The Cross Section for the Radiative Capture of Protons by C^{12} near 100 Kev

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(Received September 15, 1949)

A low voltage accelerator and high current ion source has been used to determine the cross section of the reaction $C^{12}(p\gamma)N^{13}$ over the energy range from 88 to 128 kev. A counter arrangement is described which detects 26 percent of all the positrons from the decay of the N^{13} produced in the reaction and which has a low background rate of 5.5 counts per minute. With this accelerator and detector, yields of the order of 10^{-16} positron per proton and cross sections as low as 10^{-10} barn or 10^{-34} cm² can be measured with errors of the order of ±20 percent. The cross section for the $C^{12}(p\gamma)N^{13}$ reaction has been found to fit the semi-empirical expression $\sigma=0.0024E^{-1}\exp[-6E^{-\frac{1}{2}}]$ barn with E in Mev over the energy range measured. This is in satisfactory agreement with the Breit-Wigner one-level dispersion formula using constants determined at the 456-kev resonance. The astrophysical implications of these results in connection with the carbon-nitrogen cycle of nuclear reactions in stellar interiors are discussed.

INTRODUCTION

IN 1938 H. A. Bethe[1] showed that the most important source of energy in ordinary stars is the nuclear reactions of carbon and nitrogen with protons. These reactions which form a cycle in which the original nucleus is reproduced are as follows:

$$\begin{array}{ll} C^{12}+H^1\rightarrow N^{13}+\gamma, & N^{13}\rightarrow C^{13}+\beta^++\nu, \\ C^{13}+H^1\rightarrow N^{14}+\gamma, & \\ N^{14}+H^1\rightarrow O^{15}+\gamma, & O^{15}\rightarrow N^{15}+\beta^++\nu. \\ N^{15}+H^1\rightarrow C^{12}+He^4 & \end{array} \tag{1}$$

Thus the carbon and nitrogen isotopes serve as catalysts for the combination of four protons into an alpha-particle, two positrons (β^+) and presumably two neutrinos (ν). The mass energy difference of 28 Mev per cycle (0.8 percent of the mass energy of the four protons involved) appears in the form of gamma-radiation (γ) or as kinetic energy of the alpha-particle, positrons, and neutrinos. Only 1.5 Mev is lost as kinetic energy of the neutrinos and is thus not convertible by presently known mechanisms except inverse β-decay into the general energy content of the star.

Of the nuclear reactions which might serve as sources of nuclear energy those involving hydrogen are favored by its large abundance, by its large mass excess which makes a considerable energy evolution possible, and by its small charge and mass which enable it to penetrate easily through nuclear potential barriers. Neutrons, with no barriers, react too rapidly to yield observed stellar lifetimes! Of the reactions involving hydrogen, the carbon-nitrogen processes are unique in their cyclical character. In general nuclei lighter than carbon are permanently destroyed by alpha-particle emission in reactions with protons and those heavier disappear through radiative capture of the protons and the formation of heavier stable nuclei.

On the basis of relatively meager experimental evidence available at the time on nuclear reaction rates or cross sections, Bethe showed that the process described above is the only one consistent with the known evolution of energy in the bright stars of the main sequence, including the sun, and with the central temperature of these stars as calculated by integration of the Eddington equations ($\sim 2\times 10^7$ degrees). For fainter stars with lower central temperatures the reaction

$$H^1+H^1\rightarrow D^2+\beta^++\nu, \tag{2}$$

and the reactions following it were suggested[2] as being mainly responsible for the energy production.

The essential point of Bethe's argument cannot be questioned in spite of the fact that nuclear reaction rates at stellar temperatures can be only roughly estimated from existing experimental data at laboratory energies. However, it was felt that additional and more accurate experimental evidence should be obtained on these reactions, in particular at energies as close as possible to the effective stellar energies. It is the purpose of this paper to present such evidence on the first reaction given above, namely $C^{12}(p\gamma)N^{13}$.

EXPERIMENTAL ARRANGEMENTS

The proton energy at which a thermo-nuclear reaction is most probable in the carbon-nitrogen cycle is determined by the maximum in the product of the Maxwellian velocity distribution among the protons multiplied by the cross section for the reaction. This product reaches its maximum at roughly 30 kilovolts proton energy for stellar temperatures of 2×10^7 degrees when the charge of the capturing nucleus is 6 or 7. The cross sections (10^{-40} cm²) at this energy are far too small to be measured in the laboratory with existing techniques, so the data obtained at higher energies must be extrapolated to 30 kev. A low voltage accelerator shown in Fig. 1 powered by a conventional x-ray tube transformer-condenser rectified supply[3] has been used to extend the measurement of the cross section of $C^{12}(p\gamma)$

[1] H. A. Bethe, Phys. Rev. **55**, 103, 434 (1939); Astrophys. J. **94**, 37 (1940).

[2] H. A. Bethe and C. L. Critchfield, Phys. Rev. **54**, 248, 862 (1938).

[3] R. N. Hall, Rev. Sci. Inst. **19**, 905 (1948). The results reported in this paper were described briefly by W. A. Fowler and R. N. Hall, Phys. Rev. **74**, 1558(A) (1948).

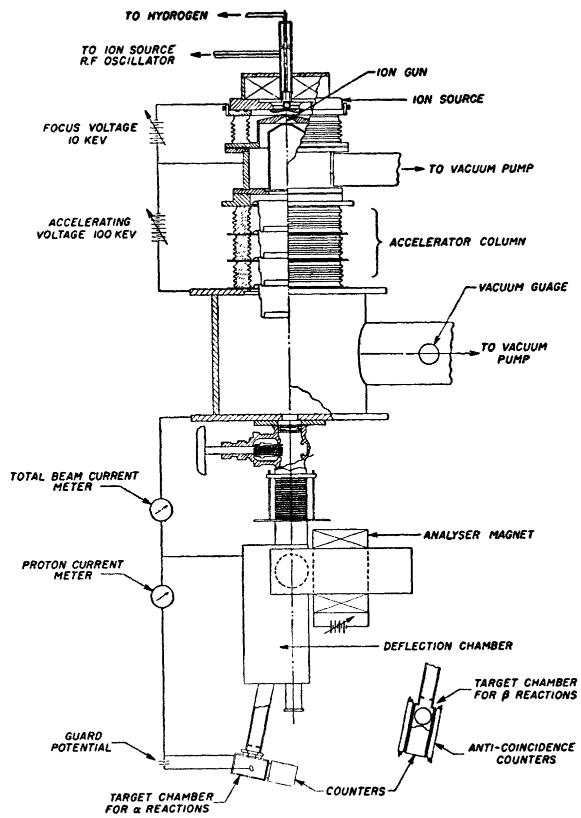

FIG. 1. Schematic arrangement of the 100-kev accelerator, high frequency ion source, magnetic analyzer, and detector used in these experiments.

down to bombarding energies as low as 88 kev, whereas previously reported data had been taken in the neighborhood of the 456-kilovolt resonance and above. This extension was made possible by the large current ($\sim$250 μa) of focused and monoenergetic protons which could be obtained from the radiofrequency ion source especially constructed for this purpose.[3]

The $C^{12}(p\gamma)N^{13}$ reaction has been studied by several investigators[4] in the energy region from 0.3 to 1.3 Mev. A well defined resonance exists at 456$\pm$2 kev having a proton width of 35 kev and a gamma-ray width of 0.63 electron volt.[5] The proton width corresponds to the measured breadth at half-maximum of the resonance, while the gamma-ray width, which is calculated from the cross section at resonance, represents the probability that radiation will follow capture of the incident proton. This information may be used in the dispersion formula to calculate the low energy cross section, but the results are open to question since they involve a considerable extrapolation from resonance. In the first place, higher energy resonances may contribute appreciably to the cross section. In the second place, the simple one-level dispersion formula itself is not necessarily valid except in the neighborhood of a resonance.

The N^{13} produced in this reaction has a positron radioactivity with a 10.13$\pm$0.1 minute half-life.[6] The recent results of W. F. Hornyak, C. B. Dougherty, and T. Lauritsen[7] show that the positron spectrum has a simple Fermi distribution with a maximum energy of 1.202$\pm$0.005 Mev. No gamma-radiation other than that from the annihilation of the positrons has been established with certainty, indicating the absence of K-capture or of any other positron groups. The presence of C^{13} in the target does not give rise to any radioactivity since the N^{14} formed by the $C^{13}(p\gamma)$ reaction is stable. It follows that for every proton captured by C^{12}, there is emitted one positron which is accompanied by no other activity.

In order to obtain a measurable yield at as low an energy as possible, a large bombardment current, together with a high counting efficiency in the presence of a low background, is necessary. Using the proton source described in reference 3, magnetically analyzed proton currents of 250 μa were focused upon an area of about two square millimeters of the target.

Since the counting efficiency for the 2.8-Mev gamma-rays produced in the $C^{12}(p\gamma)N^{13}$ is low ($\sim$1.5 percent) the yield was determined by counting the positrons emitted by the N^{13}. It is essentially the fact that a radioactive isotope is produced in this reaction which makes establishment of the cross section at a very low value possible. Due to the high background produced by x-rays and sparking during bombardment, it was necessary to measure the induced radioactivity after the beam was turned off.

The target and detector arrangement shown in Fig. 2 was used for these measurements. The carbon target, consisting in a few cases of a soot deposit and in most cases of a graphite disk, was mounted on the water-cooled brass target sphere. After the bombardment period, the sphere was rotated 180° so as to bring the target before the opening of a mica-window beta-ray counter sealed into the vacuum system. With this arrangement, a large solid angle was obtained with a minimum amount of absorbing material between the target and the sensitive volume of the counter.

The counter used was made by the Radiation Counter Laboratory, Chicago, Illinois, and was constructed with the sensitive volume reduced in length relative to that of their standard pressure seal mica window counter, Mark 1, Model 2. This reduction was accomplished by extending the glass sleeve surrounding the central wire to within a half-inch of the mica window (See Fig. 2). This modification reduced the background

[4] Cockcroft, Gilbert, and Walton, Proc. Roy. Soc. **A148**, 225 (1935); L. R. Hafstad and M. A. Tuve, Phys. Rev. **48**, 306 (1935); R. B. Roberts and N. P. Heydenburg, Phys. Rev. **53**, 374 (1938); Curran, Dee, and Petrzilka, Proc. Roy. Soc. **A169**, 269 (1938); Fowler, Lauritsen, and Lauritsen, Rev. Mod. Phys. **20**, 236 (1948); R. Tangen, Kgl. Norske Vid. Sels. Skrifter NR 1 (1946).

[5] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **76**, 314 (1949).

[6] K. Seigbahn and H. Slatis, Arkiv. f. Ast. Math. Fys. **32A**, No. 9 (1945).

[7] Hornyak, Dougherty, and Lauritsen, Phys. Rev. **74**, 1727 (1948).

counting rate by a factor of two without appreciably altering the sensitivity of the counter to beta-radiation incident on the window. Although the "plateau" of these counters extends only over an interval of 100 volts, they behave very stably under standard voltage regulation techniques.

To reduce the cosmic-ray background, the counter was surrounded by 2 inches of lead on the ends and underside, 4 inches on the sides, and $4\frac{1}{2}$ inches on top. A further reduction in the background was made by using a belt of ten anti-coincidence counters mounted as shown in Fig. 2. These measures reduced the background counting rate to $5\frac{1}{2}$ counts per minute. This extremely low rate was maintained in several counters over an extended period of time. Counting rates appreciably above background were measured above 90-kev bombarding energy. Bombarding energies were limited to a maximum of 128 kev by the high voltage rectifier arrangement used, and by breakdown of the accelerating column.

YIELD MEASUREMENTS

The thick target yield curve obtained is shown in Fig. 3. Different runs were made with different bombardment periods and proton currents. In order that all of the results might be plotted on the same curve for comparison, the data have been corrected to give the values which would have been obtained if the bombardment had been made for an infinite length of time using 100 μa of proton current. The results thus give the saturation yield in disintegrations per minute for this current or the counts per 6000 microcoulombs of protons. The probable error indicated represents the statistical fluctuation due to the small numbers of counts recorded. The curve which is drawn through the points is a calculated thick target yield curve fitted near 110 kev. It will be discussed in more detail in a later section.

Since the counting rates measured were only slightly above background, the statistical errors involved were large, and considerable thought was given to the problem of making the measurements in such a way as to obtain the greatest accuracy. A fundamental consideration was the fact that observations could be made only with the high voltage supply completely cut off. Methods of rapidly alternating bombarding and observing periods were not considered practical because of the necessity of stabilizing the voltage and focusing the beam for each run. It was arbitrarily decided to bombard for a reasonable period of time, usually three half-lives with the maximum available current. The problem then reduced to choosing the time of observation, T, so that the statistical error in the yield measurement was a minimum when background was taken into account by subtracting the average number of background counts expected during T from the observed number of counts. That T has an optimum value is clear from the fact that for small T a large error arises from the large random fluctuations in the true counts while for large T the absolute (not relative!) statistical fluctuations in the background introduce an excessive error.

To make the calculation more explicit we assume an average exponential decay rate $\bar{n}$ during observation

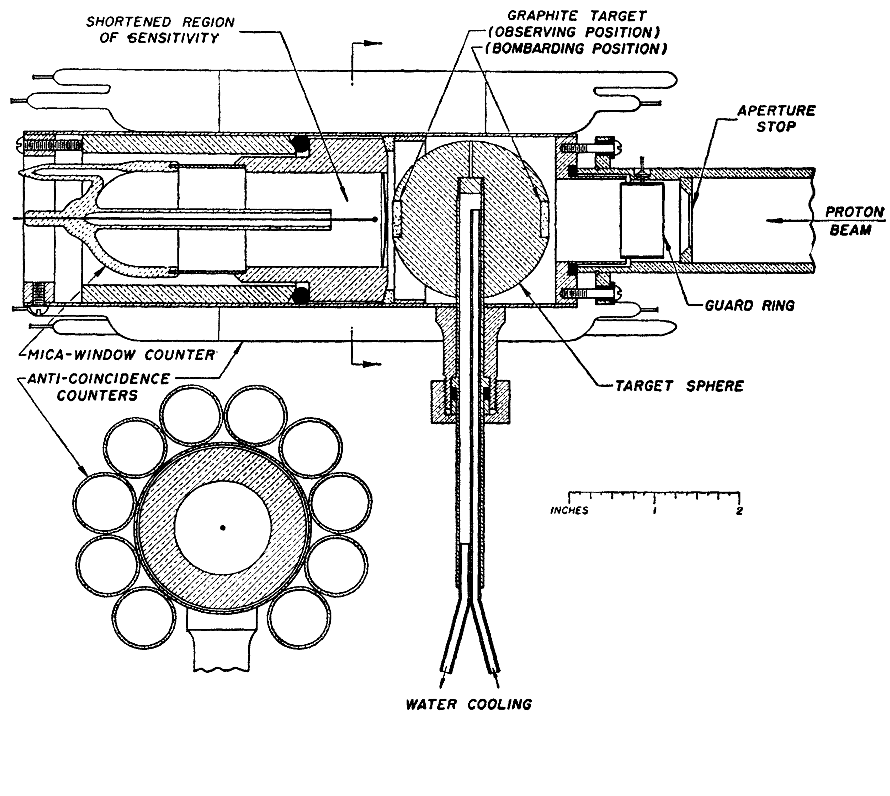

FIG. 2. Target support and detecting arrangement.

given by

$$\bar{n} = \bar{n}_0 e^{-t/\tau} \quad (3)$$

in the presence of an average background rate $\bar{n}_b$, which has been accurately determined in auxiliary measurements. The mean life of the activity, 14.6 minutes, is designated by τ. It is desired to determine as accurately as possible the average counting rate, $\bar{n}_0$, at the end of bombardment or the beginning of observation. The time elapsed between shutting off the high voltage and starting the counters was negligible. If $\bar{N}$ is the average total number of counts observed in time T, then $\bar{n}_0$ is given by

$$\bar{n}_0 = [\bar{N} - \bar{n}_b T]/[\tau(1-e^{-T/\tau})]. \quad (4)$$

The relative probable error, δ, in $\bar{n}_0$ is given by the square root of the sum of the squares of the errors in $\bar{N}$ and $\bar{n}_b T$ divided by the counts above background and is thus given by probability theory as

$$\delta = 0.6745 \frac{(\bar{N} + \bar{n}_b T)^{\frac{1}{2}}}{\bar{N} - \bar{n}_b T} = 0.6745 \frac{[2\bar{n}_b T + \bar{n}_0 \tau(1-e^{-T/\tau})]^{\frac{1}{2}}}{[\bar{n}_0 \tau^{\frac{1}{2}}(1-e^{-T/\tau})]}. \quad (5)$$

Choosing T so as to make the relative error a minimum, results in the equation

$$\frac{\bar{n}_0}{\bar{n}_b} = 2\frac{e^x - 1 - 2x}{1-e^{-x}}, \quad (6)$$

where x is the optimum T expressed in mean lives. The minimum error is given by

$$\delta_{\min} = 0.6745 \left[\frac{1}{2\bar{n}_b \tau} \frac{e^x - 1 - x}{(e^x - 1 - 2x)^2}\right]^{\frac{1}{2}} = 0.6745 \left[\frac{1}{\bar{n}_0 \tau} \frac{(e^x - 1 - x)(1-e^{-x})}{(e^x - 1 - 2x)}\right]^{\frac{1}{2}}. \quad (7)$$

The optimum counting time expressed in mean lives is determined by the initial decay rate divided by the background counting rate. Solution of the above equation for x indicates that for initial counting rates less than or equal to several times the background the minimum error is obtained if counts are recorded for approximately 1.4 mean lives or two half-lives. In this time three-quarters of the maximum possible number of counts will be recorded. Under these conditions the percentage error is approximately equal to 1.2 $(\bar{N} - \bar{n}_b T)^{-\frac{1}{2}}$ for $\bar{n}_0 \sim \bar{n}_b$ or approximately twice that expected for no background.

In order to obtain the thick target yield in disintegrations per proton, a number of corrections must be made. The initial decay rate given by Eq. (4) must be increased by a factor

$$\bar{n}_0(\infty)/\bar{n}_0(t) = 1/(1-e^{-t/\tau}) \quad (8)$$

to account for the fact that the equilibrium concentration of N^{13} nuclei was not attained during the finite bombardment time t. For t equal to three half-lives as used in most of these measurements

$$\bar{n}_0(\infty)/\bar{n}_0(t) = 1.14. \quad (8')$$

In what follows a factor f will be used to represent the fraction of the disintegration positrons which are registered by the counting circuits. This factor takes into account the solid angle subtended by the counter, the counter efficiency, backscattering in the target and the fraction of the positrons which are missed due to absorption in the mica window and to the N^{13} nuclei which escape from the target before disintegrating. Because of the low counting rates involved, a resolving time correction is not necessary. The equilibrium decay rate is, therefore, given by

$$\bar{n}_0(\infty) = \frac{\bar{N}_t - \bar{n}_b T}{\tau(1-e^{-T/\tau})(1-e^{-t/\tau})}, \quad (9)$$

and the thick target yield by

$$Y = \bar{n}_0(\infty)/fI \times 0.267 \times 10^{-14}, \quad (10)$$

where I is the analyzed proton current in micro-amps.

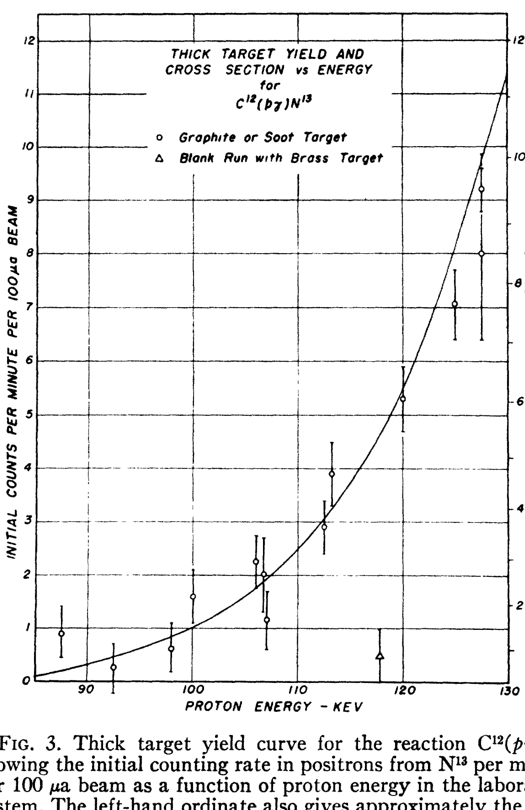

FIG. 3. Thick target yield curve for the reaction $C^{12}(p\gamma)N^{13}$ showing the initial counting rate in positrons from N^{13} per minute per 100 μa beam as a function of proton energy in the laboratory system. The left-hand ordinate also gives approximately the yield in positrons per 10^{16} protons over the whole sphere while the right-hand ordinate has been adjusted to give the cross section of the reaction.

It will be noted that $\bar{n}_0(\infty)$ is expressed in counts per minute. The left-hand ordinate of Fig. 3 gives the yield, Y, in disintegrations per 10^{16} protons to a good approximation.

DETERMINATION OF THE COUNTING EFFICIENCY

The factor f representing the fraction of disintegrations which are observed will now be discussed. The positron spectrum[7] shows that less than 1 percent of the positrons have energies below 100 kev. The mica window of the counter has a thickness of 2 mg/cm^2 which is capable of stopping only a small fraction of the positrons having this energy. Absorption in the counter window may, therefore, be neglected.

There is considerable evidence that only a small amount of N^{13} leaves the target. Roberts and Heydenburg[4] estimate that the counting rate is decreased about 10 percent by the loss of active nitrogen. In their investigation of the carbon reaction above 300 kev, Fowler, Lauritsen, and Lauritsen[4] found no evidence for any loss greater than 10 percent as long as the targets remained at room temperature under bombardment. In order to reduce any loss of nitrogen in the low voltage experiments, the target sphere was water-cooled. The same yields were obtained within experimental error for targets made of graphite, soot, and the black deposit formed on the brass surface of the sphere by prolonged bombardments. The fact that the yield is independent of the physical form of the carbon is some evidence that no large amount of active nitrogen leaves the target.

Evidence concerning the N^{13} loss can also be obtained from the rate of rise of activity during bombardment and from the rate of decay after bombardment. In these experiments most of the bombardment times were 30 minutes but these gave results not significantly different than the few taken at 10 and 20 minutes on the assumption of a 10-minute half-life. Records were kept of the counts after bombardment *versus* time in all individual measurements with the results that 649 ± 17 positrons were counted in the first ten minutes after bombardment, 283 ± 11 in the second ten minutes and 141 ± 8 in the third ten minutes. These results establish a half-life of 9 ± 1 minutes again consistent within 10 percent of the known value.

Back-scattering of a fraction of those positrons which leave the surface layer in the direction of the target away from the counter will increase the number of counts. This subject has been investigated experimentally for RaE β-rays by Schonland for normal angles of incidence and corrected by Chalmers for all angles as in an active surface layer on a backing. The results indicate reflection corrections of 26 percent for Al and 58 percent for Cu.[8] Since the RaE β-rays are not markedly different than those from N^{13} we estimate a correction of 15 percent for N^{13} in the carbon targets. As this correction is of the same order of magnitude and opposite to that for N^{13} loss we have not made any over-all correction for the two effects.

The effective aperture and counter efficiency were examined in the experiment illustrated in Fig. 4. The beta-particles from a RaD source were collimated by a lead canal so as to produce a beam having a width of about one millimeter. The alpha-particles were stopped by a thin aluminum foil so that the resulting beam consisted only of beta-particles having an energy comparable to that of the N^{13} positrons. The source and collimator could be rotated about an axis perpendicular

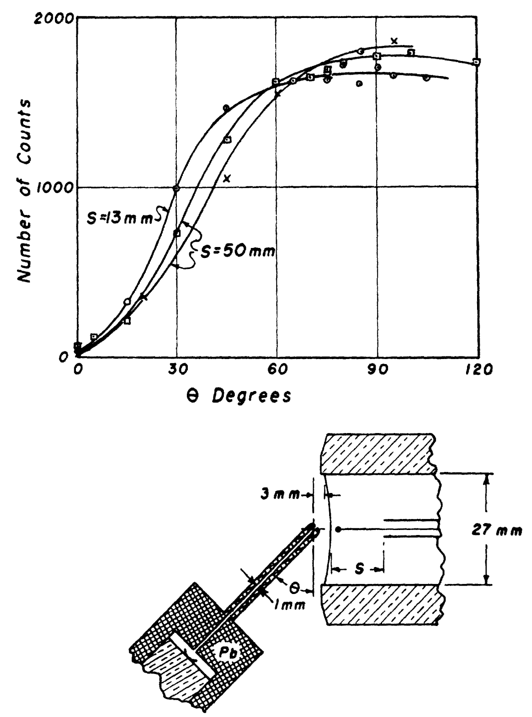

FIG. 4. Measurement of the counting efficiency. The sensitivity of mica-window beta-ray counters was explored with a collimated beam of RaD beta-particles. The special low background counter used in measuring the N^{13} activity had 13 mm of exposed central wire. Sensitivity curves for two standard counters having 50 mm of exposed wire are shown for comparison.

to the beam and to the axis of the counter. This axis was located 3 mm from the plane of the periphery of the mica window so that the particles from the beam entered the counter at the same angles as did the N^{13} positrons. The counting rate was measured as a function of the angle of incidence of the beam, giving the curves shown in Fig. 4. Three counters were measured in this fashion. The special low background counter used in the measurements of the carbon reaction and having 13 mm of exposed central wire was compared with two standard counters having 50 mm of exposed wire.

The maximum counting rate is nearly the same for each of the standard counters but is slightly less for the special low background counter. This difference indicates that the active volume of the low background

[8] Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1930), p. 420.

counter is sufficiently shallow that an appreciable fraction of the beta-particles pass through without being counted. It is also to be noted that the curve of the low background counter rises more sharply than do those of the standard counters. This arises from the fact that this counter was filled to slightly higher gas pressure. The curve for the low background counter reaches half-sensitivity at an angle of 27° corresponding to a path length within the counter of 7 mm. Since the active length of this counter is likely to be not much greater than the 13 mm length of exposed center wire, it is not unreasonable that an appreciable fraction of the particles pass through this counter without being counted. On the other hand, particles entering along the axis of the standard counters are almost certain to be counted.

The curve for the low background counter represents its counting efficiency as a function of the angle of incidence, the maximum of the standard counter curves corresponding to 100 percent efficiency. Using this curve as a weight function and integrating over all angles gives the effective aperture of the counter. A solid angle of 3.3 steradians is thus obtained corresponding to a counting efficiency of 26 percent on the assumption that the positrons are emitted with intensity independent of angle. The curves of Fig. 4, and hence the efficiency, were found to be nearly independent of the counter voltage within the limits of the plateau.

CROSS-SECTION CALCULATIONS

For purposes of comparison with the results obtained from the resonance in the $C^{12}(p\gamma)N^{13}$ reaction at 456 kev we use the Breit-Wigner one-level dispersion formula for the cross section, *viz*:

$$\sigma=\pi\lambda\!\!\!^{-2}\frac{\omega\Gamma_\gamma\Gamma_p}{(E_R-E)^2+\Gamma^2/4}, \tag{11}$$

where $\lambda\!\!\!^{-}$ = wave-length of incident protons in CM system, Γ_γ = gamma-ray width, Γ_p = proton width, $\Gamma=\Gamma_p+\Gamma_\gamma$, E = proton energy, E_R = proton energy at resonance.

The statistical factor ω is given by

$$\omega=[2J+1]/[(2s+1)(2i+1)],$$

where J = spin of compound nucleus, s = spin of incident particle, i = spin of target nucleus. For C^{12}, $i=0$ and hence for s wave protons, $J=s$ and $\omega=1$.

At low energies both Γ_p and Γ_γ are negligible compared with E_R and so

$$\sigma=\pi\lambda\!\!\!^{-2}[(\omega\Gamma_\gamma\Gamma_p)/(E_R-E)^2]. \tag{11'}$$

Γ_γ is nearly independent of E since it depends essentially on the quantum energy which increases only slowly with E. However Γ_p is proportional to the Gamow barrier penetration factor, P, and the proton velocity and can be written as

$$\Gamma_p=\gamma_p E^{\frac{1}{2}}P, \tag{12}$$

where γ_p is the width without barrier evaluated at a velocity corresponding to an energy of 1 Mev if E is in Mev.[9] It is independent of E.

For low energies P is given approximately by

$$P=(g/g+1)(E_B/E)^{\frac{1}{2}}\exp(-\pi g[(E_B/E)^{\frac{1}{2}}-4/\pi]), \tag{13}$$

where

$$g=\left(\frac{2\mu Z_1Z_0e^2R}{\hbar}\right)^{\frac{1}{2}}=0.312\left(Z_1Z_0A_1A_0\frac{A_1^{\frac{1}{3}}+A_0^{\frac{1}{3}}}{A_1+A_0}\right)^{\frac{1}{2}}$$

$$\approx 0.55Z_0^{\frac{1}{2}} \text{ for light elements under proton bombardment, and}$$

$$E_B=2m_0c^2\frac{Z_1Z_0}{A_0}\frac{A_1+A_0}{A_1^{\frac{1}{3}}+A_0^{\frac{1}{3}}}=1.02\frac{Z_1Z_0}{A_0}\frac{A_1+A_0}{A_1^{\frac{1}{3}}+A_0^{\frac{1}{3}}}\text{ Mev}$$

$$\approx 0.33Z_0 \text{ Mev for light elements under proton bombment.}$$

In the above expressions subscripts zero refer to target nuclei, subscripts one to incident particle, μ is the reduced mass of the system, R the radius at which nuclear forces set in, E_B is the barrier height or Coulomb potential at R and g is a function independent of E but not of the charge Z and atomic weight A of the interacting nuclei. E_B and E are measured in the laboratory system.

For any given reaction we can thus write Γ_p as a function of energy of the proton as

$$\Gamma_p=a\gamma_p\exp(-bE^{-\frac{1}{2}}), \tag{14}$$

where

$$a=\frac{g}{g+1}e^{4g}E_B^{\frac{1}{2}}\approx\frac{0.32Z_0}{0.55Z_0^{\frac{1}{2}}+1}\exp(2.2Z_0^{\frac{1}{2}}),$$

$$b=\pi gE_B^{\frac{1}{2}}\approx Z_0 \text{ Mev}.$$

Both a and b are independent of E. Since the term multiplying the exponential in a is ~ 1 for light elements under proton bombardment, we can write as a very rough approximation

$$\Gamma_p\approx\gamma_p\exp(-Z_0E^{-\frac{1}{2}}+2.2Z_0^{\frac{1}{2}}) \tag{14'}$$
$$=\gamma_p\exp(-6E^{-\frac{1}{2}}+5.4) \quad \text{for} \quad C^{12}+H^1.$$

The quantity $\pi\lambda\!\!\!^{-2}$ is given by

$$\pi\lambda\!\!\!^{-2}=(\pi\hbar^2/2\mu E)(A_1/A_0+1) \tag{15}$$
$$=0.65/A_1E(A_1/A_0+1)^2 \text{ barns},$$

so that

$$\sigma=(k\Gamma_\gamma\gamma_p/E)\exp(-bE^{-\frac{1}{2}}), \tag{16}$$

where

$$k=\frac{0.65}{A_1}\left(\frac{A_1+A_0}{A_0}\right)^2\frac{\omega a}{(E_R-E)^2}\sim\frac{\omega e^{2.2Z_0^{\frac{1}{2}}}}{E_R^2}.$$

[9] R. F. Christy and R. Latter, Rev. Mod. Phys. 20, 185 (1948).

k is substantially independent of E for E_R not in the stellar range.

The thick target yield is obtained by integrating the ratio of the disintegration cross section to the stopping cross section per disintegrable nucleus, ϵ, over the energy of the incident particle, *viz*:

$$Y=\int_0^E \frac{\sigma}{\epsilon} dE. \tag{17}$$

In the region near 100 kev the stopping cross section of protons in air is approximately constant and the stopping power of carbon relative to air is not greatly dependent on E so that ϵ can be taken as a constant and one obtains, substituting $b \approx Z_0$,

$$Y=2\sigma(E^{\frac{3}{2}}/\epsilon Z_0)(1-E^{\frac{1}{2}}/Z_0+\cdots). \tag{18}$$

From the yield one can thus determine σ as

$$\begin{aligned}\sigma&=Y(\epsilon Z_0/2E^{\frac{3}{2}})(1+E^{\frac{1}{2}}/Z_0+\cdots)\\ &=3Y(\epsilon/E^{\frac{3}{2}})(1+E^{\frac{1}{2}}/6+\cdots) \qquad \text{for } C^{12}+H^1 \text{ with } E \text{ in Mev}\end{aligned} \tag{18'}$$

and ϵ in Mev-cm^2. The stopping cross section for air[10] at 100 kev is 1.51×10^{-20} Mev-cm^2 while the stopping power of carbon is 0.96. The stopping cross section of carbon per C^{12} nucleus (98.9 percent) is thus $1.51\times0.96/0.989\times10^{-20}=1.46\times10^{-20}$ Mev-cm^2. In Fig. 3 the right-hand scale has been corrected to read directly in cross section using the coefficients of Y in the above expression for σ. The curve drawn through the data is of the form of (18) for Y or (16) for σ. It has been fitted over the lower half of the energy range and can be expressed with numerical coefficients as:

$$Y=5.6E^{\frac{1}{2}}(1-E^{\frac{1}{2}}/6)\exp(-6E^{-\frac{1}{2}})\times10^{-8}\beta^+/p, \tag{19}$$

or

$$\sigma=(0.0024/E)\exp(-6E^{-\frac{1}{2}}) \text{ barn}. \tag{20}$$

The cross section is 1.0×10^{-10} barn at 96 kev and rises to $8.5\pm1\times10^{-10}$ barn at 128 kev. The empirical curve is slightly high at 128 kev.

The cross section constants evaluated at the 456-kev resonance[5] are $\Gamma_\gamma=0.63$ ev, $\Gamma_p=35$ kev and $\gamma_p=1680$ kev. Using these constants one calculates a numerical coefficient of 0.0014 in Eq. (20). This agreement within a factor of two is satisfactory considering the single level approximation made above. As we have neglected the dependence of Γ_γ on E as well as the resonance denominator $(E_R-E)^{-2}$ in the approximations leading to (20) it is clear that the variation of σ with E is not completely described by the single level dispersion formula. The data are consistent with the inclusion of an interference term $(E_R-E)/[(E_R-E)^2+\Gamma^2/4]$ in σ but is not accurate enough to give quantitative details. Extrapolating expression (20) to lower energies, it is found that $\sigma=10^{-16}$ barn$=10^{-40}$ cm^2 at 30 kev.

EXPERIMENTAL ERRORS

A number of sources of error are involved, most of which may be placed within fairly well defined limits. Certain other sources of error are believed to be small but are not easily estimated; these are all such as to give too small a computed cross section.

Examination of Fig. 3 shows that the yield is very sensitive to the proton energy; a 1 percent error in the voltage measurement is equivalent to a 10 percent error in the yield. A Type K potentiometer was used to measure the voltage developed across a precision resistor by the current from a 120-megohm stack of high voltage resistors in parallel with the accelerator. The calibration depends upon the accuracy of the 20-megohm precision resistor (<1 percent error) which was used as a standard to measure the 120-megohm stack. This 120-megohm stack was constructed from 1 percent accuracy 1-megohm wirewound resistors and the measurement of each section agreed within a few tenths of one percent of the nominal value. The voltage was continuously adjusted during each run so as to maintain a steady value. Corona currents to junctions between the sections of the 120-megohm stack were eliminated by maintaining large clearances between the stack and other conductors, and by wrapping the junctions with dental dam. It is believed that errors in cross section due to inaccuracy in the voltage measurement amount to less than 10 percent.

The proton current meter was calibrated and introduces negligible uncertainty. The stopping cross section for carbon may be in error by 10 percent according to Livingston and Bethe.[10] The measurement of the fraction of counts recorded would appear to introduce only a few percent error.

Blank runs have been made using the brass target sphere as a target instead of carbon. It was necessary to install a dry ice trap in the vacuum system to reduce the deposition of carbon upon the brass. The trap decreased the pressure indicated by the ion gauge by a factor of four, and bombardment periods of over ten minutes could be used without forming a detectable carbon deposit. One such blank run is shown in Fig. 3.

The positron radiation is not believed to be accompanied by any other kind of radioactivity which could cause spurious counts. The counter pulses were watched on an oscilloscope to check on the possibility of counter breakdown. The counter plateau was checked at frequent intervals to make sure that the counter was performing properly. The negative results obtained from the blank runs further indicate that the counters were functioning properly. The guard-ring potential of the target holder was adjusted so as to be well above the value required to return all secondary electrons to the target.

The statistical errors involved are indicated in Fig. 3. If it is assumed that the slope of the thick target yield curve is given correctly by Eq. (18′) over the energy range explored, the cross section at 100 kev may be evaluated with an uncertainty of about 20 percent arising from statistical errors.

[10] M. S. Livingston and H. A. Bethe Rev. Mod. Phys. **9**, 245, (1937).

DISCUSSION

Any discussion of the astrophysical implications of the results reported here must begin by emphasizing the fact that these results constitute but a small fragment of the data which must be obtained in order to reach a detailed understanding of the operation of the carbon-nitrogen cycle in stellar interiors. Ideally one requires for such an understanding the variation of all the cross sections with energy over the entire range of energies which may be significant in the stellar problem say from a few kilovolts up to 100 kev. Only in this way will the contributions of resonances lying in this region be determined with certainty. This aspect of the problem has been discussed by Gamow and Teller.[11] In these experiments an attempt has been made to extend the direct cross-section measurements for one of the reactions of the carbon-nitrogen cycle to the lowest possible energies at which statistically reliable results could be obtained with the techniques developed and to compare these results with those calculated from constants determined at the 456-kev resonance using the Breit-Wigner single level dispersion formula. The measured cross section is somewhat less than twice that calculated and taking all possible sources of uncertainty into consideration one cannot exclude the possibility that the 456-kev resonance alone contributes to the yield near 100 kev. At the same time it is clear that in this region the effect of other resonances is at most of the same order of magnitude as that of the 456-kev resonance. Professor R. F. Christy has considered the implications of this fact in regard to stellar cross sections, bearing in mind that resonances in a limited region above the mean stellar energies may contribute markedly to stellar processes and yet not be more important in the thick target yield at 100 kev than the 456-kev resonance. He concludes that resonances which would completely invalidate the extrapolation of Eq. (20) to stellar temperatures cannot be excluded in the region from 10 to 40 kev. Since the average level spacing in the compound nucleus, N^{13}, for the low excitations available from $C^{12}+H^1$ is at least as great as 1 Mev experimentally, and with good theoretical justification, it can be argued that the chance of a level falling within this narrow region is small but this small chance must not be overlooked in discussions of the astrophysical implications. The experimental evidence is that no resonance levels other than the 456-kev resonance (2.34-Mev excitation in N^{13}) have been found in the $C^{12}(p\gamma)N^{13}$ reaction from 0.3 to 1.3 Mev* and no evidence of excited states in N^{13} has been found in the $C^{12}(dn)N^{13}$ reaction in spite of the fact that neutrons could be emitted accompanying excited states up to 2.0 Mev.

[11] G. Gamow and E. Teller, Phys. Rev. **53**, 608 (1938).

* A level at 1.697 Mev has been recently found by the MIT Nuclear Cross Sections Group (Progress Report, July 1, 1949). This level has a width of 74±9 kev and a thick target yield 1.3 times that at 456 kev. These results give $\gamma_p=127$ kev and $\Gamma_\gamma=1.3$ ev. The contribution to low energy cross sections is <1 percent that of the 456-kev level.

It has been pointed out by Fowler, Lauritsen, and Lauritsen that the single level resonance formula using constants determined at the 456-kev resonance gives a stellar cross section for $C^{12}(p\gamma)$ about 40 times that used by Bethe in his original calculations. The results reported here confirm this high value and actually indicate a somewhat larger cross section. The lifetime of the individual C^{12} nucleus under Bethe's conditions in the center of the sun is to be taken as $\sim 5\times 10^4$ years rather than 2.5×10^6 years.

The reaction $C^{13}(p\gamma)N^{14}$ shows resonance[5] at 554 kev with constants $\Gamma_\gamma=15$ ev, $\Gamma_p=40$ kev, and $\gamma_p=880$ kev. With these constants the *single-level* formula predicts a rate for this reaction 8.5 times that for $C^{12}(p\gamma)N^{13}$ at stellar temperature and thus an abundance ratio of 8.5:1 for $C^{12}:C^{13}$ in stellar interiors. The results reported here would lower this figure somewhat. It has not been found possible to measure the $C^{13}(p\gamma)$ cross section at low voltages because it is not followed by radioactivity. It will remain for investigations of such reactions as $C^{13}(dn)N^{14}$ with 3.0-Mev deuterons and $B^{11}(\alpha n)N^{14}$ with 10-Mev alpha-particles to reveal details of the energy levels in N^{14} in the relevant energy range. It is probable, however, that the single-level expression will not give correct results since the average level separation[12] in N^{14}, the compound nucleus, is certainly less than 0.5 Mev at the high excitation, ~8 Mev, produced by $C^{13}+H^1$. A considerably higher cross section for $C^{13}(p\gamma)$ at stellar energies than that given by the single level expression based on the level at 554 kev is not to be ruled out. Considerations of level density and resonance effects aside, the C^{13} cross section should be higher than that for C^{12} by the ratio of the cube of the quantum energies of the capture radiations assuming both to be electric dipole and with the same dipole moment. This ratio is $(8.07/2.33)^3=41$. The experimental radiation widths are in the ratio $15/0.63=24$. For this reason, as originally pointed out by Bethe, the terrestrial abundance $C^{12}:C^{13}=90$ and the abundance ratio observed in most stars $C^{12}:C^{13}\geqslant 50$ are not in contradiction with the carbon-nitrogen cycle. We must emphasize, however, that only by measurements at low energy can the cross sections and thus the abundances in stellar interiors be determined. In addition we must note that there is considerable question as to whether terrestrial abundances or those in stellar atmospheres should correspond to those in stellar interiors.

Experiments now under way in this laboratory may clarify these problems and this paper is to be considered a report on progress to date and a discussion of the experimental problems arising in the investigation of the carbon-nitrogen cycle. It is a pleasure to acknowledge discussions of the contents of this paper with Professors C. C. Lauritsen, R. F. Christy, Jesse L. Greenstein and Father J. D. O'Reilly.

This work was assisted by the Joint Program of the ONR and the AEC.

[12] T. Lauritsen, "Energy Levels of Light Nuclei," Preliminary Report No. 5, Nuclear Science Series, National Research Council.