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The Self-Stress of the Electron

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The diagonal elements of the symmetrical energy-momentum tensor density of an electron at rest are calculated. The covariant formalism of Tomonaga, Schwinger, Feynman, and Dyson is used, and it is shown that it is necessary to use a relativistic cut-off in addition to the covariant separation of the infinite renormalizations. Therefore "formalistic regulators" are used in the form of additional neutral vector fields. The integrations are carried out with the Feynman method. The resultant vanishing value for the self-stress constitutes a proof of the consistency of the relativistic formalism.

It is also shown how the Feynman-Dyson method can be used for the calculation of expectation values of operators, of which the self-stress calculation constitutes an example.

THE SELF-STRESS PROBLEM

In classical physics the finite charge distribution attributed to the electron could not be stable unless additional non-electromagnetic "cohesive" forces were postulated. The necessity of this requirement was also exhibited by the fact that a finite electron had a finite *non-vanishing* electromagnetic part of its experimental mass which spoiled the relation between kinetic energy and momentum. This is true even for the non-relativistic Abraham electron. For the relativistic Lorentz electron it can best be seen from the transformation properties of the energy-momentum tensor. Let $\Theta_{\mu\nu}$ be the energy-momentum tensor density of an electron moving with velocity v in the x direction; let the index zero indicate the rest system. Then energy and momentum transform as follows:[1]

$$E=-\int \Theta_{44}d\tau =-(1-v^2)^{-\frac{1}{2}}\left(\int \Theta_{44}{}^{(0)}d\tau_0-v^2\int \Theta_{11}{}^{(0)}d\tau_0\right), \quad (1a)$$

$$p_x=-i\int \Theta_{41}d\tau =-v(1-v^2)^{-\frac{1}{2}}\left(\int \Theta_{44}{}^{(0)}d\tau_0-\int \Theta_{11}{}^{(0)}d\tau_0\right). \quad (1b)$$

It follows from (1) and from the symmetry in the rest system that a consistent relativistic theory must yield

$$\Theta_{11}{}^{(0)}=\Theta_{22}{}^{(0)}=\Theta_{33}{}^{(0)}=0. \quad (2)$$

It is clear that the Eqs. (1) and (2) remain true in quantum theory. A direct verification of (2) for conventional quantum electrodynamics, however, so far led to difficulties because of the inherent infinities.

The recent developments in quantum field theory enable one to properly deal with these infinities. It is therefore the purpose of this paper to explicitly verify the vanishing of the self-stress and thus give a proof of the internal consistency of our present relativistic quantum electrodynamics.

In order to carry out this program it is important to realize that the formalism of Tomonaga,[2] Schwinger,[3] and Dyson[4] still suffers from divergent and undefined integrals. The covariant separation of these integrals from the infinite matrix element results in the finite expectation value of the observable in most cases. Exceptions are the calculation of self-energies, vacuum-polarization, scattering of light by light, and the self-stress. It is therefore necessary to introduce a relativistic cut-off, for example by the use of "regulators."[5] We shall consider the interaction of the electron field ψ with a set of neutral vector fields $A_\mu{}^{(i)}$ of mass M_i and coupling constant f_i ($i=0, 1, 2, \cdots$). At the end we shall pass to the limit[6] as $M_0\rightarrow 0$ and $M_i\rightarrow\infty$ ($i\neq 0$). The resulting integrals will be finite and well determined if the f_i and M_i satisfy certain conditions. In the course of the calculations the following condition will turn out to be necessary and sufficient:

$$\sum_i f_i{}^2=0. \quad (3)$$

We shall first give a simple argument which shows that the self-stress is identically zero to all orders and then carry out an explicit calculation to first order in $\alpha=e^2/(4\pi)$.[7]

A SIMPLE ARGUMENT[8]

The Lagrangian density for the system electron field —neutral vector meson fields is

[1] Here and throughout this paper we use natural units, $\hbar=c=1$.

[2] S. Tomonaga, Prog. Theor. Phys. **1**, 27 (1946); Koba, Tati, Tomonaga, Prog. Theor. Phys. **2**, 101, 198 (1947); S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3**, 1, 101 (1948).

[3] J. S. Schwinger, Phys. Rev. **74**, 1439 (1948); **75**, 651 (1949); **76**, 790 (1949).

[4] F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949).

[5] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949). References to earlier work on this subject are also found in this article.

[6] This "formalistic" use of the auxiliary fields is employed since any attempt at a consistent "realistic" theory leads to Bose-fields of negative energy and to other difficulties. See also reference 5.

[7] In the following we shall use Heaviside-Lorentz units and in general use the same notation as Schwinger and Dyson (references 3 and 4); note however that our $S_F(x)$ equals Dyson's $S_F(-x)$. We use ∂_μ for $\partial/\partial x_\mu$ and a bold letter like $\mathbf{A}$ for $\gamma_\mu A_\mu$.

[8] This type of argument was first used by A. Pais, Rev. Mod. Phys. **21**, 445 (1949) and led him to the wrong result $\Theta_{11}{}^{(0)}=(\alpha/2\pi)m\bar{\psi}\psi$. The reason for this finite non-vanishing result is

$$\mathcal{L}(x) = -\sum_i [\bar{\psi}(x)(\partial - ief_i A^{(i)} + m)\psi(x) + \tfrac{1}{4}F_{\mu\nu}{}'^{(i)}(x) - (M_i^2/2)A_\lambda{}^{(i)}A_\lambda{}^{(i)}] \quad (4)$$

from which one finds the symmetrized energy-momentum tensor density and the Hamiltonian density in the usual way. The latter is of the form

$$\mathcal{H}(x) = \mathcal{H}'(x) + m\bar{\psi}(x)\psi(x) + \sum_i (M_i^2/2)A_\lambda{}^{(i)}(x)A_\lambda{}^{(i)}(x), \quad (5)$$

where $\mathcal{H}'(x)$ does not contain the masses explicitly. From Eqs. (5) and (4) we obtain

$$\begin{aligned}\Theta_{\mu\mu}{}^{(e)} &= -m\bar{\psi}\psi = -m(\partial\mathcal{H}/\partial m),\\ \Theta_{\mu\mu}{}^{(i)} &= -M_i^2 A_\lambda{}^{(i)}A_\lambda{}^{(i)} = -M_i(\partial\mathcal{H}/\partial M_i), \quad (6)\\ \Theta_{44} &= -\mathcal{H}.\end{aligned}$$

For the rest system we have, omitting the index zero from now on

$$\Theta_{11} = \tfrac{1}{3}(\Theta_{\mu\mu} - \Theta_{44}). \quad (7)$$

The expectation value of Θ_{11} which we shall designate by $\langle\Theta_{11}\rangle$ in a state where there is one electron present and no other particle, is therefore

$$\langle\Theta_{11}\rangle = -\tfrac{1}{3}(m(\partial/\partial m) + \sum_i M_i(\partial/\partial M_i) - 1)\langle\mathcal{H}\rangle$$

but since $\langle\mathcal{H}\rangle$ is simply the self-energy of the electron we have for dimensional reasons

$$\langle\mathcal{H}\rangle = \mathcal{H}_{\text{self}}^{(e)} = mf(m/M_i) \quad (8)$$

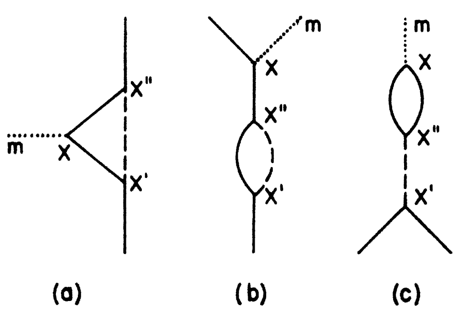

FIG. 1. Diagrams for the calculation of the interaction of an electron (full lines) with an external field (dotted lines) to second order in the coupling between the electron and its radiation field. For the calculation with regulators the photon lines (dashed lines) represent neutral vector mesons. (See Eq. (10).)

found in that he did not take into account regulators. It is vital to the calculation of the self-stress that the theory be *finite*. Otherwise, it would be necessary to circumvent the direct integration which gives $(\alpha/2\pi)m\bar{\psi}\psi$ by the explicit use of the conservation laws $\partial_\mu\Theta_{\mu\nu}=0$. The situation is completely analogous to the calculations of the photon self-energy (G. Wentzel, Phys. Rev. **74**, 1070 (1948)) were the conservation of charge $\partial_\mu j_\mu = 0$ has to be used explicitly in order to obtain the result zero.—I am grateful for a discussion on this point to Drs. S. Borowitz and W. Kohn who are calculating the self-stress along these lines.

and

$$\begin{aligned}\langle\Theta_{11}\rangle &= -\frac{m}{3}\left(m\frac{\partial}{\partial m} + \sum_i M_i\frac{\partial}{\partial M_i}\right)\left\langle\frac{\mathcal{H}}{m}\right\rangle\\ &= -\frac{m}{3}\left(\frac{\partial}{\partial \log m} - \sum_i \frac{\partial}{\partial \log M_i}\right)f\left(\frac{m}{M_i}\right)\\ &= 0. \quad (9)\end{aligned}$$

CALCULATION OF THE SELF-STRESS TO ORDER α

We shall use the Feynman[9]-Dyson[4] method to write down the matrix elements from the corresponding diagrams and evaluate the integrals with the Feynman method.

We first turn to the evaluation of $\Theta_{\mu\mu}{}^{(i)} = -m\bar{\psi}\psi$. As is shown in the Appendix, the expectation value of this operator is identical with the expectation value of the interaction energy of the ψ field with a constant external scalar field of strength m, i.e., we regard $-m\bar{\psi}\psi$ as the interaction energy of these two fields. The diagrams corresponding to that process for a state of one electron and no other particles present in first order of α are clearly these shown in Fig. 1. They are identical with those for the radiative corrections (Lamb-shift and correction to electron scattering), except that the external field is a scalar rather than a vector field. One finds[7]

$$\begin{aligned}\langle\Theta_{\mu\mu}{}^{(e)}(x)\rangle = -\tfrac{1}{2}\alpha m\pi\sum_i f_i^2\Big[&\int \bar{\psi}(x')\gamma_\lambda S_F(x'-x)\\ &\times S_F(x-x'')\gamma_\lambda\psi(x'')\Delta_F{}^{(i)}(x'-x'')dx'dx''\\ &+\int \bar{\psi}(x')\gamma_\lambda S_F(x'-x'')\gamma_\lambda\\ &\times S_F(x''-x)\psi(x)\Delta_F{}^{(i)}(x'-x'')dx'dx''\\ &+\int \bar{\psi}(x')\gamma_\lambda\psi(x')\Delta_F{}^{(i)}(x'-x'')\\ &\times Tr(\gamma_\lambda S_F(x''-x)S_F(x-x''))dx'dx''\Big], \quad (10)\end{aligned}$$

and in momentum space

$$\begin{aligned}\langle\Theta_{\mu\mu}{}^{(e)}(p)\rangle &= -(i\alpha m/4\pi^3)\\ &\times\sum_i f_i^2\Big[\int \bar{\psi}(p)\gamma_\lambda\left(\frac{ip-ik-m}{(p-k)^2+m^2}\right)^2\gamma_\lambda\psi(p)\frac{d_4k}{k^2+M_i^2}\\ &+\int \bar{\psi}(p)\gamma_\lambda\frac{ip-ik-m}{(p-k)^2+m^2}\gamma_\lambda\frac{ip-m}{p^2+m^2}\psi(p)\frac{d_4k}{k^2+M_i^2}\\ &+\int \bar{\psi}(p)\gamma_\lambda\psi(p)\frac{1}{M_i^2}Tr\left(\gamma_\lambda\left(\frac{ik-m}{k^2+m^2}\right)^2\right)d_4k\Big]. \quad (11)\end{aligned}$$

As explained in the appendix of the second paper of

[9] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).

Feynman,[9] we have to introduce the auxiliary variables x into each integral, shift the origin such that the denominator depends only on k^2 and integrate with respect to d_4k. The following integrals occur in the first term of (11).

$$\sum_i f_i^2 \int \frac{k_\lambda^2 d_4k}{(k^2+a_i^2)^3} = -i\pi^2 \sum_i f_i^2 \ln a_i^2, \qquad (12a)$$

$$\sum_i f_i^2 \int \frac{d_4k}{(k^2+a_i^2)^3} = \tfrac{1}{2} i\pi^2 \sum_i (f_i^2/a_i^2), \qquad (12b)$$

where use was made of (3). The first term of (11) yields, therefore, after an easy calculation,

$$8i\pi^2 \bar{\psi}(p)\psi(p) \sum_i f_i^2 \int_0^1 x dx \{ \ln[(M_i^2/m^2)(1-x)+x^2] + \tfrac{1}{2}(1-x+x^2)((M_i^2/m^2)(1-x)+m^2x^2) \}. \qquad (13)$$

Whereas we could use the Dirac equation $(ip+m)\psi=0$ and $p^2+m^2=0$ without running into difficulties in the first term of (11), this is not so for the second term. One has of course to subtract the renormalization integral before the use of the Dirac equation is justified. Following Feynman[9] and Dyson[4] we have for the second term of (11)

$$\begin{aligned} &2i\pi^2 \int_0^1 dx \bar{\psi}(p)[ip(1-x)+2m][(ip-m)/(p^2+m^2)]\psi(p) \\ &\quad \times \ln[(M_i^2/m^2)(1-x)+((p^2/m^2)+1)x-(p^2/m^2)x^2] \\ &\quad -2i\pi^2 \int_0^1 dx \bar{\psi}(p) m(1+x)[(ip-m)/(p^2+m^2)] \\ &\quad \times \psi(p) \ln[(M_i^2/m^2)(1-x)+x^2] \\ &= 2i\pi^2 \int_0^1 dx \bar{\psi}(p) \{ (ip+m)(1-x)[(ip-m)/(p^2+m^2)] \\ &\quad \times \psi(p) \ln[(M_i^2/m^2)(1-x)+((p^2/m^2)+1)x \\ &\quad -(p^2/m^2)x^2] + m(1+x)[(ip-m)/(p^2+m^2)]\psi(p) \\ &\qquad \times \ln[1+(p^2+m^2)x(1-x)/(M_i^2(1-x)+m^2x^2)] \}. \end{aligned}$$

We expand the second logarithm and find after use of the Dirac equation

$$2i\pi^2 \int_0^1 dx \bar{\psi}(p)[(x-1)\ln[(M_i^2/m^2)(1-x)+x^2] + m(1+x)(-2m)x(1-x)/(M_i^2(1-x)+m^2x^2)]\psi(p). \qquad (14)$$

The third term of (11) is easily seen to vanish;[9a] the "vacuum polarization" diagram (Fig. 1c) does not contribute. From (13) and (14) the expectation value (11) becomes

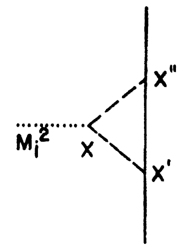

FIG. 2. Diagram for the calculation of the interaction of a virtual neutral vector meson (dashed lines) with an external field (dotted lines). (See Eq. (17a).)

$$\begin{aligned} \langle \Theta_{\mu\mu}^{(e)}(p) \rangle = \frac{\alpha}{\pi} m \bar{\psi}(p)\psi(p) \sum_i f_i^2 \int_0^1 dx \{ \tfrac{1}{2}(5x-1) \\ \times \ln[(M_i^2/m^2)(1-x)+x^2] \\ -(x^2-2x^3)/((M_i^2/m^2)(1-x)+x^2) \}. \end{aligned}$$

An elementary integration leads in the limit $M_i \rightarrow \infty$ $(i \neq 0)$ $(M_0 \rightarrow 0)$, since $f_0=1$ by definition, with (3) to

$$\langle \Theta_{\mu\mu}^{(e)}(p) \rangle = (\alpha/2\pi) m \bar{\psi}(p)\psi(p)(3l_m+9/4), \qquad (15)$$

where

$$l_m = \sum_{i \neq 0} f_i^2 \ln(M_i^2/m^2). \qquad (16)$$

We now turn to the evaluation of

$$\Theta_{\mu\mu}^{(i)} = -M_i^2 A_\lambda^{(i)} A_\lambda^{(i)}.$$

This operator is again regarded as the interaction energy of the $A_\mu^{(i)}$ field with a constant scalar field of coupling M_i^2. The corresponding diagram is shown in Fig. 2: The vector meson emitted by the electron is scattered by the external field and then reabsorbed. This process is essentially different from the processes considered in Dyson's papers. It is, however, very similar to the "Lamb-shift" process, i.e., to Fig. 1a: Instead of the P brackets of 2 pairs $\bar{\psi}\psi$ and 1 pair $A_\mu A_\nu$ we now have 1 pair $\bar{\psi}\psi$ and 2 pairs $A_\mu A_\nu$. The missing pair of anticommuting operators introduces a minus sign in front of the matrix element,[10] the P bracket of the four A_μ's yields a factor[11] 2, and otherwise the integrals for Figs 1a and 2 differ only by the interchange of the S_F functions and the $\Delta_F^{(i)}$ functions. Thus we find

$$\langle \Theta_{\mu\mu}^{(i)}(x) \rangle = \frac{e^2}{4} \sum_i f_i^2 M_i^2 \int \bar{\psi}(x') \gamma_\lambda S_F(x'-x'') \times \gamma_\lambda \psi(x'') \Delta_F(x'-x) \Delta_F(x-x''), \qquad (17a)$$

[9a] Actually, the identically vanishing traces of one and three γ_μ's are multiplied by divergent integrals. Strictly speaking it would be necessary to use a cut-off; e.g., one can add electron fields of mass m_i and coupling constants g_i. As for the vacuum-polarization the conditions on them would be $\Sigma g_i^2=0$; $\Sigma g_i^2 m_i^2=0$.

[10] See Dyson's first paper (reference 4), Eq. (51).

[11] $P\langle \psi(x')\psi(x)\psi(x)\psi(x'') \rangle = \tfrac{1}{2}\eta(x',x) S_F(x'-x) \tfrac{1}{2}\eta(x,x'') S_F(x-x'')$, $P\langle A_\mu(x') A_\lambda(x) A_\lambda(x) A_\nu(x'') \rangle = 2 \cdot \tfrac{1}{2} \delta_{\mu\lambda}\delta_{\nu\lambda} \Delta_F(x'-x) \Delta_F(x-x'')$. Note that the factor 2 would not occur if A_μ were a charged field and we had $P\langle A_\mu^*(x') A_\lambda(x) A_\lambda^*(x) A_\nu(x'') \rangle$.

$$\langle\Theta_{\mu\mu}{}^{(i)}(p)\rangle=\frac{i\alpha}{4\pi^3}\sum_i f_i{}^2M_i{}^2\int\bar{\psi}(p)\gamma_\lambda\frac{ip-ik-m}{(p-k)^2+m^2}\times\gamma_\lambda\psi(p)\frac{d_4k}{(k^2+M_i{}^2)^2}.\quad(17\text{b})$$

Again we introduce the variables x and find after a shift of origin and application of (12b)

$$\langle\sum_i\Theta_{\mu\mu}{}^{(i)}(p)\rangle=(\alpha/\pi)m\bar{\psi}(p)\psi(p)\sum_i f_i{}^2M_i{}^2\times\int_0^1(1-x^2)dx/[M_i{}^2(1-x)+m^2x^2].$$

We see that for $i=0$ we find $\Theta_{\mu\mu}{}^{(0)}(p)=0$ indicating the vanishing value of the photon self-energy. The limit of the other fields gives, because of (3),

$$\langle\sum_i\Theta_{\mu\mu}{}^{(i)}(p)\rangle=-(3\alpha/2\pi)m\bar{\psi}(p)\psi(p).\quad(18)$$

The $\langle\Theta_{44}\rangle$ term is simply the self-energy of the vector mesons and the electron, taken in a state where there is only an electron present. Therefore

$$\langle\Theta_{44}\rangle=\langle\Theta_{44}{}^{(e)}\rangle=-\mathcal{H}_{\text{self}}^{(e)}=(-\alpha/2\pi)\bar{\psi}\psi(-3l_m+\tfrac{3}{4}).\quad(19)$$

The last relation is easy to work out and agrees with Feynman's expression[12] if we use only *one* additional field[13] $M_1=\lambda$ and, because of (3), $f_1{}^2=-f_0{}^2=-1$.

The expectation value to order α of the self-stress, Eq. (6), becomes with (15), (18), and (19)

$$\langle\Theta_{11}(p)\rangle=\tfrac{1}{3}(\alpha/2\pi)m\bar{\psi}(p)\psi(p)\times(3l_m+9/4-3-3l_m+\tfrac{3}{4})=0.\quad(20)$$

Since the l_m terms cancel, the result without regulators would also be finite but would not include the term (18) and thus yield Pais' result $(\alpha/2\pi)m\bar{\psi}\psi$. It should be pointed out here that in the use of regulators one usually adds the matrix elements corresponding to the auxiliary fields ($i=1, 2, \cdots$) to the matrix element which is to be regulated ($i=0$). This method does not work here. It would again mean omitting the term (18) and we would find the same result as without regulators.

In conclusion the author would like to thank Professor R. P. Feynman for very helpful discussions, and Professor J. R. Oppenheimer for his stimulating interest and the extremely profitable time which the author spent at the Institute for Advanced Study.

[12] Reference 9, second paper, Eq. (21).

[13] If one does not carry out the limiting process $M\to\infty$ this special case is identical with the proposals by F. Bopp, Ann. d. Physik **38**, 345 (1940); **42**, 575 (1943) and A. Landé and L. Thomas, Phys. Rev. **60**, 121, 514 (1940); **65**, 175 (1944). It is to be noted that this additional vector field has negative energy, since $f_1{}^2=-1$.

APPENDIX

Expectation Values in the Feynman-Dyson Formalism

Let ψ_H and $A_{\mu H}$ be the operators representing the electron and the electromagnetic fields in the Heisenberg representation; let ψ_H and/or $A_{\mu H}$ interact with an external field such that the interaction energy is $\mathcal{H}_H{}^{\text{int}}$. Let Φ and $\Psi(\sigma)$ denote the functionals in the Heisenberg representation and in the interacting representation. Then

$$\Psi(\sigma)=U(\sigma)\Phi\quad(\text{A}1)$$

and any operator $F_H(x)$ in the Heisenberg representation is related to the same operator in the interaction representation, $F(x)$, by[14]

$$F(x)=U(\sigma)F_H(x)U^{-1}(\sigma)\quad(\text{A}2)$$

such that the expectation value

$$\langle F_H(x)\rangle=(\Phi, F_H(x)\Phi)=(U^{-1}\Psi, F_HU^{-1}\Psi)=(\Psi, F\Psi).$$

Also

$$i(\delta\Psi(\sigma)/\delta\sigma)=(\mathcal{H}^{(1)}+\mathcal{H}^{\text{int}})\Psi(\sigma),\quad(\text{A}3)$$

where $\mathcal{H}^{(1)}=-j_\mu A_\mu$ and the missing subscript H indicates the interaction representation. A further transformation $S(\sigma)$ leads to the new functional $\Omega(\sigma)$ given by[3,4]

$$\Omega(\sigma)=S(\sigma)\Psi(\sigma)\quad(\text{A}4)$$

such that

$$i(\delta\Omega(\sigma)/\delta\sigma)=S^{-1}(\sigma)\mathcal{H}^{\text{int}}(x)S(\sigma)\Omega(\sigma),\quad(\text{A}5)$$

where

$$S(\sigma)=1+(-i)\int_{-\infty}^{\sigma}dx_1\mathcal{H}^{(1)}(x_1)+(-i)^2\int_{-\infty}^{\sigma}dx_1\int_{-\infty}^{\sigma_1}dx_2\mathcal{H}^{(1)}(x_1)\mathcal{H}^{(1)}(x_2)+\cdots.$$

The expectation value of $\mathcal{H}^{\text{int}}$ is

$$\langle\mathcal{H}^{\text{int}}\rangle=(\Omega(\sigma), S^{-1}(\sigma)\mathcal{H}^{\text{int}}(x)S(\sigma)\Omega(\sigma)).\quad(\text{A}6)$$

Consider now an operator O_H in the Heisenberg representation. It follows from (A2) that this operator in the Ω representation is $S^{-1}U^{-1}O_HUS=S^{-1}OS$. The expectation value is therefore

$$\langle O\rangle=(\Omega, S^{-1}OS\Omega).\quad(\text{A}7)$$

Apparently, diagrams can only be drawn for matrix elements of interaction energies, i.e., for expectation values of the type (A6), and at first sight it seems that the Feynman-Dyson method is incapable of calculating expectation values of operators in general, i.e., it seems not possible to immediately write down the integral (A7), because there is no corresponding diagram.

The fact that (A6) actually is only a special case of (A7) suggests the following simple method. Assume first that O is a scalar operator. Since O will always be bilinear in ψ and/or A_μ we can regard it as an interaction energy between the ψ and/or the A_μ field with some external field and solve this equivalent problem which can easily be put into diagrams. Thus the operator $O_H=\Theta_{\mu\mu H}{}^{(e)}=-m\bar{\psi}_H\psi_H$ is clearly the interaction energy of the ψ field with a constant scalar field. A similar reasoning holds for $\Theta_{\mu\mu}{}^{(i)}=-M_i{}^2A_\mu{}^{(i)}A_\mu{}^{(i)}$.

If the operator is not a scalar we can construct a scalar operator by multiplication with a corresponding operator. For example, the expectation value of $O_H=\bar{\psi}_H\sigma_{\mu\nu}\psi_H$ can be calculated by finding the expectation value of $\mathcal{H}_H{}^{\text{int}}=\bar{\psi}_H\sigma_{\mu\nu}F_{\mu\nu}{}^e\psi_H$, where $F_{\mu\nu}{}^e$ is a fictitious external field contragradient to $\sigma_{\mu\nu}$.

[14] Note that Eq. (A2) in general contains additional terms if $F_H(x)$ involves differential operators (see Schwinger, reference 3). This does not effect the arguments given below.