ascent on July 11, 1947, in which events ascribable to primaries of $Z \geq 2$ were observed at altitudes exceeding 15 mm of Hg. It has since become possible, through analyses such as that described above, to ascertain that approximately one-third of the incoming cosmic rays are alpha-particles (the presence of helium atoms was originally predicted by Swann[4]), and the remainder predominantly protons, with a small percentage of particles having higher atomic numbers.

The possible influence of stars, bursts, air showers, etc., upon these experiments has been investigated by means of an arrangement whereby an out-of-line counter replaces an in-line counter in the quadruple coincidence train at predetermined altitudes during the course of a flight. The relative frequency of out-of-line events compared with that of events produced by single particles makes it improbable that spurious effects are responsible for the high efficiencies observed.

* Assisted by the Joint Program of the ONR and the AEC.
1 Progress Report, Contract N6ori-144, Bartol Research Foundation, July 15, 1947; subsequent dates. Semi-Annual Report, Contract N6ori-144, Bartol Research Foundation, September 15, 1947; September 15, 1948; March 15, 1949. See also Phys. Rev. **75**, 1316 (1949).
2 Freier, Lofgren, Ney, Oppenheimer, Bradt, and Peters, Phys. Rev. **74**, 213 (1948).
3 W. E. Danforth and W. E. Ramsey, Phys. Rev. **49**, 854 (1936).
4 W. F. G. Swann, J. Franklin Inst. **236**, 1 (1943).

Nuclear Quadrupole Moments and Shell Structure

R. D. Hill
Physics Department, University of Illinois, Urbana, Illinois
August 10, 1949

It is generally accepted that positive quadrupole moments (Q) indicate nuclei which are elongated in the direction of the spin (I) axis, and *vice versa* for negative Q. In this note it is suggested that the spatial distortion of nuclei, and hence Q values, arise in the first order of approximation from incomplete proton and/or neutron sub-shells.

In Table I are tabulated the nuclei for which Q values are known.[1,2] On the basis of nuclear shell theory, level assignments of the nucleons in the incomplete shells are given in column 4 of this table. These assignments lead to conclusions as to the signs of the quadrupole moments, which are also given in column 4. In drawing these conclusions the following assumptions were made: (1) The shapes of nuclei are determined by the probability-density distributions $\psi\psi^*$ of the independent nucleons in the incomplete shells. (2) Completely closed shells or sub-shells (which are spherically symmetrical) do not contribute to Q. Likewise, since pairs of nucleons make no contribution to I, unfilled sub-shells with even numbers of nucleons to a first order of approximation make no contribution to Q. Accordingly, only incomplete shells with odd numbers of nucleons are listed in the table. (3) In an unfilled shell having an odd number of nucleons, the "unpaired" nucleon has an m_j value equal to I, and the other nucleons distribute themselves in pairs among the remaining values of m_j (m_j is the magnetic quantum number with respect to the I field). (4) A sub-shell with only one nucleon in the state $m_j = |I|$ endows the nucleus with a negative Q value. A sub-shell with an odd number of nucleons in states $m_j = |I|, \pm\frac{1}{2}, \pm\frac{3}{2}, \cdots$ generally will produce a nucleus of positive Q value, the magnitude being the greater the more the nucleons are concentrated in the states of lower values of m_j.

The order of the sub-shells follows closely that given by Mayer.[3] There is a certain amount of arbitrariness in selecting sub-shells for the higher Z and N nuclei, but for Z or $N \leqslant 20$ the latitude of selection is fairly narrow. The case of ${}_{19}K^{39}$ is of interest because the shell interpretation indicates a positive Q value and the tabulated negative value is as yet only an estimate.[2] Other particularly rigid selections are In^{113}, In^{115}, Sb^{121}, Hg^{201} and Bi^{209}, which occur in the regions of closed shells. The experimental values in these cases, and in all other cases with the possible exception of Xe^{131}, are in accord with the shell interpretation. It would appear also that apart from Q being dependent upon the distribution of nucleons among the m_j states, the magnitude of Q increases with increase of oscillator quantum number of the unfilled nucleon level.

TABLE I. Interpretation of quadrupole moments.

Nucleus	Q	I	Nucleon configuration of unfilled shells*
${}_{1}H^{2}$	+0.0027	1	$1s_{1/2}$ proton, $1s_{1/2}$ neutron. Q = zero
${}_{3}Li^{6}$	∼0	1	$1p_{1/2}$ proton, $1p_{1/2}$ neutron. Q = zero
${}_{3}Li^{7}$	−0.020	3/2	$1p_{3/2}$ proton, $m_j = \|3/2\|$ negative Q
${}_{5}B^{10}$	+0.060	3	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ } $3p_{3/2}$ neutrons, $m_j = \pm 1/2, \|3/2\|$ } positive Q
${}_{5}B^{11}$	+0.030	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{7}N^{14}$	+0.020	1	$1p_{1/2}$ proton, $1p_{1/2}$ neutron. Q = zero
${}_{13}Al^{27}$	+0.156	5/2	$5d_{5/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \|5/2\|$ positive Q
${}_{16}S^{33}$	−0.050	3/2	$1d_{3/2}$ neutron, $m_j = \|3/2\|$ negative Q
${}_{17}Cl^{35}$	−0.079	3/2	$1d_{3/2}$ proton, $m_j = \|3/2\|$ negative Q
${}_{17}Cl^{37}$	−0.062	3/2	$1d_{3/2}$ proton, $m_j = \|3/2\|$ negative Q
${}_{19}K^{39}$	−0.030	3/2	$3d_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{19}K^{41}$	−0.020	3/2	$3d_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{29}Cu^{63}$	−0.010	3/2	$1p_{3/2}$ proton, $m_j = \|3/2\|$ negative Q
${}_{29}Cu^{65}$	−0.010	3/2	$1p_{3/2}$ proton, $m_j = \|3/2\|$ negative Q
${}_{31}Ga^{69}$	+0.232	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{31}Ga^{71}$	+0.147	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{33}As^{75}$	+0.300	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{35}Br^{79}$	+0.280	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{35}Br^{81}$	+0.230	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{36}Kr^{83}$	+0.150	9/2	$7g_{9/2}$ neutrons, $m_j = \pm 3/2, \pm 5/2, \pm 7/2, \|9/2\|$ positive Q
${}_{37}Rb^{87}$	+0.170	3/2	$3p_{3/2}$ protons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{49}In^{113}$	+1.300	9/2	$9g_{9/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \pm 5/2, \pm 7/2, \|9/2\|$ positive Q
${}_{49}In^{115}$	+1.17	9/2	$9g_{9/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \pm 5/2, \pm 7/2, \|9/2\|$ positive Q
${}_{51}Sb^{121}$	−0.90	5/2	$1d_{5/2}$ proton, $m_j = \|5/2\|$ negative Q
${}_{53}I^{127}$	−0.60	5/2	$1d_{5/2}$ proton, $m_j = \|5/2\|$ negative Q
${}_{53}I^{129}$	−0.43	7/2	$1g_{7/2}$ proton, $m_j = \|7/2\|$ negative Q
${}_{54}Xe^{131}$	∼0	3/2	$1d_{3/2}$ neutron, $m_j = \|3/2\|$ negative Q. or $3d_{3/2}$ neutrons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{57}La^{139}$	+0.20	7/2	$7g_{7/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \pm 5/2, \|7/2\|$ positive Q
${}_{63}Eu^{151}$	+1.20	5/2	$5d_{5/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \|5/2\|$ positive Q
${}_{63}Eu^{153}$	+2.50	5/2	$5d_{5/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \|5/2\|$ positive Q
${}_{70}Yb^{173}$	+3.9	5/2	$5f_{5/2}$ neutrons, $m_j = \pm 1/2, \pm 3/2, \|5/2\|$ positive Q
${}_{71}Lu^{175}$	+5.9	7/2	$3g_{7/2}$ protons, $m_j = \pm 1/2, \|7/2\|$ positive Q
${}_{71}Lu^{176}$	+7	⩾7	$3g_{7/2}$ protons, $m_j = \pm 1/2, \|7/2\|$ } $7f_{7/2}$ neutrons, $m_j = \pm 1/2, \pm 3/2, \pm 5/2, \|7/2\|$ } positive Q
${}_{73}Ta^{181}$	+6.0	7/2	$5g_{7/2}$ protons, $m_j = \pm 1/2, \pm 3/2, \|7/2\|$ positive Q
${}_{75}Re^{185}$	+2.8	5/2	$3d_{5/2}$ protons, $m_j = \pm 1/2, \|5/2\|$ positive Q
${}_{75}Re^{187}$	+2.6	5/2	$3d_{5/2}$ protons, $m_j = \pm 1/2, \|5/2\|$ positive Q
${}_{80}Hg^{201}$	+0.50	3/2	$3p_{3/2}$ neutrons, $m_j = \pm 1/2, \|3/2\|$ positive Q
${}_{83}Bi^{209}$	−0.40	9/2	$1h_{9/2}$ proton, $m_j = \|9/2\|$ negative Q

* The number in front of the angular momentum value of the nucleon state denotes the number of nucleons in this state, not the principal quantum number.

1 H. H. Goldsmith and D. R. Inglis, BNL-1-5 (October, 1948). B. T. Feld, Nuclear Science Series, Preliminary Report No. 2 (September, 1948).
2 W. Gordy, Phys. Rev. **76**, 139 (1949).
3 M. G. Mayer, Phys. Rev. **75**, 1969 (1949).

The Equation of State of Gaseous He^3

J. van Kranendonk, K. Compaan, and J. de Boer*
Institute for Theoretical Physics, University of Amsterdam, Amsterdam, Netherlands
August 16, 1949

Recent experiments[1,2] with pure He^3 show that it can be expected that in the near future measurements of the second virial coefficient $B(T)$ of the equation of state $PV = RT(1 + B/V + \cdots)$ will be available. In this connection it might be of interest to communicate briefly the results of a theoretical calculation of the second virial coefficient of He^3.

As the intermolecular forces, which are determined entirely by the electronic structure of the molecules, are the same in the cases He^3 and He^4, the second virial coefficients would be exactly equal in classical theory. In quantum theory, however, the second virial coefficients are not equal, because of the difference in de Broglie wave-length of He^3 and He^4 molecules to which must be added the influence of the fact that the wave functions in He^3 must be anti-symmetrical and those of He^4 symmetrical at permutation of the molecules.