On the Solar Origin of Cosmic Radiation. II.

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Teller-Richtmyer's theory of the origin of cosmic radiation is discussed. It is found that the magnetic field ("trapping field"), which they postulate in order to explain the isotropy of cosmic radiation, must be toroidal and have an extension of about 0.1 light year. A field of this type may be produced by the motion of the solar system relative to interstellar matter. The theory gives the energy spectrum and total intensity of cosmic radiation.

1. UPPER ENERGY LIMIT OF COSMIC RADIATION

THE idea of Teller-Richtmyer[1] that cosmic radiation may be a local solar phenomenon was developed in a recent paper.[2] Arguments were given for the existence of magnetic fields strong enough to confine cosmic radiation to the vicinity of the solar system. The acceleration of particles to cosmic-ray energies was supposed to take place in the solar magnetic field in the environment of the sun, but not on the sun itself. The supposed acceleration mechanism accounts at the same time for the magnetic storm effects on cosmic radiation.

In the present paper the theory shall be further discussed.

In the cited paper[2] the acceleration was assumed to be due to "solar activity" disturbances of the solar dipole field. In order to calculate the order of magnitude of particle energies it is reasonable to start from Störmer orbits in a dipole field. Assuming that the solar magnetic dipole has the moment a, the highest "momentum" $p_1(=H\rho)$ of a particle moving in a periodic orbit is given by

$$p_1 = aR_\odot^{-2} \tag{1}$$

($R_\odot$ = solar radius). This corresponds to a circular orbit close to the solar magnetic equator. This orbit is unstable, however, and consequently of little physical significance. The most important class of stable trajectories are trochoids in the equatorial plane. Usually the trochoids are superimposed by oscillations through the equatorial plane. The highest momentum trochoid in the equatorial plane is given by

$$p_2 = a/(R_\odot^2)(3-2\sqrt{2}) = 0.172p_1. \tag{2}$$

As this orbit requires that the velocity component perpendicular to the equatorial plane is zero, it represents a special case, and in order to find orbits which can keep a great number of particles we must go still somewhat lower. On the other hand, special combinations of sunspot fields with the general magnetic field may give stable orbits even above p_2.

The energy V, in electron volts, of a particle with charge Ze is in the relativistic range

$$V_{\text{volt}} = 300Zp. \tag{3}$$

Assuming the polar strength of the sun's dipole field to be 25 gauss (as an average between the high values and the low values reported) we have $a = 4.2\times10^{33}$ gauss cm^3 and

$$p_2 = 1.5\times10^{11} \text{ gauss cm.} \tag{4}$$

This gives for protons

$$V = 4.5\times10^{13} \text{ ev.} \tag{5}$$

Hence the highest proton energy which at present could be generated in the solar field is of the order 5×10^{13} ev. This does not necessarily mean that no protons with higher energies could be present in cosmic radiation, because the radiation which we now receive is the accumulated effect of generation processes during may be as much as 100 million years. If the solar field has been stronger than now during some part of this time, higher energies may be observed.

Higher energies could also be obtained under the present conditions by acceleration of multiply charged heavy atoms. A 30-fold ionized particle, such as has really been observed in cosmic radiation,[3] would be able to reach the energy 1.4×10^{15} ev. The very rare case of a completely stripped uranium atom would bring us up to 4×10^{15} ev.

2. MINIMUM ENERGY EMITTED FROM THE ACCELERATOR

Let us confine the discussion to orbits in the equatorial plane, which in any case gives the right order of magnitude. At a point at the solar distance R the maximum momentum of a trochoidal orbit is

$$p_3 = aR^{-2} = p_1(R_\odot/R)^2. \tag{6}$$

A small increase in energy will change the orbit so that it goes to infinity. A small disturbance of the dipole field is able to produce the same effect. The main emission of particles from the accelerator will take place from orbits of this type. Particles with momenta smaller than p_3 will of course also be accelerated, and during large disturbances some of them will have a chance of being emitted, but the main emission is likely to occur from orbits close to p_3.

[1] E. Teller, Nuclear Physics Conference, Birmingham, 1948; Alfvén, Richtmyer, and Teller, Phys. Rev. **75**, 892 (1949); R. D. Richtmyer and E. Teller, Phys. Rev. **75**, 1729 (1949).

[2] H. Alfvén, Phys. Rev. **75**, 1732 (1949).

[3] Freier, Lofgren, Ney, Oppenheimer, Bradt, and Peters, Phys. Rev. **74**, 213 (1948); H. L. Bradt and B. Peters, Phys. Rev. **74**, 1828 (1948).

Close to the sun the magnetic field is approximately a dipole field (although sometimes rather much disturbed), but at great distances the field H_T postulated by Teller and Richtmyer dominates. As the extension of the latter field should be much greater than the solar system dimensions, we may suppose it to be homogeneous within the solar system. At so large a solar distance that H_T dominates over the dipole field, particles do not drift around the sun in the way which is necessary for the acceleration process. Hence they are not accelerated or, in any case, the acceleration is not very effective. Still more important is that in the dipole field the particles oscillate through a stable equilibrium whereas outside it they are free to move away from the solar vicinity along the lines of force.

The limit R_T between H_T and the dipole field is given by

$$H_T \approx a R_T^{-3}. \tag{7}$$

Due to the finite extension of the dipole field there is an inferior limit p_4 to the particles emitted from the accelerator. This is of the order

$$p_4 = p_1(R_\odot/R_T)^2 = a R_T^{-2}. \tag{8}$$

Particles far below this limit will not be emitted if situated in the dipole field and if outside it, they will not

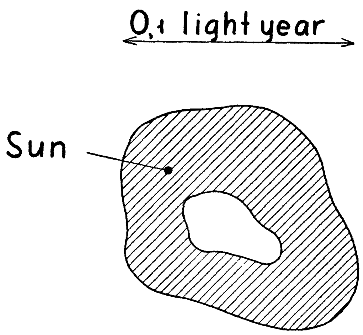

FIG. 1. Possible shape of the space in which cosmic radiation is trapped.

be accelerated very much. It seems reasonable to identify p_4 with the low energy cut-off in the cosmic-ray spectrum, which earlier has been attributed to the cut-off by the solar magnetic field.[4] This enables us to estimate H_T. Equations (7) and (8) give

$$H_T = a^{-\frac{1}{2}} p_4^{\frac{3}{2}}. \tag{9}$$

Putting the low energy cut-off equal to 5×10^9 ev ($p_4 = 1.7\times10^7$ gauss cm) we get

$$H_T \approx 10^{-6}\ \text{gauss}. \tag{10}$$

A more detailed calculation shows that this result depends very much upon the assumption of the orientation of H_T in relation to the solar dipole and may easily be in error by a factor 10.

[4] L. Jánossy, Zeits. f. Physik **104**, 430 (1937); *Cosmic Rays* (Oxford University Press, London, 1948).

The limit R_T between the dipole field and H_T goes not very far from the earth's orbit.

3. MAXIMUM VOLUME OCCUPIED BY COSMIC RADIATION

According to our assumptions the whole cosmic radiation should be generated near the sun, so that, for example, a sphere with the radius $R_\oplus$ equal to the earth's orbital radius encloses the whole region of generation. This must cause an anisotropy of the radiation observed on the earth. That this anisotropy is very small is explained by Teller and Richtmyer through the assumption of an extra-solar field H_T which traps the radiation and reflects it back again. Let us assume that the ratio of the radiation flux outward and inward through the sphere $R_\oplus$ is $(1+\gamma)/(1-\gamma)\approx 1+2\gamma$, so that γ is a measure of the degree of anisotropy. Let us further assume that in average an emitted particle will pass through the sphere $R_\oplus$ N times before being absorbed. As $(N+1)/N = 1+2\gamma$ we have

$$N = (2\gamma)^{-1}. \tag{11}$$

It is possible to find a relation between γ and the volume to which cosmic radiation is confined. The probability that a particle moving at random with velocity c within a volume U shall hit a surface $\pi R_\oplus^2$ during a small time Δt is $\pi R_\oplus^2 c\Delta t/U$. If the average life of a particle is T, the sphere $R_\oplus$ will be hit N times if $N = \pi R_\oplus^2 cT/U$, which gives

$$U = 2\pi R_\oplus^2 cT\gamma. \tag{12}$$

For the average life of a particle moving in interstellar space Fermi[5] gives $T = 6\times10^7$ years $= 2\times10^{15}$ sec. On the other hand Kane, Shanley, and Wheeler[6] find for a particle trapped in the solar magnetic field $T = 5000$ years. Our value should lie between these, but much closer to the upper value, because the particle spends most of its time in interstellar space. If we only want an upper limit to U we could put $T < 2\times10^{15}$ sec. The anisotropy γ is certainly less than 10^{-2}, probably even less than 10^{-3}. With $\gamma < 10^{-3}$ and $R_\oplus = 1.5\times10^{13}$ cm we get

$$U < 0.8\times10^{50}\ \text{cm}^3. \tag{13}$$

(The cube of a light year equals 10^{54} cm^3.)

It should be observed that the calculation of U is based on the assumption of a stationary state. If, for some reason, the output of the cosmic-ray generator has been much lower during the time we have observed cosmic radiation than the average output the last 100 million years, our calculations would be erroneous.

4. PROPERTIES OF THE TRAPPING FIELD

A sphere with the volume given by (13) would have a radius of about 3×10^{16} cm. In a field of the order

[5] E. Fermi, Phys. Rev. **75**, 1169 (1949).

[6] Kane, Shanley, and Wheeler, Rev. Mod. Phys. **21**, 51 (1949).

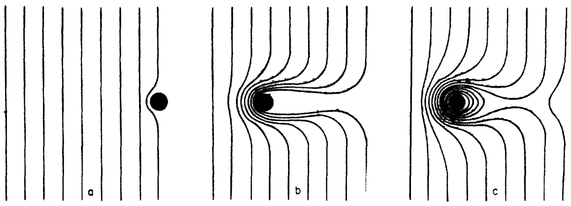

FIG. 2. Magnetic field disturbed by the motion of a body through a conducting medium.

10^{-5} gauss a particle with a momentum $p=10^{11}$ gauss cm has a radius of curvature which is not very much smaller. Hence it is obvious that scattering of the cosmic rays by an irregular magnetic field could not possibly prevent the radiation from spreading to a much larger volume.

If we assume the trapping field to be approximately homogeneous over a region of the order of 10^{16} or 10^{17} cm around the solar system, this field would no doubt prevent the radiation from escaping perpendicular to the field. In fact it would fill a circular cylinder with the radius of 2ρ. The maximum height of the cylinder must be

$$L=U/4\pi\rho^2=\tfrac{1}{2}R_{\oplus}{}^2 cT\gamma H_T{}^2/p^2. \tag{14}$$

With $\gamma=10^{-3}$, $H_T=10^{-5}$ gauss and $p=10^{11}$ gauss cm we find $L=0.8\times 10^{17}$ cm≈ 0.1 light year. As we have chosen p in the upper part of the spectrum we cannot exclude the possibility of a higher anisotropy. Measurements on the high energy component alone are still somewhat uncertain and a value as high as $\gamma=0.1$ would perhaps not be in conflict with observations. This brings us up to $L=10$ light years as an upper limit. In a homogeneous field nothing would prevent the radiation from moving parallel to the field and spreading over a much larger distance. Hence the trapping field could not be approximately homogeneous over a distance of 10^{16}–10^{17} cm around the solar system.

If we drop the assumption of a homogeneous field, the radiation will be enclosed in a tube of flux of arbitrary shape. The magnetic field prevents the radiation from leaking out through the walls of the tube but it is free to move along the tube parallel to the field. As the part of the tube which it occupies must not surpass the length L there must be some "lock" at each end of the tube, which reflects back the radiation. This lock must be very effective because during its lifetime T a particle will reach the end of the tube a number of times n which is of the order

$$n=cT/2L=p^2/(R_{\oplus}{}^2\gamma H_T{}^2). \tag{15}$$

Even if we put γ as high as 0.1 and $H_T=10^{-5}$, n surpasses 10^6 for $p=10^{11}$ gauss cm. It seems quite impossible to find a mechanism which closes the ends of the tube so effectively that a particle has less than one chance in a million to pass. Certainly, if the field strength increases along the tube, particles moving toward increasing field will turn, but in order to make all but one in 10^6 turn, the field must increase by a factor of 10^6. This would bring us up to interstellar fields of the order 10 gauss which of course is impossible.

The problem could also be considered in the following way. In order to make a particle pass N times through the earth's orbital surface $\pi R_{\oplus}{}^2$, the "hole" of the surface which locks in cosmic radiation must not be larger than $\pi R_{\oplus}{}^2/N$, because a larger hole would mean that the radiation leaks out too rapidly.

It is easily seen that a series of locks, each with a smaller efficiency, does not change the order of magnitude.

The only possibility seems to be to assume that *the lines of force are closed lines* so that the flux tubes resemble toroids.

In this case L means the circumference of the toroid. The value of L which we have found, shows that the over-all dimension of the cosmic ray "doughnut" should be smaller than about one light year. An inferior limit is given by the diameter of the tube of force, which must at least equal 4ρ. With $H=10^{-5}$ and $p=10^{11}$ this gives 4×10^{16} cm. Thus the best guess of the size of the doughnut would be the order of 10^{17} cm ($=0.1$ light year).

5. PRODUCTION OF THE TRAPPING FIELD

In the earlier paper it was supposed that the trapping field postulated by Teller and Richtmyer was produced by the magneto-hydrodynamic mechanism which should convert part of the kinetic energy of local motions into magnetic energy. In a primary magnetic field, which may be extremely weak, the polarization due to local motions produces currents, which give rise to the trapping field. It is easily seen that these local currents through space should produce magnetic lines of force which are closed. This amplifies our conclusion that cosmic radiation is trapped by a closed field.

There is a close analogy between this production of the trapping field and the production of sunspots according to the magneto-hydrodynamic theory.[7] Due

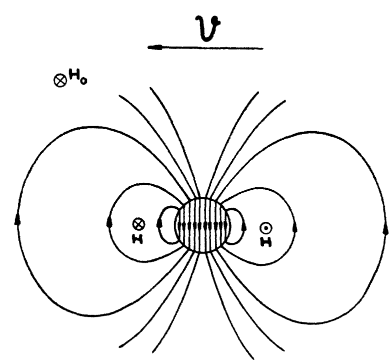

FIG. 3. Current system produced by the motion of a conducting sphere through a conducting medium in a magnetic field H_0. The current system produces a magnetic field H.

[7] H. Alfvén, M. N. R.A.S. **105**, 3, 382 (1945); Ark. f. mat., astr. o. fysik, **34** A, 23 (1948).

to the presence of a general magnetic field, local motions in the sun produce local magnetic fields (which later give rise to sunspots). The local fields are toroidal and their strength is more than 100 times larger than the general field.

The size of the trapping field as found in the preceding paragraph shows that the current system must be confined to the environment of the solar system. In any case its extension is small compared to interstellar distances. Hence it seems likely that the whole phenomenon should be associated with *the motion of the solar system relative to interstellar matter*. A mechanism which may produce the trapping field shall be proposed tentatively.

Suppose that a perfectly conducting body moves rectilinearly in a perfectly conducting medium in which a magnetic field is present. Let the field be homogeneous and perpendicular to the motion. Figure 2 shows qualitatively what occurs. The moving body drags the lines of force with it. The first result (Fig. 2b) is an amplification of the field in front of the body and a long tail of field behind it. Due to the elasticity of the lines of force the tail will break up as shown by Fig. 2c. When a stationary state is reached the moving body will be surrounded by a system of closed lines of force.

The problem can also be treated in the following way. Seen from the moving body the surrounding medium moves with a velocity which we call v. As there is a magnetic field H_0 in the medium, this is polarized seen from the body, so that there is an electric field $E=vH/c$. In the body this field produces a current, and when this current is closed through the surroundings, the current system of Fig. 3 is produced. It is easily seen that it produces a toroidal magnetic field.

In the ideal case we have discussed the motion in the surrounding medium produced by the body has been neglected. In the real case the moving body should probably not be identified with the sun itself but rather with the sun and the interstellar matter in its environment which it drags with it. Further the solar dipole field affects the phenomenon. The whole mechanism is no doubt very complicated and until a more detailed analysis has been made the results of this paragraph and the next are only tentative suggestions.

6. THE INTENSITY OF COSMIC RADIATION

As pointed out by Dungey and Hoyle[8] a magnetic field which traps cosmic radiation is subject to a pressure equal to the energy density of the radiation. This means that the magnetostatic pressure $p_H=H^2/8\pi$ outside the cosmic-ray doughnut must surpass the pressure inside it by the energy density of cosmic radiation, which amounts to $w=4\times10^{-13}$ erg cm^{-3}. The magnetostatic pressure of the trapping field may be of the same order of magnitude as w. In fact $H_T^2/8\pi=w=4\times10^{-13}$ gives $H_T=3\times10^{-6}$ gauss, which is an acceptable value for the trapping field.

When the solar system moves with the velocity v through an interstellar medium with density ϑ, the pressure p_f at the front should be of the order $p_f=\vartheta v^2$. Even this pressure may be of the same order as w and p_H. In fact, if $\vartheta=10^{-24}$ g cm^{-3}, an energy $w=4\cdot10^{-13}$ corresponds to $v=10^6$ cm/sec. which would be a possible value for the motion of the sun in relation to interstellar matter.

It is possible that the order of magnitude agreement between w, p_H, and p_f is not fortuitous. The value of H_T is not determined by the primary magnetic field, H_0, in interstellar space which was supposed to be its ultimate cause. Even an extremely weak primary field is enough to produce very large currents due to the good conductivity of interstellar matter. It is more reasonable that H_T is determined by the condition that the magnetic field in front of the advancing solar system should be strong enough to withstand the pressure due to the medium in which the motion takes place. This would give $p_H=p_f$.

The equality between p_H and w could be understood in the following way. Suppose that the sun generates so much cosmic rays that w surpasses p_H. Then the magnetic field cannot keep the radiation and the cosmic-ray doughnut will expand until w has diminished again to p_H. Thus the output of the solar generator does not determine the intensity but the volume filled by cosmic radiation. The intensity is ultimately determined by p_f, so if the resistance of interstellar matter to the solar motion changes, this would affect the cosmic-ray intensity.

7. THE ACCELERATION PROCESS

According to the earlier paper[2] the acceleration is due to electric fields produced at disturbances of the solar magnetic field, or in other words to the betatron action of magnetic field variations. It is important to observe that in average such variations produce a cumulative increase of the particle energy. This is in principle the same effect as discussed by Fermi:[5] The interaction between a variable magnetic field and a particle tends to increase the particle energy. In our case the details of the process could be understood in the following way.

In a dipole field charged particles move in Störmer orbits. In the orbits of most interest in this connection the motion consists of a trochoidal motion perpendicular to the magnetic field superimposed by oscillations parallel to the field. If the regular dipole field is disturbed in such a way that from time to time the magnetic field in some parts of it changes, the effect of this is twofold: 1. Particles passing a region with a static disturbance of the field are scattered from one Störmer orbit to another. 2. Particles moving in a region where the magnetic field in it changes, increase or decrease their energy.

[8] J. W. Dungey and F. Hoyle, Nature **162**, 888 (1948).

Consider a number of particles moving in the dipole field. The first process tends to distribute them isotropically, so that in average their momenta p_0 are distributed in such a way that the motion parallel to the field accounts for $p_0/3$ and the motion perpendicular to the field for $2p_0/3$ (in the relativistic case). Suppose that in a region, where some of the particles move, the magnetic field increases by a factor α^2. This produces an increase by a factor α of the momentum perpendicular to the field, but it does not affect the motion parallel to the field. Hence the total momentum of a particle is

$$p'=p_0(\tfrac{1}{3}+2\alpha/3). \tag{16}$$

Let the particles then be subject to scattering so that their momenta becomes equally distributed on the three degrees of freedom. Hence the "parallel" momentum is $\frac{1}{3}p'$ and the "perpendicular" momentum $\frac{2}{3}p'$. If later the field in which they move decreases by a factor $1/\alpha^2$, their "parallel" momentum remains unchanged but their "perpendicular" momentum decreases by a factor $1/\alpha$. Hence the result of an increase of the field and a subsequent decrease back to the initial field strength is that the momentum has increased from p_0 to

$$p_0'=p_0\left(\frac{1}{3}+\frac{2}{3}\alpha\right)\left(\frac{1}{3}+\frac{2}{3\alpha}\right)=\frac{5+2(\alpha+\alpha^{-1})}{9}p_0. \tag{17}$$

This shows that if the magnetic field varies up and down an accumulated increase in particle momentum will occur.

Consider a moment when the solar field is undisturbed. Suppose that a certain disturbance occurs, after which the field goes back to its original conditions. The disturbance has increased the momenta of a group of particles by

$$\Delta p=Ap. \tag{18}$$

At the same time it has thrown a number of particles out of their normal orbits in such a way that they have been emitted into interstellar space. The number of particles lost by emission is Bn, where n is the total number of particles. Further during the time between two disturbances a number of Cn particles is lost by absorption (in interplanetary matter or by collisions with celestial bodies). Hence the total change Δn is

$$\Delta n=-(B+C)n. \tag{19}$$

A, B, and C are independent of n but may be functions of p. Supposing that a series of disturbances of the same type occurs we could find the spectrum of accelerated particles. Dividing (19) by (18) we obtain after integration

$$n=n_0\exp\left(-\int\frac{B+C}{A}\frac{dp}{p}\right),$$

where n_0 is an integration constant.

The spectrum emitted from the generator is

$$n_e=Bn_0\exp\left(-\int\frac{B+C}{A}\frac{dp}{p}\right).$$

According to Kane, Shanley, and Wheeler[6] cosmic-ray particles in certain trapped orbit move some thousand years before being absorbed. As this time probably is much larger than the time of the acceleration process, it is reasonable to neglect C except very close to the sun.

If there is an orbit with momentum p at a solar distance R, there is also a similar orbit at R' with the momentum $p'=p(R/R')^2$. If we suppose that the disturbance has the same character at R' as at R, particles with the momentum p' are affected in the same was as those with momentum p. Thus it is reasonable to assume that A and B are independent of p. This gives

$$n_e=n_0'p^{-B/A},$$

where $n_0'=n_0B$ is a constant. In a stationary state this emission should compensate the absorption in a tube of constant length but with a cross section proportional to p^2. This gives the energy spectrum of cosmic radiation as

$$N=N_0p^{-2-(B/A)},$$

where N_0 is a constant. Agreement with the observational exponent -2.8 is obtained if $B/A=0.8$. A theoretical calculation of B/A is probably rather complicated, but a qualitative discussion indicates that the quantity should be of the order unity. The formula should hold in a region extending to the neighborhood of the upper and lower limits given in Sections 1 and 2.

It should be observed that particles moving in the trapping field far away from the sun could also be accelerated by a similar process if the trapping field varies. The difference is that no loss of particles is produced by the disturbance itself. Instead the loss is only due to absorption in interstellar matter. Hence this process is of the same kind considered by Fermi and should also lead to a power spectrum. The "injection" which leads to some difficulty in Fermi's theory could be accomplished by the solar generator.

If observations of cosmic rays should give energies definitely in excess of the upper limits given in Section 1 this may be due to an earlier higher value of the solar dipole moment, but it may also be due to a subsequent Fermi acceleration in the trapping field. In both cases the considerations in Section 2 of the low energy cut-off would be erroneous.