Nuclear Dipole Vibrations and the Surface Symmetry Energy

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The frequency of nuclear electric dipole vibrations of the type suggested by Goldhaber and Teller is derived on the basis of the semi-empirical nuclear model. The resulting formula, taken together with the (γ, n) and (γ, f) cross-section data of Baldwin and Klaiber, determines a lower bound for the term in the surface energy depending on $N-Z$ (surface symmetry energy). This is close to an upper bound estimated by Feenberg, and one may tentatively assume that the nuclear surface tension contains a factor $1-0.4(N-Z)^2A^{-2}$.

THE symmetry properties of the nuclear volume energy have been extensively treated but it has been customary to neglect the symmetry dependence of the surface energy. Recently, attention has been drawn to the possible dependence of the surface energy on isotopic spin, and an upper bound for the surface symmetry energy has been estimated from experimental data on isotopic abundances.[1] It is the purpose of this note to point out the connection between the nuclear electric dipole vibrations proposed by Goldhaber and Teller[2] and the semi-empirical nuclear model, and to show that this leads to a lower bound for the surface symmetry energy.

We consider first a slightly different derivation of the vibration frequency from that already given,[2] which eliminates part of the uncertainty in the nuclear constants and is also somewhat more general. The dipole vibration involves a relative displacement of the centroid of the protons from that of the neutrons. A varying gradient of the proton density is accompanied by a neutron density gradient in the opposite direction. The problem has been idealized in reference 2 to the relative motion of a homogeneous, incompressible ball of protons and a similar sphere of neutrons overlapping each other. Using this same approximation, we estimate the potential energy of the displacement from the change in the volume symmetry energy and the surface energy. The nuclear model used is based on the semi-empirical binding energy formula

$$-E_B=A\{-a_v+a_{\tau v}(N-Z)^2A^{-2}\}+A^{\frac{2}{3}}\{a_s-a_{\tau s}(N-Z)^2A^{-2}\}+a_cZ^2A^{-\frac{1}{3}}, \quad (1)$$

where the term in $a_{\tau s}$ is the surface symmetry energy and the other terms have their usual significance. The energy terms in a_v and a_c (Coulomb) do not change in the type of nuclear distortion considered.[3] The density of the volume symmetry energy is taken to be $a_{\tau v}\rho\{(\rho_N-\rho_Z)/(\rho_N+\rho_Z)\}^2$ where ρ_N, ρ_Z and ρ are the particle densities of neutrons, protons and nucleons, respectively. Consider a relative displacement ξ of the centers of the two spheres, where $|\xi|$ is small compared to the radius $R=r_0A^{\frac{1}{3}}$ but large compared to the effective thickness of surface layer ($\sim r_0$). The volume of extruded neutrons is $\pi R^2|\xi|$ and the change in the volume symmetry energy is readily found to be

$$\Delta E_{\tau v}=\pi R^2|\xi|\rho a_{\tau v}\cdot\{1-(N-Z)^2A^{-2}\}. \quad (2)$$

In the extruded regions $(\rho_N-\rho_Z)/(\rho_N+\rho_Z)\approx\pm1$, and it may seem questionable to use the binding energy formula for such an extreme departure from normal densities; in particular, higher powers of $(\rho_N-\rho_Z)/(\rho_N+\rho_Z)$ in the energy density and of $(N-Z)/A$ in the binding energy formula would seem to be required. This objection is not, however, serious, since (1) the idealized picture in which ρ_Z drops from a uniform value in the overlap region abruptly to zero in the extruded neutron layer is an approximation to an actually continuous change of density; and (2) the kinetic energy of the statistical model expanded in powers of $(N-Z)^2A^{-2}$ differs from the first two terms in the expansion by only two percent as $Z\to0$, and the potential energy in the "equal forces" approximation contains no higher power of $N-Z$ than $(N-Z)^2$.

The change in surface energy is readily seen to be

$$\Delta E_s=4\pi R^2(O'-O)+2\pi RO|\xi|, \quad (3)$$

where O and O′ are the surface tension of the undistorted nucleus and of the extruded region, respectively, and O′ is independent of ξ. The second term represents the energy of the extra area in the equatorial belt and leads to a small correction of the vibration frequency by a factor $(1+\beta A^{-\frac{1}{3}})^{\frac{1}{2}}$ where $\beta=2a_s/3a_{\tau v}\approx0.5$. This will be omitted in the following. Since the effective potential in a pure neutron fluid is very much less than (about one-fourth) that in normal nuclei, the surface tension O′ is evidently considerably smaller than O. We set $4\pi R^2(O-O')=A^{\frac{2}{3}}\delta$ and defer the analysis of δ in terms of the parameters of Eq. (1). The potential energy of deformation for $|\xi|\gg r_0$ is then given by $\Delta E_{\tau v}+\Delta E_s=\Delta E_{\tau v}-A^{\frac{2}{3}}\delta$. The potential energy for small displacements $|\xi|\lesssim r_0$ can evidently be written as $k\xi^2/2$, and the two expressions are to be joined continuously and with continuous derivative at a displacement $|\xi|=\epsilon$. The values of k and ϵ are obtained by solving these two equations simultaneously. The derivative

[1] E. Feenberg, Rev. Mod. Phys. **19**, 239 (1947).
[2] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).
[3] To the extent that a proton density gradient is set up in the vibration, there will be an accompanying change in the electrostatic energy. This is neglected. Effects due to the finite compressibility of nuclear matter are also neglected.

equation yields (after substituting $A/(4\pi R^3/3)$ for ρ)

$$k=(3Aa_{\tau v}/4\epsilon R)\cdot\{1-(N-Z)^2A^{-2}\}. \quad (4)$$

Equating the energies and using Eq. (4), one obtains

$$\epsilon=(8r_0\delta/3a_{\tau v})\cdot\{1-(N-Z)^2A^{-2}\}^{-1}. \quad (5)$$

Thus ϵ is independent of A except for the small dependence through the curly bracket. The latter is nearly canceled by a similar variation of δ (see below); hence ϵ may be considered independent of A and Z.

The vibration frequency depends on the reduced mass of the two spheres:

$$\mu=4\pi R^3M\rho_Z\rho_N/3\rho=(MA/4)\cdot\{1-(N-Z)^2A^{-2}\}, \quad (6)$$

where M is the mass of a nucleon. Combining (4) and (6) gives

$$\hbar\omega=\hbar(k/\mu)^{\frac{1}{2}}=(3\hbar^2a_{\tau v}/\epsilon Mr_0)^{\frac{1}{2}}\cdot A^{-1/6}. \quad (7)$$

This is the same formula as that given by Goldhaber and Teller, although the interpretation of the constants is different. In comparing with the experimental results, we have included the small correction introduced by the second term of Eq. (3). The maxima in the (γ, n) and (γ, f) cross sections observed by Baldwin and Klaiber[4] come at values of $\hbar\omega=30$, 22, 18 and 16 Mev for C, Cu, Th and U, respectively. Averaging the last two results, the experimental ratios are: $(\hbar\omega)_C/(\hbar\omega)_{Cu}=1.36$ and $(\hbar\omega)_{Cu}/(\hbar\omega)_{Th-U}=1.29$. The theoretical ratios are 1.37 and 1.27; the agreement is better than the experimental and theoretical uncertainties would warrant. Recent measurements[5] on Ta show a maximum cross section at 15 Mev; this value is anomalously low when compared with the preceding data. It is worthy of note that the inclusion of an additional term[6] in the energy density containing $|\rho_N-\rho_Z|$ and corresponding to a volume symmetry term in $|N-Z|/A$ in Eq. (1), yields a dependence of $\hbar\omega$ on A at variance with the experiments. In this case the frequencies are multiplied by an additional factor which is approximately $[1+8(Z/A^2)]^{\frac{1}{2}}$ and the ratio $(\hbar\omega)_C/(\hbar\omega)_{Cu}$ is increased to 1.53.

In order to confirm the absolute magnitude of $\hbar\omega$ predicted by Eq. (7), it is necessary to know the value of ϵ the joining distance (the semi-empirical value of $a_{\tau v}=20$ Mev is well known[7]). Obviously ϵ is of the order of magnitude of $r_0\approx1.4\times10^{-13}$ cm, but a more precise estimate would seem dubious. Turning the problem around, we can determine ϵ from experiment and then use it to obtain further information about the nuclear surface energy. The data of Baldwin and Klaiber give $\epsilon=(3/4)r_0\approx1.1\times10^{-13}$ cm; this can be inserted in Eq. (5) to determine δ. The relation between δ and the constants of Eq. (1) will now be considered.

The surface energy of the undeformed nucleus is (from Eq. (1))

$$\begin{aligned}E_s&=a_sA^{\frac{2}{3}}-a_{\tau s}A^{\frac{2}{3}}(N-Z)^2A^{-2}\\&=a_sA^{\frac{2}{3}}\{1-C_A(N-Z)^2A^{-2}\}\\&=a_sA^{\frac{2}{3}}\{1-C_\sigma(\sigma_N-\sigma_Z)^2(\sigma_N+\sigma_Z)^{-2}\}\end{aligned} \quad (8)$$

where σ_N and σ_Z denote the surface densities of neutrons and protons in the undeformed nucleus. Because of the electrostatic repulsion, the proton density must increase from the center to the surface of the nucleus, hence $(\sigma_N-\sigma_Z)/(\sigma_N+\sigma_Z)=\lambda(N-Z)/(N+Z)$ with $\lambda<1$. The departures from uniform density have been calculated by Feenberg;[8] his results lead to a value of λ (independent of Z and A) of 0.86. Hence $C_A=(0.86)^2C_\sigma$ for the undeformed nucleus. The surface energy of the deformed nucleus is then*

$$E_s'=a_sA^{\frac{2}{3}}\{1-C_\sigma(\sigma_N'-\sigma_Z')^2(\sigma_N'+\sigma_Z')^{-2}\}$$

neglecting possible higher powers of $\kappa=|\sigma_N'-\sigma_Z'|/(\sigma_N'+\sigma_Z')$. Hence

$$\begin{aligned}\delta&=a_sC_\sigma\{(\sigma_N'-\sigma_Z')^2(\sigma_N'+\sigma_Z')^{-2}-(\sigma_N-\sigma_Z)^2(\sigma_N+\sigma_Z)^{-2}\}\\&=a_{\tau s}(\kappa/\lambda)^2\{1-(\lambda/\kappa)^2(N-Z)^2A^{-2}\}.\end{aligned} \quad (9)$$

The idealized model of Goldhaber and Teller implies that $\lambda=\kappa=1$ since the proton density is uniform and the extruded layer of neutrons contains no impurity of protons. In this case ϵ is exactly independent of A and Z and we obtain from Eqs. (5) and (9) with $\epsilon=(3/4)r_0$ the result that $a_{\tau s}/a_{\tau v}=0.28$. Actually, one would expect a value of κ somewhat less than unity and in that case one obtains

$$\frac{a_{\tau s}}{a_{\tau v}}=\frac{3\epsilon}{8r_0}\cdot\frac{\lambda^2}{\kappa^2}\cdot\left\{\frac{1-(N-Z)^2A^{-2}}{1-(\lambda/\kappa)^2(N-Z)^2A^{-2}}\right\}. \quad (10)$$

The factor in brackets is nearly independent of κ and may be neglected. Hence for $\kappa<1$

$$a_{\tau s}/a_{\tau v}=0.28(0.86)^2\kappa^{-2}>0.21. \quad (11)$$

An upper bound for $a_{\tau s}/a_{\tau v}$ has been found by Feenberg.[9] The most stable value of A for each Z is estimated from nuclear abundance data and $a_{\tau v}$ calculated. The resulting values of $a_{\tau v}$ are not strictly constant but show a small monotonic variation with Z, which is attributed in part to the surface symmetry term. In this way Feenberg finds that $0<a_{\tau s}/a_{\tau v}<0.6$. On introducing certain specific assumptions about the semi-empirical constants, the upper bound can be reduced to 0.4. One may tentatively set $a_{\tau s}/a_{\tau v}=0.3$ so that $a_{\tau s}\approx6$ Mev. Since $a_s\approx15$ Mev, it follows that the nuclear surface tension undergoes a decrease of about two percent from one end of the periodic table ($N=Z$) to the other (U^{238}) because of symmetry effects. This justifies the previous neglect of the surface symmetry term in Eq. (1).

[4] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **71**, 3 (1947); **73**, 1156 (1948).

[5] McElhinney, Hanson, Becker, Duffield, and Diven, Phys. Rev. **75**, 542 (1949).

[6] This term is discussed in R. D. Present, Phys. Rev. **72**, 7 (1947); footnote 7 of this paper is relevant to the application to C^{12}.

[7] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[8] E. Feenberg, Phys. Rev. **59**, 593 (1941).

* It is assumed that σ_N' and σ_Z' are constant over each hemisphere and that their values are exchanged at the equator.

[9] E. Feenberg, Rev. Mod. Phys. **19**, 239 (1947).