Selection Rules for the Dematerialization of a Particle into Two Photons

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Selection rules governing the disintegration of a particle into two photons are derived from the general principle of invariance under rotation and inversion. The polarization state of the photons is completely fixed by the selection rules for initial particles with spin less than 2. These results which are independent of any specific assumption about the interactions may possibly offer a method of deciding the symmetry nature of mesons which decay into two photons.

I. INTRODUCTION

It has been pointed out[1] that a positronium in the ^3S state cannot decay through annihilation with the emission of two photons. Recent calculation[2] shows that also a vector or a pseudovector neutral meson cannot disintegrate into two photons. It is the purpose of the present paper to show that these facts are immediate consequences of certain selection rules which can be derived from the general principle of invariance under space rotation and inversion.

These selection rules also yield information on the polarization state of the two photons emitted. In particular, one concludes that the two photons resulting from the annihilation of slow positrons in matter always have their planes of polarization perpendicular to each other.[3] This has been pointed out by Wheeler who also proposed a possible experimental verification.[4]

An especially interesting consequence of these selection rules is that they could conceivably offer a means of studying the nature of particles which dematerialize into two photons. If, for example, neutral mesons are found which disintegrate into two photons,[5] one would conclude that they cannot be vector or pseudovector mesons. Besides, as will be apparent from the selection rules, the two disintegration photons from a scalar particle always have parallel planes of polarization while those from a pseudoscalar particle always have mutually perpendicular planes of polarization. An experimental determination of the relative orientation of the planes of polarization of the two disintegration photons would therefore decide whether the neutral meson is a scalar or a pseudoscalar meson.

In Section II we shall give a simple but mathematically somewhat incomplete treatment of the symmetry nature of a state of two photons propagating in opposite directions, the detailed mathematical treatment being discussed in Section IV. The selection rules are derived in Section III and are based on the symmetry nature of the two photon states discussed in Section II. In the last section the parity of mesons and of the positronium is discussed.

TABLE I. Eigenvalues of the rotations $\mathcal{R}_\varphi$, $\mathcal{R}_\xi$, and the inversion $\mathcal{P}$ for the four polarization states.

	$\Psi^{RR}+\Psi^{LL}$	$\Psi^{RR}-\Psi^{LL}$	Ψ^{RL}	Ψ^{LR}
$\mathcal{R}_\phi$ (rotation around z axis through an angle φ)	1	1	$e^{2i\varphi}$	$e^{-2i\varphi}$
$\mathcal{R}_\xi$ (rotation around x axis through 180°)	1	1		
$\mathcal{P}$ (inversion)	1	-1	1	1

II. BEHAVIOR OF THE STATE OF TWO PHOTONS UNDER ROTATION AND INVERSION

Consider two photons of equal wave-length λ_0 propagating in opposite directions along the z axis. There are four such states which we shall denote by Ψ^{RR}, Ψ^{RL}, Ψ^{LR} and Ψ^{LL}. The first index refers to the circular polarization state of the photon propagating in the $+z$ direction, the second to that of the other photon. E.g., Ψ^{RL} would represent a state with a right circularly polarized photon propagating along the $+z$ axis and a left circularly polarized photon propagating along the $-z$ axis.

In order to investigate the behavior of these four states under a space rotation or an inversion let us first write down the electric field for a right circularly polarized electromagnetic wave propagating along the z axis,

$$(E_x)_+{}^R = E_0\cos(kz-\omega t+\delta_+{}^R),\qquad (E_y)_+{}^R = E_0\sin(kz-\omega t+\delta_+{}^R). \tag{1}$$

TABLE II. Circular polarization of disintegration photons.

parity \ J	0	1	2, 4, 6 ···	3, 5, 7 ···
even	$\Psi^{RR}+\Psi^{LL}$	forbidden	$\Psi^{RR}+\Psi^{LL}$, Ψ^{RL}, Ψ^{LR}	Ψ^{RL}, Ψ^{LR}
odd	$\Psi^{RR}-\Psi^{LL}$	forbidden	$\Psi^{RR}-\Psi^{LL}$	forbidden

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[1] J. A. Wheeler, Ann. N. Y. Acad. Sci. **48**, 219 (1946).
[2] S. Sakata and Y. Tanikawa, Phys. Rev. **57**, 548 (1948); R. Finkelstein, Phys. Rev. **72**, 415 (1947); J. Steinberger, Phys. Rev. **76**, 1180 (1949).
[3] They are not individually plane polarized. But if they are analyzed into plane polarized waves their planes of polarizations show the stated correlation.
[4] J. A. Wheeler, reference 1. See also M. H. L. Pryce and J. C. Ward, Nature **160**, 435 (1947), and Snyder, Pasternack and Hornbostel, Phys. Rev. **73**, 440 (1948). Experimental verification has been reported by E. Bleuer and H. L. Bradt, Phys. Rev. **73**, 1398 (1948).
[5] Bjorklund, Moyer, and York, Phys. Rev. **77**, 213 (1950).

TABLE III. Correlation of the planes (see reference 3) of polarization of disintegration photons ($\perp$ =planes of polarization perpendicular, $\|$=planes of polarization parallel).

parity \ J	0	1	2, 4, 6 ···	3, 5, 7 ···
even	$\|$	forbidden	$\|\geqq 50\%$ $\perp\leqq 50\%$	$\|$ 50% $\perp$ 50%
odd	$\perp$	forbidden	$\perp$	forbidden

For a right circularly polarized wave propagating in the opposite direction,

$$\begin{aligned}(E_x)_-{}^R&=E_0\cos(-kz-\omega t+\delta_-{}^R),\\(E_y)_-{}^R&=-E_0\sin(-kz-\omega t+\delta_-{}^R).\end{aligned}\tag{2}$$

Under a rotation through an angle φ around the z axis,

$$\begin{aligned}x&=x'\cos\varphi+y'\sin\varphi,\\y&=-x'\sin\varphi+y'\cos\varphi,\\z&=z'.\end{aligned}\tag{3}$$

We have

$$\begin{aligned}(E_x)_+{}^{R'}&=E_0\cos(kz-\omega t+\delta_+{}^R+\phi),\\(E_y)_+{}^{R'}&=E_0\sin(kz-\omega t+\delta_+{}^R+\phi);\end{aligned}\tag{4}$$

$$\begin{aligned}(E_x)_-{}^{R'}&=E_0\cos(-kz-\omega t+\delta_-{}^R-\varphi),\\(E_y)_-{}^{R'}&=-E_0\sin(-kz-\omega t+\delta_-{}^R-\phi).\end{aligned}\tag{5}$$

Thus the phase of a right circularly polarized wave along the z axis changes by $+\varphi$ while that of a right circularly polarized wave along the $-z$ axis changes by $-\varphi$ under the rotation. For the quantum state Ψ^{RR} the total phase factor is the product of the two phase factors of the two photons. (This will become evident in Section IV.) Hence we conclude that the state Ψ^{RR} is an eigenstate of the rotation (3) with the eigenvalue 1.

Mathematically the states are changed under the rotation (3) by a unitary transformation which we shall call $\mathcal{R}_\phi$. We conclude that:

$$\mathcal{R}_\varphi\Psi^{RR}=\Psi^{RR}.\tag{6}$$

Similar conclusions are reached for a rotation around the x axis through 180° and for an inversion. We summarize the results in Table I.

It is of course evident that the angular momentum along the z axis for the different states is related to the eigenvalue of $\mathcal{R}_\varphi$ in the usual way.

III. SELECTION RULES

The selection rules governing the disintegration of a particle into two photons follow immediately from Table I. We take the center-of-mass reference system and take the z axis along the direction of one of the outgoing photons.

(i) For an odd initial state the only possible mode of disintegration is to go into the state $\Psi^{RR}-\Psi^{LL}$. For an even initial state the three final states Ψ^{RL}, Ψ^{LR}, and $\Psi^{RR}+\Psi^{LL}$ are possible.

(ii) For an initial state with total angular momentum $J=1, 3, 5\cdots$ the only possible final states are Ψ^{RL} and Ψ^{LR}. This is so because $\Psi^{RR}+\Psi^{LL}$ and $\Psi^{RR}-\Psi^{LL}$ are both simultaneous eigenstates of $\mathcal{R}_\varphi$ and $\mathcal{R}_\xi$ with eigenvalues one, while the initial state that is an eigenstate of $\mathcal{R}_\varphi$ with eigenvalue one has the rotation properties of the spherical harmonics Y_{J0} and therefore changes sign under $\mathcal{R}_\xi$ for $J=1, 3, 5\cdots$.

(iii) For an initial state with $J=0$, 1 the only possible final states are $\Psi^{RR}+\Psi^{LL}$ and $\Psi^{RR}-\Psi^{LL}$, because the states Ψ^{RL} and Ψ^{LR} have angular momentum values $\pm 2\hbar$ along the z axis, which is too big for $J=0$ or 1.

These results are summarized in Table II.

It will be shown in the next Section that

(i) $\Psi^{RR}+\Psi^{LL}$ represents two photons with their planes of polarization always[3] parallel;

(ii) $\Psi^{RR}-\Psi^{LL}$ represents two photons with their planes of polarization always perpendicular; and

(iii) Ψ^{RL} and Ψ^{LR} both represent two photons with their planes of polarization 50 percent of the time parallel and 50 percent of the time perpendicular.

These facts combined with Table II lead to the conclusions summarized in Table III concerning the correlation of the planes of polarization of the disintegration photons.

IV. SPACE ROTATION AND INVERSION IN QUANTUM ELECTRODYNAMICS

In the electromagnetic field and the meson field, the number of particles is not a constant of motion. A complete formulation of the principle of invariance can only be made with the quantized field theory. Let us first consider the electromagnetic field described by the vector potential $\mathbf{A}(xyz)$. These are operators operating on state vectors Ψ which are usually represented as functions of occupation numbers. Under a space rotation ρ defined by

$$x_i=\sum_{j=1}^{3}\rho_{ij}x_j'\quad(x_1=x,\,x_2=y,\,x_3=z),\tag{7}$$

the operators $\mathbf{A}(xyz)$ and the wave function Ψ undergo a unitary transformation $\mathcal{R}_\rho$ and the invariance under rotation requires that

$$\mathcal{R}_\rho A_i(xyz)\mathcal{R}_\rho{}^{-1}=\sum_{j=1}^{3}\rho_{ij}A_j(x'y'z').\tag{8}$$

It is of course further required that the $\mathcal{R}$'s form a group isomorphic to the group of rotations.

To see in detail what this means let us expand the vector potential $\mathbf{A}$ into plane waves as usual:

$$\mathbf{A}(xyz)=\sum_{\mathbf{k}}\sum_{\lambda=1}^{2}(2\pi\hbar c/vk)^{\frac{1}{2}}\mathbf{e}_{k\lambda}(a_{k\lambda}e^{i\mathbf{k}\cdot\mathbf{r}}+a_{k\lambda}{}^*e^{-i\mathbf{k}\cdot\mathbf{r}}),\tag{9}$$

where $\mathbf{e}_{k1}$ and $\mathbf{e}_{k2}$ are two unit vectors forming with $\mathbf{k}/k$ a right-handed orthogonal system of unit vectors. It will be more convenient to use circular polarization for the study of rotation and we define

$$(\mathbf{e}_{k1}+i\mathbf{e}_{k2})/\sqrt{2}=\mathbf{e}_k{}^L,\quad(\mathbf{e}_{k1}-i\mathbf{e}_{k2})/\sqrt{2}=\mathbf{e}_k{}^R;\tag{10}$$

$$(a_{k1}-ia_{k2})/\sqrt{2}=a_k{}^L,\quad(a_{k1}+ia_{k2})/\sqrt{2}=a_k{}^R.\tag{11}$$

TABLE IV. The parity of particles at rest.

Scalar meson	Vector meson	Pseudovector meson	Pseudoscalar meson	Positronium in ${}^1S$ and ${}^3S$ states
even	odd	even	odd	odd

Evidently

$$\mathbf{A}(xyz)=\sum_{\mathbf{k}}(2\pi\hbar c/vk)^{\frac{1}{2}}(\mathbf{e}_k{}^L a_k{}^L e^{i\mathbf{k}\cdot\mathbf{r}}+\mathbf{e}_k{}^{L*}a_k{}^{L*}e^{-i\mathbf{k}\cdot\mathbf{r}})$$
$$+\sum_{\mathbf{k}}(2\pi\hbar c/vk)^{\frac{1}{2}}(\mathbf{e}_k{}^R a_k{}^R e^{i\mathbf{k}\cdot\mathbf{r}}+\mathbf{e}_k{}^{R*}a_k{}^{R*}e^{-i\mathbf{k}\cdot\mathbf{r}}). \quad (12)$$

The operators $a_{k\lambda}$, $a_k{}^R$ and $a_k{}^L$ all satisfy the usual commutation relations

$$a_k{}^R a_k{}^{R*}-a_k{}^{R*}a_k{}^R=1, \quad \text{etc.}$$

We are particularly interested in those modes of electromagnetic waves propagating along the $+z$ or the $-z$ direction with a definite wave-length, There are four such modes and we shall write their a operators as $a_+{}^R$, $a_+{}^L$, $a_-{}^R$ and $a_-{}^L$; $+$ and $-$ meaning the direction $+z$ or $-z$ of propagation. For definiteness we choose the phases of those modes such that the $\mathbf{e}_{k1}$, $\mathbf{e}_{k2}$ vectors in Eqs. (10) have as their xyz components:

$$\mathbf{e}_{+,1}=\mathbf{e}_{-,1}=(1,0,0), \quad \mathbf{e}_{+2}=-\mathbf{e}_{-,2}=(0,1,0). \quad (13)$$

With the operators a one can express in a very convenient form the states Ψ^{RR}, Ψ^{RL}, Ψ^{LR} and Ψ^{LL} defined in Section II.

$$\begin{aligned}\Psi^{RR}&=a_+{}^{R*}a_-{}^{R*}\Psi_{00\cdots}, & \Psi^{RL}&=a_+{}^{R*}a_-{}^{L*}\Psi_{00\cdots},\\ \Psi^{LR}&=a_-{}^{R*}a_+{}^{L*}\Psi_{00\cdots}, & \Psi^{LL}&=a_+{}^{L*}a_-{}^{L*}\Psi_{00\cdots},\end{aligned} \quad (14)$$

where $\Psi_{00\cdots}$ is defined to be the state with no photons.

We make a digression here to prove the statement at the end of the last section about the correlation in the planes of polarization of the two photons for the states $\Psi^{RR}+\Psi^{LL}$, $\Psi^{RR}-\Psi^{LL}$, Ψ^{RL} and Ψ^{LR}. By (14) and (11),

$$\begin{aligned}\Psi^{RR}+\Psi^{LL}&=(a_+{}^{R*}a_-{}^{R*}+a_+{}^{L*}a_-{}^{L*})\Psi_{00\cdots}\\ &=(a_{+,1}{}^*a_{-,1}{}^*-a_{+,2}{}^*a_{-,2}{}^*)\Psi_{00\cdots},\\ \Psi^{RR}-\Psi^{LL}&=-i(a_{+,1}{}^*a_{-,2}{}^*+a_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots},\\ \Psi^{RL}&=\tfrac{1}{2}(a_{+,1}{}^*a_{-,1}{}^*+a_{+,2}{}^*a_{-,2}{}^*\\ &\quad+ia_{+,1}{}^*a_{-,2}{}^*-ia_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots},\\ \Psi^{LR}&=\tfrac{1}{2}(a_{+,1}{}^*a_{-,1}{}^*+a_{+,2}{}^*a_{-,2}{}^*\\ &\quad-ia_{+,1}{}^*a_{-,2}{}^*+ia_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots}.\end{aligned} \quad (15)$$

Noticing that e.g., $a_{+,1}{}^*a_{-,1}{}^*\Psi_{00\cdots}$ represents a state with two photons with parallel planes of polarization one completes the proof with no difficulty.

Returning to the investigation of the behavior of the states Ψ^{RR}, Ψ^{RL}, etc., under rotation let us consider the rotation around the z axis through an angle φ, as defined by (3). Substitution of (13) and (12) into (8) shows[6] that

$$\begin{aligned}\mathcal{R}_\varphi a_+{}^R\mathcal{R}_\varphi{}^{-1}&=e^{-i\varphi}a_+{}^R, & \mathcal{R}_\varphi a_+{}^L\mathcal{R}_\varphi{}^{-1}&=e^{+i\varphi}a_+{}^L,\\ \mathcal{R}_\varphi a_-{}^R\mathcal{R}_\varphi{}^{-1}&=e^{+i\varphi}a_-{}^R, & \mathcal{R}_\varphi a_-{}^L\mathcal{R}_\varphi{}^{-1}&=e^{-i\varphi}a_-{}^L.\end{aligned} \quad (16)$$

These and similar equations for the other annihilation operators and for a general rotation ρ determine the operators $\mathcal{R}_\rho$ if the additional condition is imposed that $\mathcal{R}_\rho$ form a group. It is not difficult to prove that

$$\mathcal{R}_\rho\Psi_{00\cdots}=\Psi_{00\cdots}. \quad (17)$$

We can now prove Eq. (6) which asserts that Ψ^{RR} is an eigenstate of $\mathcal{R}_\varphi$ with eigenvalue 1. Take the Hermitian conjugate of (16) and multiply from the right by $\mathcal{R}_\varphi$:

$$\mathcal{R}_\varphi a_+{}^{R*}=e^{i\varphi}a_+{}^{R*}\mathcal{R}_\varphi, \quad \mathcal{R}_\varphi a_-{}^{R*}=e^{-i\varphi}a_-{}^{R*}\mathcal{R}_\varphi.$$

Hence,

$$\mathcal{R}_\varphi a_+{}^{R*}a_-{}^{R*}=a_+{}^{R*}a_-{}^{R*}\mathcal{R}_\varphi.$$

Operating on $\Psi_{00\cdots}$ with this last equation and making use of (17) and (14) one proves (6).

The other conclusions tabulated in the first row of Table I can be obtained in similar ways. For the rotation ξ:

$$x=x', \quad y=-y', \quad z=-z',$$

we have

$$\begin{aligned}\mathcal{R}_\xi a_+{}^R\mathcal{R}_\xi{}^{-1}&=a_-{}^R, & \mathcal{R}_\xi a_+{}^L\mathcal{R}_\xi{}^{-1}&=a_-{}^L,\\ \mathcal{R}_\xi a_-{}^R\mathcal{R}_\xi{}^{-1}&=a_+{}^R, & \mathcal{R}_\xi a_-{}^L\mathcal{R}_\xi{}^{-1}&=a_+{}^L.\end{aligned} \quad (18)$$

These lead to the results in the second row of Table I. Under an inversion the states are transformed by a unitary transformation $\mathcal{P}$ satisfying

$$\mathcal{P}\mathbf{A}(xyz)\mathcal{P}^{-1}=-\mathbf{A}(-x,-y,-z). \quad (19)$$

It is further required that

$$\mathcal{P}^2=1. \quad (20)$$

Expanding (19) into Fourier components we obtain

$$\begin{aligned}\mathcal{P}a_+{}^R\mathcal{P}^{-1}&=-a_-{}^L, & \mathcal{P}a_+{}^L\mathcal{P}^{-1}&=-a_-{}^R,\\ \mathcal{P}a_-{}^R\mathcal{P}^{-1}&=-a_+{}^L, & \mathcal{P}a_-{}^L\mathcal{P}^{-1}&=-a_+{}^R.\end{aligned} \quad (21)$$

It is to be noticed that (19) and (20) together do not completely determine the operator $\mathcal{P}$, as a change of sign of $\mathcal{P}$ does not affect either equation. However, changing the sign of $\mathcal{P}$ merely means a change in the name-calling of the even and odd states and is of no physical consequence. For definiteness we shall fix the sign by calling the vacuum an even state:

$$\mathcal{P}\Psi_{00\cdots}=\Psi_{00\cdots}. \quad (22)$$

Equations (21) and (22) lead to the third row of Table I.

[6] Actually since the Maxwell's equations are of the second order in $\partial/\partial t$, one should write together with (8)

$$\mathcal{R}_\rho\frac{\partial A_i(xyz)}{\partial t}\mathcal{R}_\rho{}^{-1}=\sum_j \rho_{ij}\frac{\partial A_j(x'y'z')}{\partial t}. \quad \text{(A)}$$

The operators $\partial A_i/\partial t$ are expanded into Fourier series similar to (12). Equation (A) together with (8) give Eq. (16).

V. PARITY OF MESONS AND THE POSITRONIUM

The above method for obtaining the symmetry nature of photon states can be easily extended to the meson field. In particular, if a_0 is the annihilation operator for a scalar meson at rest it is easy to see in analogy with Eq. (21) that

$$\mathcal{P}a_0\mathcal{P}^{-1}=a_0. \tag{23}$$

There is no change of sign of a_0 (see Eq. (21)) because the scalar meson field, unlike the vector potential of the electromagnetic field, retains its sign under an inversion. If we again call the state of no meson an even state, it is evident that a state with one scalar meson at rest is also even. This and similar conclusions concerning the parity of the vector, the pseudoscalar and pseudovector mesons are summarized in Table IV.

With the electron-positron field the situation is quite similar. The behavior of the field $\psi_i(xyz)$ under rotation and inversion is evidently given by

$$\mathcal{R}_\rho\psi_i(xyz)\mathcal{R}_\rho{}^{-1}=\sum_{j=1}^{4} S_{ij}(\rho)\psi_j(x'y'z'), \tag{24}$$

$$\mathcal{P}\psi_i(xyz)\mathcal{P}^{-1}=\sum_{j=1}^{4} \beta_{ij}\psi_j(x'y',z'), \tag{25}$$

where $S_{ij}(\rho)$ represents the spinor transformation corresponding to the rotation ρ and β_{ij} are the elements of Dirac's β-matrix.

If we expand ψ into plane waves and consider the particular mode representing an electron at rest in a positive or negative energy state it is evident that (25) shows

$$\mathcal{P}b_{0+}\mathcal{P}^{-1}=b_{0+},\quad \mathcal{P}b_{0-}\mathcal{P}^{-1}=-b_{0-}, \tag{26}$$

where b_{0+} and b_{0-} are the annihilation operators for an electron at rest with positive and negative energy values, respectively. The negative sign in (26) comes from the operation on ψ with the β-matrix. It is therefore evident that an electron-positron pair, both at rest, has an odd parity. Here, as before, we adopt the convention that the state of vacuum is to be called even.

Extension of the above argument to the ^1S and ^3S states of the positronium is evident. One gets for both states an odd parity. As mentioned in the introduction, Wheeler has pointed out[1] that the annihilation photons from the ^1S state of the positronium always have mutually perpendicular[3] planes of polarization. We see from Table III that the assignment of an odd parity to the ^1S state of positronium leads directly to the same conclusion.

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Note added in proof.—Some of the results of this paper have been obtained by L. D. Landau, Dokl. Akad. Nawk., USSR **60**, 207–209 (1948). See a summary in English in Phys. Abstracts **A52**, 125 (1949).