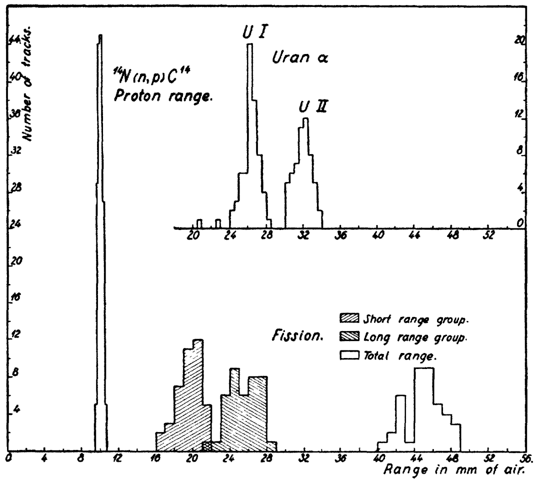

FIG. 2. Histogram of the range distribution of 40 complete pairs of ^{235}U fission fragment tracks and of the $^{14}N(n,p)^{14}C$ tracks used as controls. The range statistics of the UI and UII alpha-groups is given in a separate histogram. Mean range of the combined $^{14}N(n,p)^{14}C$ tracks 10.3 mm; UI and UII 26.3 and 31.9 mm, respectively; heavy and light fragment group 19.5 and 25.4 mm, respectively, and mean total range of paired fragment 44.9 mm.

fission fragments. Since the discrepancy here is ascribed to non-uniformity of the plutonium layer used in the work of Finkle *et al.*, similar defects might possibly be responsible for the diverging results of their and our values.

A closer comparison of our values with those of Katcoff *et al.* is hardly feasible as the two fission processes are somewhat different; however, it is noteworthy that their mean range values of 19.3 and 25.1 mm for the masses 131 and 94, respectively, in the ^{239}Pu fission are rather close to our values for the two groups of ^{235}U fission fragments, *viz.*, 19.5 and 25.4.

Measurements of the mean total range of ^{235}U fission fragments in photographic emulsions were made by various observers.[5] In Ilford nuclear research emulsions B1 and C2, the mean total track length was found to be about 24–25μ, which is about 1835 times smaller than the total range in air of 44.9 mm given above. The corresponding figures for UII alpha-particle tracks in these emulsions, as examined by Green and Livesey[5] and Lattes, Fowler, and Cuer,[6] were found to be about 1680 times smaller than the range in air. This means that the relative stopping power of the emulsion is somewhat different for fission fragments and for UII alpha-particles and, as an average over the range, about 8 percent larger for the fragments.

Keeping in mind that the stopping characteristics of the emulsion due to its large content of silver and bromine is similar to those of heavy gases, this is in agreement with present and previous cloud-chamber studies. Here the range of fission fragments in argon and xenon as compared with the range of polonium alpha-particles was found to be shorter than in air by about 3.5 and 9 percent, respectively. Incidentally, it may also be noted that the content of hydrogen is expected to contribute moderately to the diminution of the fragment ranges in the emulsion, since the fragment ranges relative to the range of alpha-particles were found shorter in hydrogen than in any other substance.

The authors wish to express their hearty thanks to Professor Niels Bohr for his continual interest and advice during these investigations.

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Erratum: Bremsstrahlung in High Energy Nucleon-Nucleon Collisions

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THROUGH an unfortunate oversight one set of intermediate states was omitted in the matrix element for the γ-emission accompanying proton-proton or neutron-neutron collisions. With the approximations of neglecting the nucleonic recoil energies in the energy denominators and of neglecting the γ-ray momentum compared to $\mathbf{p}_0 \pm \mathbf{p}$ in the Fourier transforms of the nuclear potential, there is found to be *no* γ-emission in the collision of like particles. (Equations (8) and (9) are incorrect.) For the magnetic emission this might have been anticipated from the fact that in the neutron-proton collision the effect is proportional to $(\mu_P - \mu_N)^2$. For the electric emission in the collision of two protons it is evident that in the center of mass system the total electric dipole moment is initially zero and remains zero if the recoil due to the γ-emission is neglected. There is therefore no electric dipole emission with the approximations made.

This classical argument in the electric case can be justified by the fact that the formula (6) given for the electric emission in the neutron-proton collision can be deduced from a completely classical treatment of the bremsstrahlung. According to classical electromagnetic theory the total energy radiated by a charged particle with trajectory $\mathbf{x}(t)$ (velocity $\mathbf{v}(t)$) is distributed in frequency and direction according to

$$dQ_{\nu,\,\Omega} = \nu^2 d\nu d\Omega / 4\pi^2 c^3 |\mathbf{I}|^2,$$

where

$$\begin{aligned}\mathbf{I} &= \int\int d\mathbf{r} dt \mathbf{n} \times e\mathbf{v}\delta(\mathbf{r} - \mathbf{x}(t)) \exp[i\nu(t - \mathbf{n}\cdot\mathbf{r}/c)] \\ &= e\int_{-\infty}^{\infty} dt \mathbf{n} \times \mathbf{v} \exp[i\nu(t - \mathbf{n}\cdot\mathbf{x}/c)] \\ &= \frac{ie}{\nu}\int_{-\infty}^{\infty} ds \exp[i\nu s]\frac{d}{ds}\left[\frac{\mathbf{n}\times\mathbf{v}}{1-\mathbf{n}\cdot\mathbf{v}/c}\right].\end{aligned}$$

Here $\mathbf{n}$ is the direction in which the radiation propagates. A change from t to the variable $s = t - \mathbf{n}\cdot\mathbf{x}/c$ and a partial integration yields the last expression for $\mathbf{I}$. Let the trajectory describe a collision of short duration in a region of dimensions a (the range of the nuclear forces). If we restrict ourselves to frequencies for which $\nu a/c \ll 1$ or to wave-lengths larger than a (γ-ray energies smaller than 137 $mc^2 = 70$ Mev) the integral $\mathbf{I}$ becomes

$$\mathbf{I} = \frac{ie}{\nu}\Delta\left[\frac{\mathbf{n}\times\mathbf{v}}{1-\mathbf{n}\cdot\mathbf{v}/c}\right],$$

where Δ means the change in the quantity as a result of collision. The formula (6) for the electric emission results if the classical radiation formula (with the $\mathbf{n}\cdot\mathbf{v}/c$ in the denominator neglected) is multiplied by the quantum mechanical cross section for a given change in velocity as a result of the nuclear collision. $\mathbf{n}\times\Delta\mathbf{v}$ becomes $\mathbf{n}\times(\mathbf{p}_0 - \mathbf{p})$ times $1/M$ and its square gives directly the angular dependence expressed in (6).

For a collision of two protons the integral $\mathbf{I}$ is a sum of two terms corresponding to the change in velocity of each proton. Neglecting the recoil due to the photon and the $\mathbf{n}\cdot\mathbf{v}/c$ in the denominator, we obtain zero for the total. These approximations are clearly no longer valid for relativistic energies of the nucleons.

The Existence of Stable Nuclei as Related to the Principle of Regularity and Continuity of Series and the Ends of Nuclear Shells

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IN 1916, when the atomic nucleus of Rutherford was only five years old, Harkins submitted a paper[1] which introduced a new

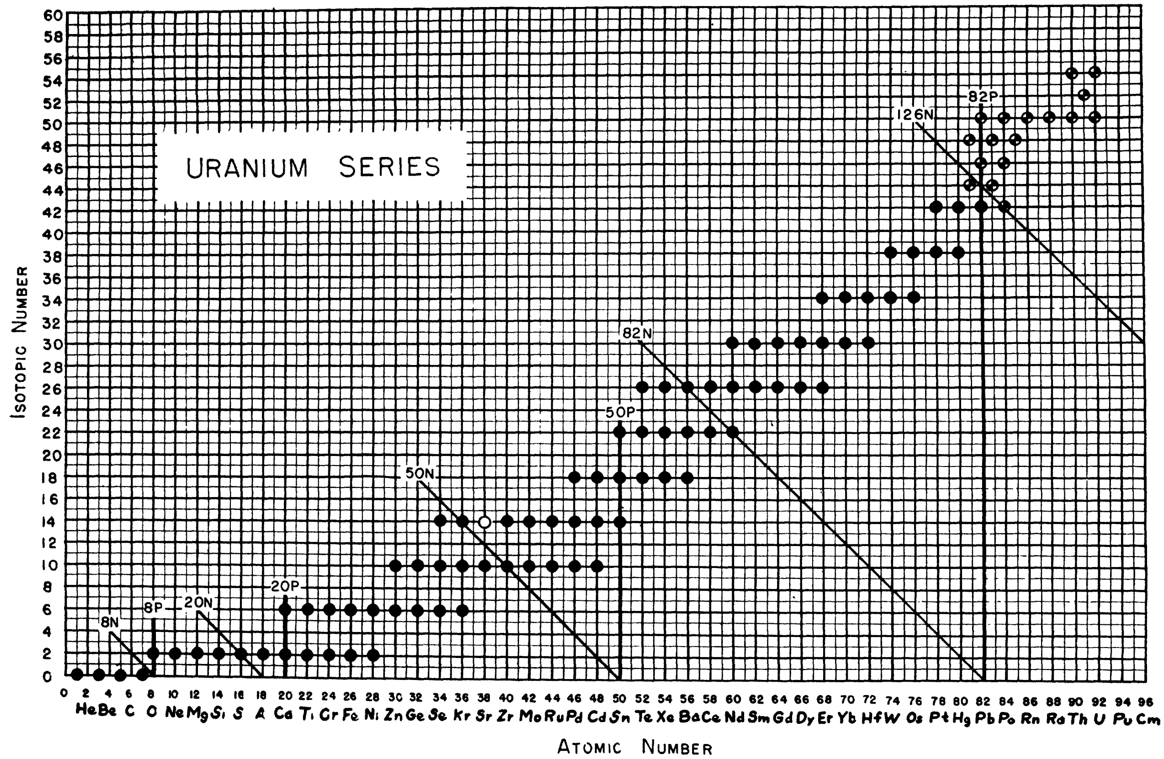

FIG. 1. Uranium series. The ends of neutron and proton shells are indicated by heavier lines at values 8, 20, 50, 82 and 126. The one unstable species is strontium of isotopic number 14, designated by an open circle.

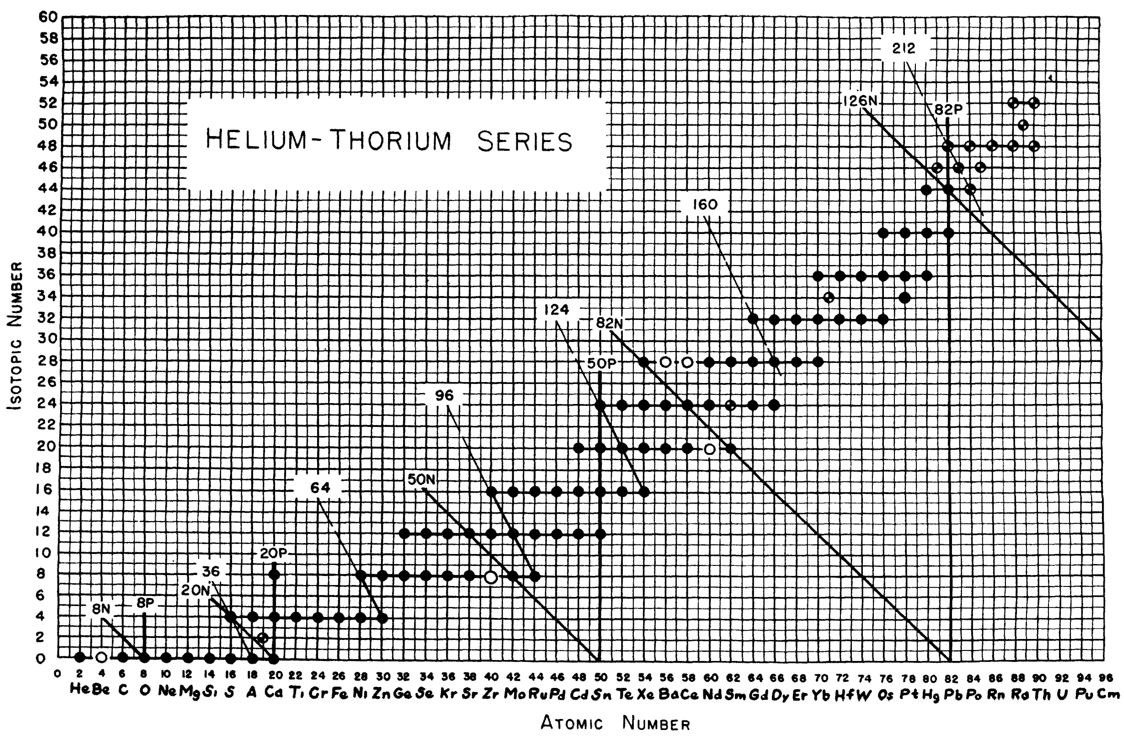

FIG. 2. Helium-thorium series. Symbols same as Fig. 1.

general principle into nuclear science. This indicated that the abundance of elements in the meteorites, earth and stars, taken as a whole, is dependent on nuclear relations, and not upon those which are chemical. It was shown that in the meteorites and in the rare earths each element of even number is much more abundant than either of the two adjacent elements of odd number. In a series of papers[2–14] (1920–23) the same was shown for neutrons. With respect to the number of neutrons (N) and of protons (P) the abundance is related to four classes.

4. Every species specified by a principle of continuity, considered later, exists and is stable, except in the relatively few cases in which some other principle intervenes and the nucleus is unstable (5 and 6 below).

5. For the even–even (N and P) series the stability and abundance are in general higher at the end of nuclear shells than for adjacent species not at the end of shells. The least stable and abundant species are adjacent in proton and neutron content to the ends of shells. For example in the uranium or $n+2$ series 83

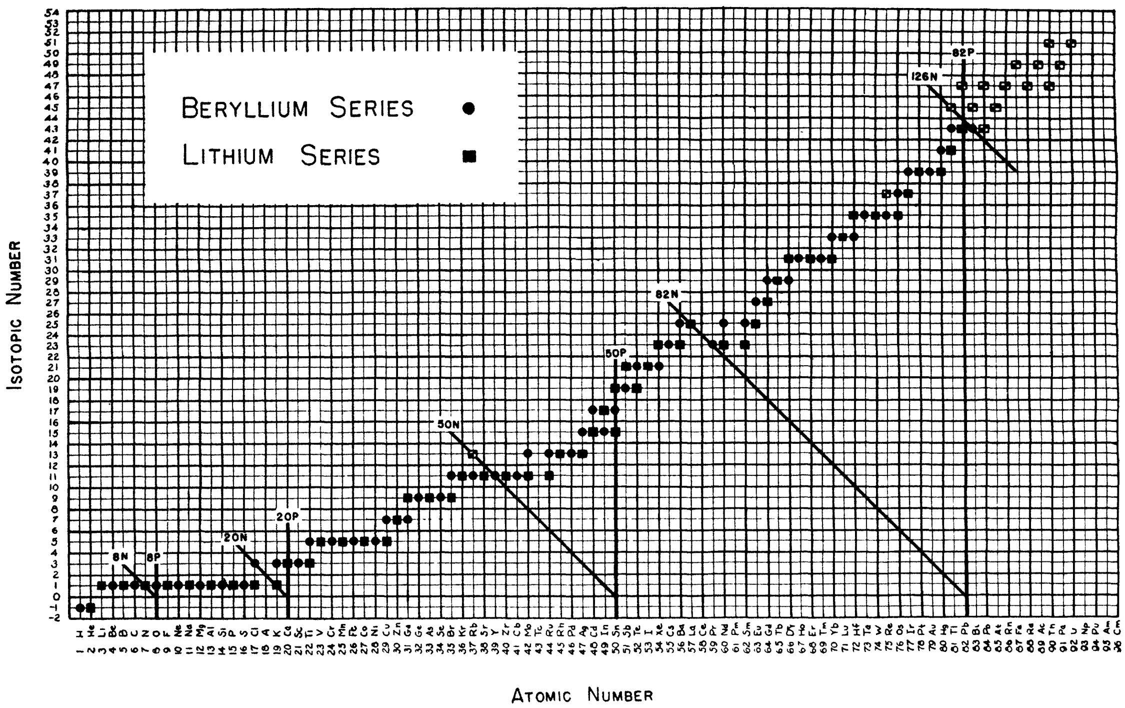

FIG. 3. Odd series of stable species. Radioactive species designated by a different symbol.

With respect to cosmic abundance almost all atoms (hydrogen excluded) belong to I (Even N–Even P), relatively few to II (Even N–Odd P) and III (Odd N–Even P), and none to IV (Odd N–Odd P). An exception to IV is that if $N=P$ odd-odd nuclei occur up to $P=7$ for nitrogen, an abundant atomic species in the universe. Similarly, for $N=P$ Ca^{40}, $N=20$, $P=20$ is the highest species.

It was shown also that all stable nuclei exist in a relatively narrow valley of stability. This is prominent when N/P *vs.* P is plotted.

These relations are basic for what is termed here the principle of regularity and continuity of series. This is illustrated here by Figs. 1 and 2, which represent the two even series of masses $4n+2$ and $4n$. These occupy a much wider region of the valley of stability than the even–odd and odd–even series of Fig. 3, which represents the $4n+3$ and $4n+1$ series.

Now it is extremely remerkable that:

1. No stable nuclear species is known *outside* the limits of these series in terms of proton (P) and neutron (N) content.

2. No stable species is known *inside* the series, except those specified by the series.

3. For even mass (M) both the number of protons and of neutrons is always even. No exception is known except for 4 species in which $N=P$ with P very small (7 or less). For odd mass ($P+N$) either P or N may be odd, but not both. All other nuclei are excluded from the system of stable nuclei.

positions are available, 82 are filled, and the one unfilled position is adjacent to the end of a shell.

6. In the even–odd and odd–even series, to the non-occurrence of adjacent isobars. In these series 106 positions are filled, 10 are unfilled by the non-occurrence of adjacent isobars. Elements 42 and 61 are excluded, as are the odd isotopes of argon (see their peculiar pattern in Fig. 3). Several of these are also close to the end of an 82 neutron shell.

7. In Fig. 2 (helium-thorium series) it appears as though three atomic species predicted by the theory of continuity, of isotopic number 8, and with atomic numbers 22, 24, and 26, are missing as stable isotopes. However, these are not predicted by the theory, since the irregularity is due to the occurrence of the highest isotope of calcium of isotopic number 8, whose stability is related to the fact that it lies at the end of a 20-proton shell.

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