On the Treatment of Quantum Electrodynamics without Eliminating the Longitudinal Field

NING HU
University of Wisconsin, Madison, Wisconsin
November 16, 1949

THE aim of the present note is to clarify certain points in a recent paper under the same title.[1] It should first be pointed out that the original derivation of equivalence of (34) and (36) in I is incomplete.* The correct derivation which applies only to the case $\sigma=\infty$ will be given as follows. $H_2(x)$ given by (25) may be written in the following form by using (8a) and (9a)

$$H_2(x)=H_2^{(+)}(x)+H_2^{(-)}(x),$$

$$H_2^{(\pm)}(x)\Omega_0=-(-i)^2 j_\mu(x)A_\mu^{(l)(\pm)}(x)\int_{\sigma(x)} j_\alpha(x')B^{(\mp)}(x')d\sigma_\alpha'\Omega_0$$
$$=-\frac{1}{2\sqrt{2}(2\pi)^3}\int\int j_\mu(x)j_\alpha(x')n_\mu^{(l)}(k)\times e^{\pm i(k,x-x')}\frac{d^3k}{k^2}d\sigma_\alpha'\Omega_0, \quad (1)$$

where $A_\mu^{(\pm)}(x)$ and $B^{(\pm)}(x)$ denote the $(\pm)$-frequency part of $A_\mu(x)$ and $B(x)$. $I_n(i)$ given by (30) (with $\sigma=\infty$) can accordingly be written $I_n(i)=I_n^{(+)}(i)+I_n^{(-)}(i)$ where $I_n^{(\pm)}(i)$ contains the integral $\int H_2^{(\pm)}(x')dx'$. (32) will still hold if $A_\mu^{(\kappa)}(x)$ and $B(x)$ are replaced by $A_\mu^{(\kappa)(\pm)}(x)$ and $B^{(\pm)}(x)$. We shall call the relation thus obtained (32±) and use (32+) to push the surface integral in $I_n^{(+)}(1)$ to the right side of the last factor $\int H_1(x_n)dx_n$ as in I and then replace it by a volume integral from $-\infty$ to σ_n, the surface integral at $-\infty$ being equal to zero if we assume as in all collision problems that the interaction between the particles vanishes at $-\infty$. We have similar to (33)

$$I_n^{(+)}(i)\Omega_0=\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times A_\mu^{(l)(+)}(x')A_\alpha^{(\kappa)(-)}(x'')\Omega_0$$
$$=-\frac{1}{2(2\pi)^3}\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times\int n_\mu^{(l)}(k)n_\alpha^{(\kappa)}(k)e^{i(k,x'-x'')}\frac{d^3k}{k}\Omega_0, \quad (2)$$

where $\cdots$ are the other factors in (33) except that the upper limit of integration of the first integral factor is ∞ instead of σ. For $I_n^{(-)}(i)$ we shall push the surface integral to the *left* side instead of the right side using the relation (32−) in the reverse direction. The last surface integral at the extreme left side is then replaced by a volume integral from $-\infty$ to $+\infty$, the surface integral at $+\infty$ being equal to zero by the same argument as at $-\infty$. Here we see the reason for considering the case $\sigma=\infty$ since otherwise we would still have a surface integral over σ. We obtain after letting $x'\rightleftarrows x''$ and $\mu\rightleftarrows\alpha$,

$$I_n^{(-)}(i)\Omega_0=-\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times A_\alpha^{(l)(-)}(x'')A_\mu^{(\kappa)(+)}(x')\Omega_0$$
$$=-\frac{1}{2(2\pi)^3}\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times\int n_\mu^{(\kappa)}(k)n_\alpha^{(l)}(k)e^{i(k,x'-x'')}\frac{d^3k}{k}\Omega_0, \quad (3)$$

Equations (2) and (3) should further be summed over $i=0, 1, 2, \cdots, n$. We shall now pick out from each sum on the right-hand side of $\Sigma_i I_n(i)=\Sigma_i I_n^{(+)}(i)+\Sigma_i I_n^{(-)}(i)$ the terms which have $j_\mu(x')$ and $j_\alpha(x'')$ in the same position. We have then

$$\Sigma_i I_n(i)\Omega_0=-\frac{1}{2(2\pi)^3}\Sigma_i\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times\int[n_\mu^{(0)}n_\alpha^{(0)}-n_\mu^{(3)}(k)n_\alpha^{(3)}(k)]e^{i(kx'-x'')}\frac{d^3k}{k}, \quad (4)$$

where we have used the relation

$$n_\mu^{(l)}(k)n_\alpha^{(\kappa)}(k)+n_\mu^{(\kappa)}(k)n_\alpha^{(l)}(k)=n_\mu^{(0)}n_\alpha^{(0)}-n_\mu^{(3)}(k)n_\alpha^{(3)}(k).$$

If we define a new state vector by the condition

$$A^{(0)}(k)\Omega_0'=0, \quad A^{(3)}(k)\Omega_0'=0, \quad (5)$$

it can easily be seen that $\Sigma_i I_n(i)\Omega_0$ is equal to

$$-\frac{1}{2(2\pi)^3}\Sigma_i\Sigma\left[\cdots\int j_\mu(x')dx'\cdots\int j_\alpha(x'')dx''\cdots\right]\times\langle\sum_{s=0,3}A_\mu^{(s)}(x')\sum_{t=0,3}A_\alpha^{(t)}(x'')\rangle_0'\Omega_0, \quad (6)$$

where $\langle\ \rangle_0'$ represents the "vacuum expectation value" with "vacuum" defined by (5). Ω_0' might formally be considered as the state with no longitudinal and scalar photons if $A^{(0)}(k)$ instead of $A^{(0)*}(k)$ would be considered as the absorption operator as whatsoever the application of "hole theory" to Einstein-Bose particles may mean. This shows that (34) and (36) are equivalent if (36) is understood as containing only the virtual emission and absorption processes with vacuum defined by (5). This Ω_0' is the state "with no longitudinal photons" referred to in the footnote on p. 395, and is just the vacuum state vector used by Feynman and Schwinger. It should be noted that in the present derivation we need not concern ourselves with the true vacuum expectation value of (36). Equation (36) can now be extended to include the virtual transverse photons and the same argument leading from (37) to (38) and (40) can be used when more $\int H_2(x)dx$ factors are added. We obtain finally the same result (40) *except that σ should be put equal to infinity.*

The situation will be complicated if σ is finite. Then $S[\sigma]$ will be given by (40) *plus* terms containing the surface integrals that have been pushed over to the left side of $I_n^{(-)}(i)$, $I_n^{(-)}(i, j)$, etc., by the relation (32−). These additional terms vanish when $\sigma=\infty$; this means that they contain no matrix elements over the states of the same energy. The appearance of these terms means at least that the application of Feynman's method must be cautious when we are considering the problems of bound states in which the knowledge of $S[\sigma]$ for finite σ is necessary. It should be noted that the matrix elements between states of different energies are usually ambiguous depending on the methods of perturbation. For instance, each term of $S[\sigma]$ obtained by the method of canonical transformation is hermitian, while that obtained by the straightforward perturbation is not. Therefore the present result cannot decide whether (40) is right or wrong. Further investigation will be necessary to decide this point. Finally we wish to point out some important misprints and mis-statements: (1) The last equation of (8a) should have a plus sign on the right-hand side, (2) the left-hand side of (14) should read $[\partial A_\mu/\partial x_\mu, x]$, (3) the first sentence below (30) should be omitted.

The author is indebted to Drs. F. J. Belinfante and S. T. Ma for the helpful discussions. He also wishes to thank Wisconsin Alumni Research Foundation for the financial support.

[1] Ning Hu, Phys. Rev. **76**, 391 (1949). This will be referred to later as I.
* The validity of the condition $\bar{\Omega}_0 A_\mu^{(k)}(x')A_\alpha^{(k)}(x'')\Omega_0=0$ used in the derivation is questionable. Furthermore the correctness of the statement on p. 395 has to be shown.

On the Infra-Red Absorption of the Hydrogen Molecule

MASATAKA MIZUSHIMA
Physics Department, Faculty of Science, Tokyo University, Tokyo, Japan
November 10, 1949

THE hydrogen molecule, being a homonuclear diatomic molecule, has no dipole infra-red absorption; but recently, 0–2, 0–3 vibrational absorption lines were observed by Herzberg,[1] and 0–1 lines by Welsh, Crawford, and Locke.[2]

There are two reasons to expect these absorptions: The quadrupole absorption and the pressure absorption[3] (absorption induced by the intermolecular forces). The 0–1 lines observed by Welsh, Crawford, and Locke are apparently due to pressure, while there are many reasons to believe that the absorption lines observed by Herzberg are due to the change of the quadrupole moment.[4]

The present author proposed a theory of the pressure absorption[3] by which the intensity of this absorption is given by

$$I_P=\frac{192\pi^4\nu P^2}{25hcR_0^5}n^2(a|\alpha|b)^2, \tag{1}$$

where ν is the frequency of the absorption line (which corresponds to the transition $a\rightarrow b$), P is the quadrupole moment of the molecule, n is the number of molecules in unit volume, $(a|\alpha|b)$ is the ab-matrix element of the polarizability α, and R_0 is the shortest distance to which the molecules can approach to each other.

For the Q-branch of 0–1 lines, the experimental results of Welsh and others[2] gives $I=3.41\times10^7$ cm^{-1} at 1 atmos. In this case $\nu=1.246\times10^{14}$ sec.$^{-1}$, $n=2.98\times10^{19}$ cm^{-3}, and $P=5.85\times10^{-27}$ c.g.s. e.s.u.[5] From the transport phenomena of hydrogen gas we obtain $R_0=2.72$A. Thus $(0|\alpha|1)$ must be 1.83×10^{-25} in order to explain the experimental results by Eq. (1). Although the theoretical estimation[6] by the Heitler-London wave function yields $(0|\alpha|1)=0.53\times10^{-25}$, it is not an absurd value when compared to the value of polarizability (an average of the diagonal elements of α) 8.02×10^{-25}.

The intensity of the pressure absorption in the harmonic lines 0–2, 0–3 due to the anharmonisity were estimated as follows.

The frequencies of the lines observed by Herzberg[1] fit well with the theoretical one, if we put $D=8.2496\times10^{-12}$ erg, $a=1.8553\times10^8$ cm^{-1} and $r_e=7.501\times10^{-8}$ cm in the Morse potential

$$U=D(1-\exp[-a(r-r_e)])^2.$$

If we put $\alpha(r)=\alpha_e+\alpha_e'(r-r_e)+\cdots$ and neglecting the terms higher than α_e'', it can be shown that the matrix elements of α calculated by the Morse wave function[7] are

$$\begin{aligned}(0|\alpha|1)&=\alpha_e'\frac{(k-3)^{\frac{1}{2}}}{a}\{\psi(k-1)-\psi(k-2)\},\\ (0|\alpha|2)&=\alpha_e'\frac{1}{\sqrt{2}a}\left(\frac{k-5}{k-2}\right)^{\frac{1}{2}}\{-(k-2)\psi(k-1)\\ &\qquad+2(k-3)\psi(k-2)-(k-4)\psi(k-3)\},\\ (0|\alpha|3)&=\alpha_e'\frac{1}{\sqrt{6}a}\left\{\frac{k-7}{(k-2)(k-3)}\right\}^{\frac{1}{2}}\{-(k-2)(k-3)\psi(k-1)\\ &\qquad+3(k-3)(k-4)\psi(k-2)-3(k-4)(k-5)\psi(k-3)\\ &\qquad+(k-5)(k-6)\psi(k-4)\},\end{aligned} \tag{2}$$

where $\psi(x)$ is the digamma function, and

$$k=4\pi[(2\mu D)^{\frac{1}{2}}/ah].$$

By the above obtained numerical values we obtain

$$\begin{aligned}(0|\alpha|1)&=9.281\cdot10^{-10}\alpha_e',\\ (0|\alpha|2)&=-1.039\cdot10^{-10}\alpha_e',\\ (0|\alpha|3)&=-2.034\cdot10^{-11}\alpha_e'.\end{aligned}$$

Thus the intensity ratio of the absorption lines calculated by Eq. (1) is

$$I_{01}:I_{02}:I_{03}=1:0.0225:0.00094. \tag{3}$$

The intensities of the pressure absorption for the Q-branch of each band calculated by it, is shown in Table I, together with the values of intensity of the quadrupole absorption calculated by the James-Coolidge value.[5]

TABLE I. Intensity of each absorption. (Q-branch, at 1 atmos., 300°K, in cm^{-1}).

Absorption lines	0–1	0–2	0–3
$I_Q\times10^{-3}$	1.8	1.65	0.34
$I_P\times10^{-7}$	3.4	0.086	0.0022
$(I_Q/I_P)\times10^5$	5	190	1500

The ratio I_Q/I_P is always less than 10^{-1}, the pressure absorption being larger. We must, however, take note of the fact that what is measured is not the integrated intensity I itself, but rather the absorption coefficient. The maximum absorption coefficient of each line is proportional to I, and inversely proportional to the width of the line. The width of the pressure absorption is about 10^2 cm^{-1}, while that of the quadrupole absorption may be assumed to be about 10^{-3} cm^{-1}. Thus the ratio of the maximum absorption coefficient of two absorptions for each line may be given by the figures in the last row of Table I, from which we can see that even at 10 atmos. the quadrupole absorption may be observed in the harmonic lines, while in the 0–1 lines the pressure absorption is predominant as we see in the experiments.

I wish to thank Professors G. Herzberg and M. Kotani for advice and encouragement.

[1] G. Herzberg, Nature **163**, 170 (1949).
[2] Welsh, Crawford, and Locke, Phys. Rev. **76**, 580 (1949).
[3] M. Mizushima, Phys. Rev. **76**, 1268 (1949).
[4] G. Herzberg (private communication).
[5] H. M. James and A. S. Coolidge, Astrophys. J. **87**, 447 (1938).
[6] B. Mrowka, Zeits. f. Physik **84**, 448 (1933).
[7] P. M. Morse, Phys. Rev. **34**, 57 (1929).

Search for Photons Emitted by Long-Life Species of Nickel

D. G. THOMAS AND J. D. KURBATOV
Ohio State University, Columbus, Ohio
November 23, 1949

AVAILABLE information concerning disintegrations of two species of nickel, Ni^{59} and Ni^{63}, is confined to their half-lives. The $T_{\frac{1}{2}}$ for Ni^{59} disintegrating by orbital electron capture was estimated to be about $(2–3)\times10^5$ yr. and that for Ni^{63}, disintegrating by negatron emission, several hundred years.[1,2] The beta-disintegration u.e. of Ni^{63} was given as ~50 kev.

The present study of long-life isotopes of nickel was made with 9.7 g of nickel activated at Oak Ridge National Laboratory of AEC. The nickel salt was chemically purified by repeated precipitation of nickel in the presence of carriers for radioactive impurities.

Due to the low specific activity of the nickel species and to their very long lives, the study was made with a G-M counter and with a cloud chamber, both mainly for the emission of photons. A specially constructed large aluminum window gamma-tube was employed. To secure low background and small fluctuations of the tube, a sample of active nickel was shielded and the emitted radiation was collimated.

The results obtained showed the presence of photons of the following energies: 7.5±1 kev, 15±2 kev, and 38±3 kev, by absorption in aluminum. Absorption of radiation in copper verified the presence of photon of 38±3 kev and indicated the existence of photon of 80±5 kev energy. The order of relative intensities of photons of 7.5±1 kev, 15±2 kev, 38±3 kev, and 80±5 kev was estimated as 6:1:1:1, respectively.

The very low intensity of radiation did not allow the establishment of the presence of higher energy photons. Absorption in lead showed the existence of at least one photon of energy higher than 500 kev.

If it is assumed that the 7.5±1-kev photon is due to x-rays of orbital electron capture and to internal conversions, the emitted gamma-rays can be ascribed to Ni^{59}.

Electron tracks in the cloud chamber with and without a magnetic field revealed the presence of monochromatic electron groups.

An attempt to obtain a continuous spectrum of beta-disintegration of Ni^{63} in the cloud chamber was not successful.

Since Ni^{59} disintegrates into Co^{59} and Ni^{63} into Cu^{63} efforts were made to obtain information on the already known excited