved to be very useful in approximate solution of eigenvalue problems of various kinds. For instance, they give much more narrow range of errors than hitherto supposed in the calculation of eigenvalues by the relaxation method.[2] Also we hope that they can successfully be applied to problems of quantum mechanics. Their application to the eigenvalue problem of the deuteron with the meson potential is now in progress.

Detailed account will appear shortly in Journal of the Physical Society of Japan.

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Neutrons from $Li^7(p,n)Be^7$ *

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If nuclear forces are charge independent, one expects Be^7 to have a low lying level equivalent to the well-known 478-kev level in its mirror nucleus, Li^7. Earlier work by one of us[1] on the neutrons from $Li^7(p,n)Be^7$ failed to detect such a level. However, with the old Ilford halftone emulsions then available, the observed neutron group was very asymmetric and was approximately 700 kev in half-width; hence neutrons of energy corresponding to Be^7 excited could easily have been missed if of low intensity.

Recently, Grosskreutz and Mather[2] reported from this same reaction neutron groups corresponding to levels in Be^7 at 205 kev, 470 kev, and 745 kev, and comparable in intensity to the main group. Because of the importance of this result on the charge independence hypothesis of nuclear forces, we have re-examined this spectrum with the improved post-war nuclear emulsions. Thin (30–60 kev thick) targets of metallic lithium evaporated upon a tantalum backing were bombarded by 3.34-Mev and 3.96-Mev homogeneous protons from the Wisconsin electrostatic generator. 100 micron thick Eastman NTA emulsions (glass-backed) were mounted 15 cm from the target and at 0° and 60° to the proton beam.

After processing, the tracks were measured in a microscope with an oil immersion objective to achieve minimum depth of focus. The observed recoil proton energy, E_p, is equal to $E_n \cos^2\theta$, where E_n is the neutron energy and θ is the angle of recoil with respect to the incident neutron. If the incident neutron is along the x axis and y is in the plane of the emulsion, then

$$\tan\theta=(R_z^2+R_{xy}^2\sin^2\chi)^{\frac{1}{2}}/R_{xy}\cos\chi,$$

where R_z is the z projection of the track, R_{xy} is the projection on the xy plane, and χ is the angle R_{xy} makes with the x axis. R_z is the only one of these quantities difficult to measure accurately. Experimentally we observed the uncertainty in R_z to be about 0.5 micron, which uncertainty must be multiplied by a factor 2.5 because of the shrinkage of these concentrated emulsions upon processing. To minimize this resultant uncertainty in θ, we therefore accepted only tracks with a dip angle in the processed emulsion of $\leqq 3°$. Recoils out to $\chi=15°$ were, however, accepted since