intercept $\beta/S$. $S$ is finally determined from the peak structure and $\alpha$ and $\beta$ are calculated.

The first simplification of this procedure and the exact solution has come independently from Hubbard and Stewart,[3] and Fox and Hunter.[4] It must be used in regions of large $\alpha$ and $\beta$ for which the approximation $\tanh(\alpha r+\beta) \doteq (\alpha r+\beta)$ is invalid. They introduced the measurement of the currents $i_{00}$ when $P=P_{00}=0$ (for a vacuum) and $i_\infty$ when $P=P_\infty=1$ (for the fluid without reflection). These give simple determinations of $\sigma_{00}=i_{00}/I_0$ and of $S=\phi_\infty$ without precise knowledge of peak structure. The $P=\phi/\phi_\infty$ are then calculated, and $\alpha$ and $\beta$ are determined from the slope and intercept respectively of $\tanh^{-1}P_{0n}$ and $1/\tanh^{-1}P_{mn}$ plotted as functions of $r$.

It should be emphasized that $\sigma_{00}$ and $S$ are constants of the interferometer only if the acoustic impedance surrounding the source is constant. Thus $\sigma_{00}$ is a function of the acoustic impedance of the media contacting the unused surfaces of the source, while $S$ is a function of the acoustic impedance of the fluid contacting the used surface of the source.

A second simplification of the procedure is suggested when it is observed that the $\phi$ may be expressed as $\phi=E\delta$ where $\delta=i-i_{00}$ is the excess of the current $i$ over the background current $i_{00}$ and

$$1/E=(i_{00}/I_0)(I_0-i_{00})[1-(i-i_{00})/(I_0-i_{00})].$$

In the region of low acoustic impedance $(i-i_{00})/(I_0-i_{00})$ is negligible, and $E$ is a constant for a given run. Consequently $\phi$ and therefore $P$ are proportional to $\delta$, and $P=\delta/\delta_\infty$. It is possible to record only the $\delta$ on a linear meter, divide each $\delta$ by $\delta_\infty=i_\infty-i_{00}$, and obtain the corresponding $P$. $\alpha$ and $\beta$ are then determined from the $P$ as above. No measurement of $I_0$ or of the absolute magnitude of $i$ is necessary.

In practice $i_\infty$ can be measured with the reflector at such a distance that the reflected waves are completely absorbed. $i_{00}$ need not be measured if the current excesses $\delta'$ over the current of the first minimum $i_{01}$ are known. Thus $\delta'=i-i_{01}$, and $\delta=i-i_{00}=\delta'+\delta_{01}$. Since $P_{m1}P_{01}=P_{m2}P_{02}=\cdots=P_\infty{}^2=1$, we have $\delta_{m1}\delta_{01}=\delta_{m2}\delta_{02}=\cdots=\delta_\infty{}^2$. This yields $\delta_{01}=(\delta_\infty')^2/(\delta_{m1}'-2\delta_\infty')$ which can be added to each $\delta'$ to give the corresponding $\delta$.

The above simplification is particularly applicable to the number of investigations in gases at low pressures now in progress.

* This work was supported by the Rutgers University Research Council.

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## Cosmic Radiation and the Maintainance of a Potential Gradient in the Atmosphere

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November 14, 1949

It has been well known for over a century that the earth's atmosphere exhibits a potential gradient. The total difference of potential between the ionosphere and the surface of the earth is thought[1] to be around 400,000 volts. The atmosphere is rendered partially conducting by ionization, produced mainly by cosmic radiation and by radioactive contamination in the lower portions. Taking the resistance of a square cm column of the atmosphere as about $10^{21}$ ohms, the average current in this column will be about $4\times10^{-16}$ amperes or 1800 amperes total, and the total power developed is $7.2\times10^8$ watts, over the entire earth. The current would of course rapidly neutralize the potential difference if there were no mechanism which maintains it. It has been postulated that thunderstorms provide the mechanism for maintaining this gradient, but to date no satisfactory theory has been proposed as to just how this operates.

Cosmic radiation is thought today to consist in the main of positively charged primaries, probably protons. A total of $5.7\times10^{17}$ rays arrive over the entire earth per second, corresponding to a total current of 0.09 ampere. The influx of energy is estimated[2] at $9.8\times10^{18}$ Bev per sec., or about $1.4\times10^9$ watts. The primary cosmic radiation would, if it consists of only positively charged particles, cause the earth to charge up at the rate of $dV/dt=dQ/Cdt=i/C$ or about $10^7$ volts per day. It is evident that this effect would soon shut off the influx of additional primary radiation. It is therefore necessary to suppose that an equal number of electrons accompany the primary protons, although these need not have high energies and in this case will not penetrate far into the earth's atmosphere. Indeed if the electrons share the average velocity of the protons they will have average energies lower by three orders of magnitude.

It is evident that if the positive incoming radiation penetrates on the average to a lower depth than does the negative, charged layers will be set up. In the steady state, the current, consisting of electrons or ions drifting through the air to neutralize the potential difference produced by the penetrating primary radiation, will equal the current initially incident, minus the amount lost by the absorption of the incident beam at any depth. Therefore the amount of current flowing through the atmosphere will be a function of depth in the atmosphere. If, because of absorption processes, the net unbalanced number of charged particles surviving at any depth decreases exponentially, then the current may be expected to vary with depth in a similar manner.

The primary radiation produces many secondaries, both charged and uncharged, in the atmosphere. Since some of these secondaries, the mesons, are more penetrating than the primary particles, it is actually these which propagate the intensity to lower depths in the atmosphere. It is known that there is a small positive excess in the number of charged cosmic-ray particles (mostly mesons) arriving at sea level. This situation is consistent with the view that charge is conserved in secondary-production processes, and that if one primary positive particle produces $n$ negative secondaries it will produce $n+1$ positives. Thus, in any event, the positive charges are projected down directly, and negatives must drift downward or other positives drift upward at the same total rate to enable a steady state to be maintained.

However, it should be pointed out that whereas the incoming radiation undoubtedly does establish and maintain a potential difference in the atmosphere, it is not possible to ascribe all the observed current through the air to cosmic radiation. Also, whereas the cosmic radiation does bring to the earth more than enough power to maintain the observed potential difference, most of the energy is used up in producing nuclear and atomic transformations. The actual number of charges per sq. cm per second arriving in the form of cosmic radiation does not equal the number required to account for observed currents through the atmosphere. Therefore some other mechanism is also at work.

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## Dispersion in $NH_3$ in the Microwave Region[1]

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November 14, 1949

It is well known that $NH_3$ strongly absorbs centimeter-wave radiation due to the inversion of the molecule in its ground vibrational state. The contribution of this inversion to the electric susceptibility has been demonstrated by recent measurements which have shown a small but significant difference in susceptibility between two frequencies, 2802 Mc (0.0934 $cm^{-1}$) and 9270 Mc (0.309 $cm^{-1}$), in the pressure range 10 to 76-cm Hg. In the

TABLE I. Values of parameters in (1) which fit (1) to $NH_3$ absorption data. Values in ( ) are for another fit.

| | Pressure, cm Hg 10 | 30 | 76 |
|---|---|---|---|
| $\pi\gamma p \times 10^4$ | 10.4 (9.5) | 31.2 (28.4) | 79 (72) |
| $\nu_0$ ($cm^{-1}$) | 0.77 (0.77) | 0.72 (0.72) | 0.40 (0.46) |
| $\Delta\nu$ ($cm^{-1}$) | 0.13 (0.125) | 0.31 (0.28) | 0.50 (0.47) |

TABLE II. Experimental and calculated difference in susceptibility of $NH_3$ between 0.0934 and 0.309 $cm^{-1}$. Values in ( ) were calculated from the values in ( ) in Table I.

| $[\epsilon'(0.0934)-\epsilon'(0.309)] \times 10^4$ | Pressure, cm Hg 10 | 30 | 76 |
|---|---|---|---|
| Experimental | $-0.34\pm0.17$ | $-0.93\pm0.50$ | $+2.9\pm1.3$ |
| Calculated | $-0.53(-0.48)$ | $-0.94(-0.98)$ | $+2.1(+1.1)$ |

present work, this difference in susceptibility is shown to be consistent with the absorption.

When certain analytic conditions are satisfied by the complex dielectric constant, $\epsilon(\nu)=\epsilon'(\nu)-i\epsilon''(\nu)$, the real or imaginary part is uniquely determined if the other is known over the entire frequency range. This statement is usually expressed in the form of two integral transformations, first given by Kramers and Kronig.[2]

In the calculation of $\epsilon'(\nu)$ from the data on absorption ($\alpha(\nu)=2\pi\nu\epsilon''(\nu)$ $cm^{-1}$), the absorption is first represented analytically. Above 10-cm Hg, the fine structure of the $NH_3$ inversion spectrum is completely obliterated by the merging of the various collision-broadened lines. At these pressures, several observers investigating collision-broadening effects have found it possible to represent the resultant absorption curve by

$$\frac{\alpha(\nu)}{\nu^2}=\pi\gamma p\left(\frac{\Delta\nu}{\Delta\nu^2+(\nu+\nu_0)^2}+\frac{\Delta\nu}{\Delta\nu^2+(\nu-\nu_0)^2}\right), \tag{1}$$

provided that empirical values of $\nu_0$ ($cm^{-1}$), and $\Delta\nu$ ($cm^{-1}$) are used to give the best fit at each pressure $p$ (cm Hg). In the present work, values were obtained for these parameters and for $\pi\gamma$ treated as a constant, by visually fitting (1) to absorption data obtained by Weingarten[3] in the frequency range 0.31 to 1.2 $cm^{-1}$. These values are given in Table I which also shows an anomalous result reported previously: $\nu_0$ decreases with increasing pressure.[3,4] The values in parentheses are for another fit, and are included to give an idea of the accuracy with which (1) represents the absorption data.

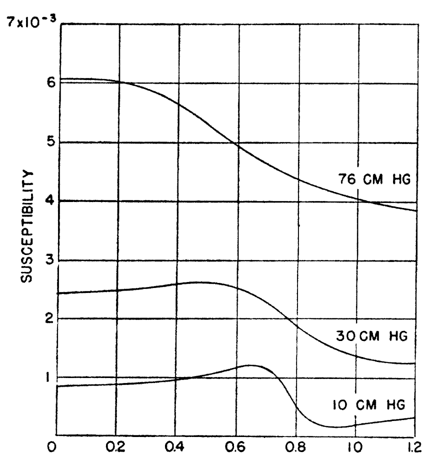


FIG. 1. Calculated dispersion in $NH_3$ ($T=25°C$). Values of parameters, not in parentheses in Table I, were used.

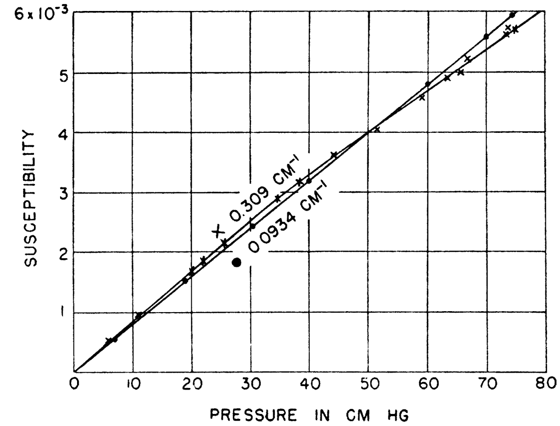


FIG. 2. Susceptibility of $NH_3$ as a function of pressure. The data were taken at room temperature and were corrected to 25°C.

Equation (1) and

$$\epsilon'(\nu)-1=\frac{\gamma p}{2}\left(\frac{\Delta\nu^2+\nu_0(\nu+\nu_0)}{\Delta\nu^2+(\nu+\nu_0)^2}+\frac{\Delta\nu^2-\nu_0(\nu-\nu_0)}{\Delta\nu^2+(\nu-\nu_0)^2}\right) \tag{2}$$

have been derived to describe the absorption and susceptibility for spectral lines broadened by collisions.[5] Semi-empirical significance has been attached to (1) in the present type of application by treating the absorption as if due to a single line of resonance frequency $\nu_0$, with a line width $\Delta\nu$ and an intensity $\pi\gamma p$ which is equal to the sum of the intensities of the individual lines. However, since (1) and (2) are consistent with the Kramers-Kronig relations, $\epsilon'(\nu)$ as calculated from (2) by using the values of the parameters given in Table I is actually independent of any special theory of line shape.

To obtain the behavior of the total susceptibility shown in Fig. 1, the contributions of all polarizations other than that due to the microwave inversion must be added to (2). These contributions which are taken as independent of frequency from 0 to 1.2 $cm^{-1}$ are given by $(\gamma'-\gamma)p$, where $\gamma p=\epsilon'(0)-1$ is given by (2) for $\nu=0$, and where $\gamma'=8.02\times10^{-5}$ per cm Hg is averaged from susceptibility measurements at 1 Mc.[6] A noteworthy feature of the curves of Fig. 1 is the disappearance of the maximum and minimum points in the curve at 76 cm Hg. Analytically, this occurs at a pressure such that $\Delta\nu=\nu_0$.

Dispersion in $NH_3$ was checked by susceptibility measurements at 0.0934 and 0.309 $cm^{-1}$, where apparatus was readily available. The experimental results shown in Fig. 2 were determined with approximately 1.5 percent error from the change in resonance frequency of a cavity resonator due to the admission of the gas. The variation of susceptibility with pressure, shown in Fig. 2, is evidently in qualitative agreement with this variation obtained from the curves of Fig. 1. Adequate agreement between experimental and calculated values of the difference in susceptibility between 0.0934 and 0.309 $cm^{-1}$ is shown in Table II.

It is planned to make susceptibility measurements in the 0.8 $cm^{-1}$ region,[7] where dispersion should be pronounced.

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