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Nuclear Impulse in Electron Pair Creation*

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The nuclear impulses were studied for electron pairs produced in an air-filled cloud chamber by the continuous spectrum of x-rays from a 22-Mev betatron. Less than one-third of the momentum transfers to the nucleus were found to consist of values less than *mc*. The probability $\Phi(P_n)$ of a momentum transfer P_n was found to decrease rapidly with decreasing P_n for $P_n<mc$.

INTRODUCTION

IN pair production the momentum K of the photon is always greater than the sum of the momenta of the pair electron and positron, i.e.,

$$K>P_1+P_2. \tag{1}$$

A certain momentum $\mathbf{P}_n$ is transferred to the nucleus,[1] where

$$\mathbf{P}_n=\mathbf{K}-\mathbf{P}_p \quad \text{and} \quad \mathbf{P}_p=\mathbf{P}_1+\mathbf{P}_2. \tag{2}$$

The nuclear impulse will have its smallest value when all momenta are parallel; in this case, P_n is equal to

$$(P_n)_{\min}=\delta=K-P_P=K-P_1-P_2. \tag{3}$$

The distribution of the values of the nuclear momenta can be calculated theoretically.[2-4] These calculations indicate that most of the pair creations involve momentum transfer values between δ and mc to the nucleus and that the probability $\Phi(P_n)$ of a momentum transfer P_n is proportional to $1/P_n$ in this region. Groshev and Frank[5] and Groshev[6,7] investigated the formation of pairs in N, Kr, and Xe in a Wilson chamber by gamma-rays from ThC″ and computed the nuclear momentum for 76 of the pairs formed in N and for 29 formed in Kr. No other experimental work on nuclear impulse during pair formation has been reported in the literature.

METHOD

Photons from a betatron operating with a peak x-ray energy of 19.5 Mev were used to create electron pairs in an air-filled cloud chamber. The determination of the momentum of each pair particle from the cloud-chamber photographs is described in the preceding paper,[8] henceforth referred to as A. From the momenta of the pair components the energy of the incident photon was determined. Since the momenta of three of the four particles involved in each pair creation were known, the momentum of the fourth particle, the nucleus, could be calculated from (2).

For each pair, a rectangular set of axes (Fig. 1(a)) was employed having its origin at the origin of the pair, its y axis along the photon direction, and its z axis parallel to the magnetic field of the cloud chamber. The diagram in Fig. 1(a) shows the vectors $\mathbf{P}_1$ and $\mathbf{P}_2$ representing the momenta of the electron and the positron, respectively. The angles α and β are identical with those described in A. Also shown (Fig. 1(b)) are the nuclear momentum $\mathbf{P}_n$, the pair momentum vector $\mathbf{P}_p$, the angle θ_p between $\mathbf{K}$ and $\mathbf{P}_p$, and the angle θ_n between $\mathbf{K}$ and $\mathbf{P}_n$. The components of $\mathbf{P}_n$ in terms of α and β are:

$$\begin{aligned}(P_n)_x&=-(P_1\cos\alpha_1\sin\beta_1+P_2\cos\alpha_2\sin\beta_2),\\(P_n)_y&=K-(P_1\cos\alpha_1\cos\beta_1+P_2\cos\alpha_2\cos\beta_2),\\(P_n)_z&=-(P_1\sin\alpha_1+P_2\sin\alpha_2).\end{aligned} \tag{4}$$

From these components the values for P_n and θ_n were computed.

An important source of possible error in determining P_n was the uncertainty of the order of ±1° involved in the measurements of α and β. Since P_1 and P_2 increase with increasing x-ray energies, an error in angle measurement becomes more serious with increasing photon energies. A statement of the angle errors will be given in the discussion section.

Another source of possible error was introduced by

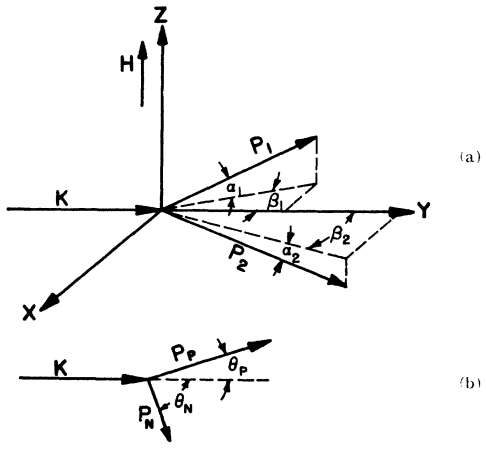

FIG. 1. Momentum vector diagrams.

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[1] Pair production in the field of the electron ("triplet" production) will not be considered here.

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TABLE I. Number of pairs as a function of photon energy.

Photon energy range (Mev)	Mean value of $h\nu$ (Mev)	Number of pairs Observed	Number of pairs Corrected for α-discrimination
1.02–2.99	2.4	41	91
3.00–4.99	4.1	129	209
5.00–7.99	6.5	266	362
8.00–10.99	9.6	263	313
11.00–13.99	12.5	216	244
14.00–16.99	15.4	154	166
17.00–19.50	18.2	54	56
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TABLE II. Numerical data obtained from Figs. 2 and 4.

Photon energy range (Mev)	Percentage of events with: $P_n < mc$	Percentage of events with: $P > 2.3mc$	Median value of P_n
1.02–2.99	16	13	1.6
3.00–4.99	25	32	1.6
5.00–7.99	29	27	1.5
8.00–10.99	27	25	1.6
11.00–13.99	31	32	1.5
14.00–16.99	23	32	1.6
17.00–19.50	27	29	1.6
1.02–19.50	27	28	1.6

the practice of discarding all pairs with either α_1 or α_2 greater than 20° (see the "α-discrimination" discussion in A). The fraction of pairs discarded increases with decreasing energy as shown in Table I, which lists the total number of pairs in each energy range corrected for the α-discrimination together with the number of pairs for which measurements were made. Thus, although the values of P_n computed by (4) were more accurate for low energies because of the smaller effect of possible angle errors, a greater portion of the total number of momentum transfers had to be computed by another method.

The values of P_n for the discarded pairs were obtained by observing the distribution of nuclear momenta for which β_1 or β_2 was greater than 20°. There is no reason to believe that the α- and β-distribution should be dissimilar. Values of nuclear momentum for pairs having more than one angle (α_1, α_2, β_1, or β_2) greater than 20° were more difficult to estimate, but they represented a small portion of the total (except for the lowest x-ray energy range) and had little effect.

The uncorrected curves (Figs. 2 and 3) represent those values of P_n computed from (4), whereas the corrected curves show, in addition, values of P_n corrected for α-discrimination as has been described. Other sources of discrimination described in A have no effect on the nuclear momentum distribution.

These arguments indicate that the values of P_n are most reliable in the middle energy ranges (at x-ray energies of the order 10 Mev).

RESULTS

Values of P_n and θ_n were computed for x-ray energies ($h\nu$) between 1.02 and 19.5 Mev. Table I lists the mean value of $h\nu$ for each interval and the number of pairs, corrected and uncorrected, in each interval. The distributions of P_n and θ_n are plotted for seven different ranges of $h\nu$ in Figs. 2 and 3. Corrected curves showing the nuclear momentum distribution for the entire $h\nu$ range from 1.02 to 19.5 Mev have been plotted in Fig. 4. Similarities in the nuclear momentum distributions (Figs. 2 and 4) may be observed with the aid of Table II. With the exception of the lowest energy range, each group fell within ±4 percent of having 25 percent of the momentum transfers below mc, and within ±3 percent of having 29 percent of the P_n values above 2.3 mc. With one other exception, each energy interval had 45±2 percent of the nuclear momentum between mc and 2.3 mc. In the range $h\nu = 11.0$–13.99 Mev, this

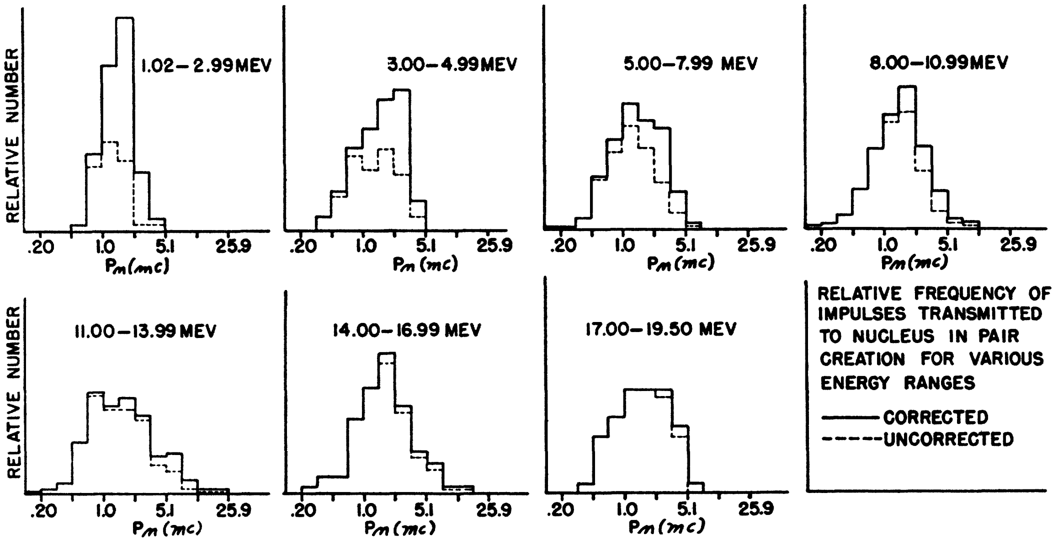


FIG. 2. Nuclear momentum distribution.

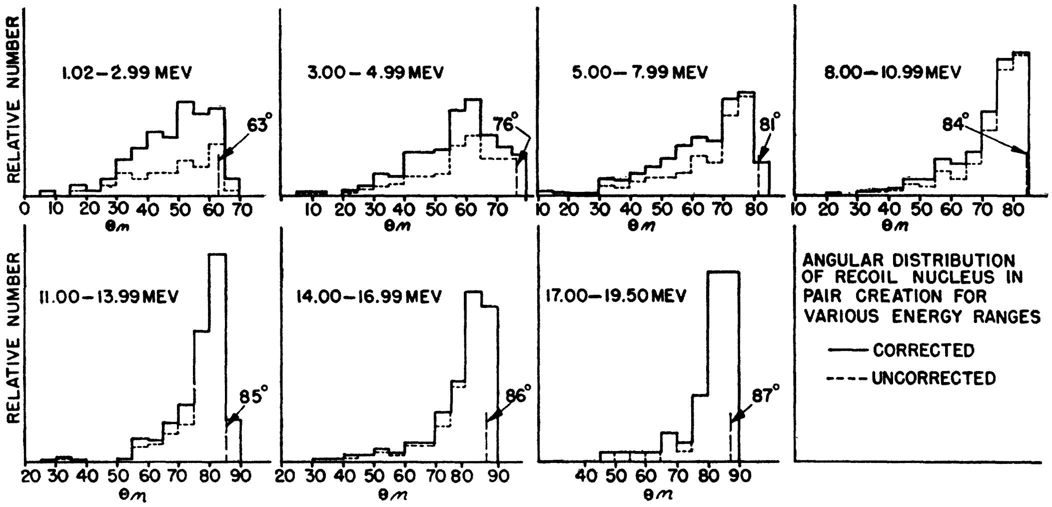


FIG. 3. Angular distribution of recoil nucleus.

figure mounted to 36 percent. The interval $h\nu = 1.02$–2.99 Mev is more peaked than the others, with 71 percent of the values of P_n falling between *mc* and 2.3 *mc*. The median value of P_n was within 0.1 *mc* of 1.6 *mc* in every group.

The average values of P_n in each of the intervals (Fig. 5(a)) seemed to vary more than would be expected from the data in Table II. The experimental curve for P_n becomes uncertain at both energy extremes because of the large effect of α-discrimination at the low energies and because of the importance of angle uncertainties at high energies. The downward trend at the high energies is probably not significant, but the values of P_n must approach 2 *mc* at the low energies. This can be seen from the table of minimum and maximum values of nuclear momenta given with the curve. The minimum value ($\delta_{\min}$) can be obtained from (3) with $P_1 = P_2$. The maximum value of the nuclear momentum will arise when the pair electrons have equal energies and are directed opposite to the incident photon direction. The mean value of all momentum transfers in the x-ray range from 1.02 to 19.5 Mev was 2.0 *mc* (1.7 *mc* when uncorrected for α-discrimination). The most probable value was near *mc* (Fig. 4(b)).

When studying the values of θ_n, the angle between $\mathbf{K}$ and $\mathbf{P}_n$ (Figs. 3 and 5(b)), one should note that there is an upper limit to these values for a given photon energy depending upon the energy distribution and direction of emission of the pair particles. The angle θ_n will be at a maximum when

$$\theta_P = \cos^{-1}(P_P/K), \tag{5}$$

where θ_P is the angle between $\mathbf{K}$ and $\mathbf{P}_P$. When $P_1 = P_2$, the angle θ_n will have its largest possible value. Calling this upper limit τ, we have

$$\tau = \tan^{-1}(P_P/2\mu), \tag{6}$$

where $\mu = mc$. The smooth curve in Fig. 5(b) shows τ as a function of $h\nu$. The vertical dotted lines indicated on each histogram of Fig. 3 show the average value of τ for the energy range being considered.

The values of θ_n (Fig. 3) show a definite trend toward becoming more peaked near the maximum possible angle (τ) with increasing energies. The effect is more pronounced than would appear from the curves as drawn because some of the angles in each energy interval are greater than the average τ for the interval. The values of θ_n plotted in Fig. 5(b) show the same tendency for θ_n to approach τ with increasing x-ray energies, as would be expected from Fig. 3. The dotted

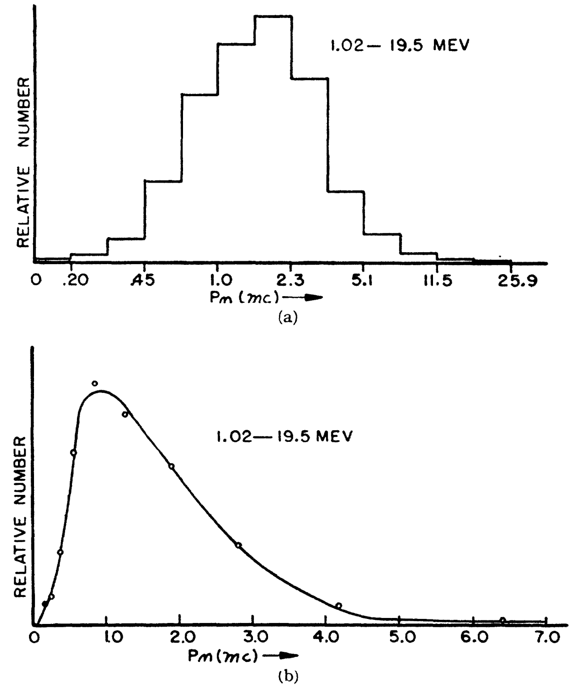


FIG. 4. Nuclear momentum distribution.

part of the experimental curve (Fig. 5(b)) shows that $\theta_n \to 0°$ as $h\nu \to 1.02$ Mev.

DISCUSSION OF RESULTS

Since screening of the atomic field due to the atomic electrons is unimportant in this experiment,[9] the probability that a momentum between P_n and $P_n + dP_n$ will be transferred to the nucleus in pair production for $h\nu \gg mc^2$ is theoretically given as [2,3]

$$\Phi(P_n)dP_n = \text{const.}\frac{dP_n}{P_n}\left(\frac{P_n-\delta}{P_n}\right)^2 \qquad (P_n \sim \delta), \tag{7}$$

$$\Phi(P_n)dP_n = \frac{c}{P_n}dP_n \qquad (\delta \ll P_n \ll \mu), \tag{8}$$

$$\Phi(P_n)dP_n = a\frac{dP_n}{P_n^3}\left(\log\frac{P_n}{\mu}+b\right) \qquad (P_n \gg \mu), \tag{9}$$

where a, b, and c are constants. The equations indicate that the main part of the pair creations is connected with a momentum transfer between δ and mc to the nucleus, and that the probability of a momentum transfer P_n is proportional to $1/P_n$ in this region.

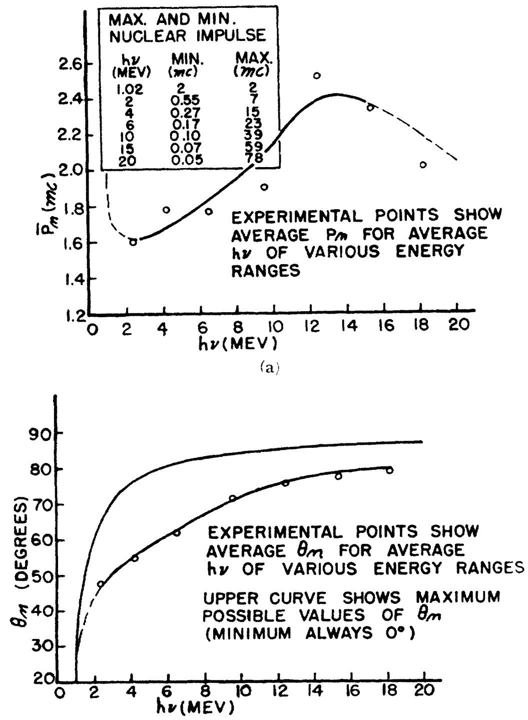
The experimental results indicated a most probable value of momentum transfer near mc and a rapid decrease in probability in both directions away from mc. In no instance was a probability proportional to $1/P_n$ noted for momentum transfers below mc. This disagreement is emphasized by the use of equal logarithmic intervals for P_n (as in Figs. 2 and 4(a)). Equation (8) predicts an equal number of momentum transfers in each of the logarithmic intervals in a region between δ and mc.

TABLE III. Comparison of two experiments at 2.6 Mev.

	Groshev and Frank		Present experiment
Gas	N	Kr	Air
Percentage of values of $P_n < mc$	24	10	20
Mean value of P_n	1.5 mc	1.7 mc	1.6 mc
Mean value of θ_n	49°	not given	48°

TABLE IV. Mean values of θ and $(P_n)_s$.

Mean particle momentum (mc)	$\frac{\bar{\theta}_1+\bar{\theta}_2}{2}$	Mean sideways component of particle momentum (mc)
1.4	30°	0.7
2.0	27°	0.9
9	7.0°	1.1
14	5.8°	1.4
21	4.1°	1.5



(a)

(b)

FIG. 5. Mean values of (a) nuclear momentum and (b) angle of recoil.

The theoretical expressions (7)–(9) are not applicable to energies of the order mc^2. For example, at the threshold energy for electron pair production ($h\nu = 2\,mc^2 = 1.02$ Mev), only one momentum transfer ($2\,mc$) is possible, and near threshold the distribution in values of P_n is necessarily more peaked. This could explain the shape of the nuclear momentum distribution in Fig. 2 for the lowest energy interval ($h\nu = 1.02$–2.99 Mev). Between $h\nu = 3.00$ Mev and $h\nu = 19.50$ Mev, the distributions of the experimental values of P_n have been shown to be similar for each energy interval.

The results obtained agree very well with the work done by Groshev and Frank, who investigated pairs formed by photons from the principal line of ThC″ ($h\nu = 2.6$ Mev).[5–7] They computed P_n for 29 pairs in Kr and 76 pairs in N and found approximately the same distributions of nuclear impulses for the two gases. They also calculated the values of θ_n for the 76 pairs formed in N. The distributions of P_n and θ_n which they found were similar to those reported here. The percentages of values of $P_n < mc$ indicated by the experiments of Groshev and Frank are compared with the percentage found in the present experiment at 2.6 Mev (Table III). Also compared are mean values of P_n and θ_n.

Perhaps with the exception of the lower energy ranges, the present experiment seems to show larger values of momentum transfers than predicted by theory. Some increase in the nuclear impulses above the predicted values was anticipated before calculations were begun because the experimental values of θ_1 and θ_2

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[9] Screening is unimportant when $S \gg 1$, where $S = (100mc^2h\nu/(E_+E_-Z^{\frac{1}{3}}))$.

were larger than theoretically predicted (see A). The components of nuclear momentum both along and perpendicular to the photon direction increase with increasing θ_1 and θ_2.

A qualitative indication of the effect of larger θ_1 and θ_2 values on P_n can be obtained by considering only the sideways momentum of the pair particles, since the forward component of nuclear momentum decreases with increasing energy and is small compared with mc for $h\nu \gg mc^2$. According to Bethe, the average sideways momentum of each of the pair particles is of the order mc for $h\nu \gg mc^2$. The experimental mean values of the sideways momenta of the pair particles were obtained from the average of the mean values of θ_1 and θ_2 and are listed in Table IV.

In addition, for nuclear momentum transfers near δ it is in general necessary that the sideways momenta of the pair particles be directed nearly opposite to each other. The sideways momentum of the nuclear recoil equals

$$(P_n)_s = [P_1^2(1-\cos^2\theta_1)+P_2^2(1-\cos^2\theta_2) + 2P_1P_2 \sin\theta_1 \sin\theta_2 \cos\phi]^{\frac{1}{2}}, \quad (10)$$

where ϕ is the angle between the planes ($\mathbf{K}$, $\mathbf{P}_1$) and ($\mathbf{K}$, $\mathbf{P}_2$). If one assumes a favorable case for producing a small value of $(P_n)_s$ by setting the sideways momentum of the pair particles equal to each other, Eq. (10) may be reduced to

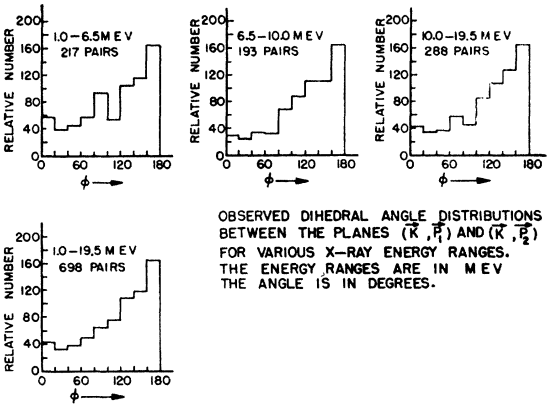
$$(P_n)_s = \sqrt{2}(P_1)_s(1+\cos\phi)^{\frac{1}{2}}, \quad (11)$$

where $(P_1)_s$ is the sideways momentum of the pair electron (equal to $(P_2)_s$).

Equation (11) shows that even in favorable cases it is necessary that the values of ϕ be near π. Groshev[7] found good agreement between the theoretical and experimental distributions of ϕ in the experiment previously discussed. His experiment showed 50 percent of the values of ϕ falling between 120° and 180° for pairs formed in N and Kr. The distributions of ϕ found in the present experiment (Fig. 6 and Table V) are very similar to the results given by Groshev. The fact that the experimental values of ϕ (Fig. 6) are not concentrated near π is consistent with the large values of nuclear momentum found in the present experiment.

TABLE V. Experimental dihedral angle data.

X-ray energy range (Mev)	1.0–5.5	5.5–9.0	9.0–18.5	1.0–18.5
Percentage of ϕ between 120° and 180°	53	58	57	57



OBSERVED DIHEDRAL ANGLE DISTRIBUTIONS BETWEEN THE PLANES $(\vec{K}, \vec{P}_1)$ AND $(\vec{K}, \vec{P}_2)$ FOR VARIOUS X–RAY ENERGY RANGES. THE ENERGY RANGES ARE IN MEV THE ANGLE IS IN DEGREES.

FIG. 6. Angular distributions.

For comparable energy ranges (2.6 Mev) the present experiment agreed in every field of comparison with the work done by Groshev and Frank, and it has been shown that no significant changes in the distribution of the values of nuclear momentum were found for x-ray energy ranges up to 19.5 Mev.

No quantitative estimate of errors has been given, although a brief statement of the causes of errors was presented in the method section. Specifically, the error in P_n increases approximately as the square of the ratio P_1/P_n and as the first power of $\theta_1\Delta\theta_1$. For the case of $P_1 = P_2 = 10\ mc$, $P_n = 2\ mc$, $\theta_1 = \theta_2 = 6°$, $\Delta\theta_1 = \Delta\theta_2 = \pm 1°$, and $\Delta P_1/P_1 = 0.05$, the probable error in P_n is 12 percent. In no energy range is the probable error great enough to explain the apparent disagreement with the theoretical prediction of an increasing probability $\Phi(P_n)$ with decreasing values of momentum transfers for $P_n < mc$.

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