excited hydrogen gas driven out of an electrodeless discharge into a side tube by the sudden pressures generated during the discharge. He found a lifetime for this radiation of the order of 10^{-5} sec. Zanstra[13] has offered electronic recombination as the explanation of these phenomena. It may be therefore that in these experiments Rayleigh has observed a process similar to the one proposed by the authors. There are, however, certain difficulties with this view, notably that the intensity of the exploded beam is continuously variable from the original intensity of the discharge tube at one end to extinction at the other. Direct excitation should be visible in the form of a discontinuity in intensity at the mouth of the side tube. Further investigation of Rayleigh's phenomenon is currently in progress.

CONCLUSION

The simplest explanation which can be given of the foregoing set of experiments is that a process exists for the production of radiation which is slow enough to permit diffusion during the lifetime of the systems from which the radiation originates. Estimates of the order of magnitude of the lifetime are of the order of 10^{-5} to 10^{-6} sec. This hypothesis is not in discord with any known experimental facts, and may account for some theoretical difficulties such as the discrepancy between experimental and theoretical cross sections for excitation of the triplet states in helium, which are in disagreement by a factor of about 10^6.

The indications are that the process is one in which the active systems are charged electrically, suggesting that they are ions. Conventional recombination of ions moving at random cannot be accorded with the facts, and a new type of recombination is proposed as a concomittant of the orderly processes now known as ambipolar diffusion. It is suggested that it is impossible to neglect the perturbing fields of the entire ion cloud in the theory of the recombination process, and perhaps even in the production of ionization.

[13] H. Zanstra, Proc. Roy. Soc. **186**, 237 (1946).

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Radiation Accompanying Meson Creation

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Electromagnetic radiation is expected to accompany the creation of charged mesons in nucleon-nucleon collisions. The amount and character of this radiation is calculated for mesons of spin 0, $\frac{1}{2}\hbar$, and $\hbar$, in each case with normal magnetic moment. The ratio of radiated energy to initial meson energy increases logarithmically for high meson energy in the first two cases, and is of the order of 1 percent for the highest energies of interest in cosmic radiation. For vector (spin $\hbar$) mesons, this ratio is $(11\alpha/4860\pi)(E/mc^2)^4$ in the relativistic region, where α is the fine-structure constant, and m and E are the rest mass and the initial energy of the meson in the coordinate system in which the center of mass of the colliding nucleons is at rest. This is so large at high energies that the assumption that π-mesons have spin $\hbar$ would, with a plausible theory of the multiplicity of meson production, account for the observation of soft radiation in conjunction with energetic nuclear events.

I. INTRODUCTION

THERE appears to be good evidence that a substantial part of the soft component of cosmic radiation originates in events that are identical with or closely related to those collisions of primary cosmic rays with air nuclei that give rise to the hard component.[1] A possible mechanism for the production of both components in nuclear collisions assumes that charged and neutral mesons are strongly coupled to nucleons, and that the neutral mesons decay quickly into electron-positron pairs or gamma-rays.[2] At the present stage of nuclear force theory, however, there is little reason to assume the existence of such neutral mesons, and the decay process is based on the further assumption that negative protons exist at least in intermediate states.[3,4] It seems of interest, therefore, to consider in some detail the electromagnetic radiation that, according to currently accepted theory, is a necessary by-product of the creation of any charged particle.[4a]

[1] B. Rossi, Rev. Mod. Phys. **20**, 537 (1948), interprets the data and gives a bibliography of experimental results; see especially Sections 15 and 18.

[2] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).

[3] R. J. Finkelstein, Phys. Rev. **72**, 415 (1947).

[4] High energy gamma-rays recently observed by Moyer, York, and Bjorklund [Phys. Rev. **75**, 1470 (1949)] cannot be accounted for by the results of the present paper. If not explained in some other way (bremsstrahlung), they may eventually provide evidence for the existence of unstable neutral mesons.

[4a] S. Hayakawa and S. Tomonaga, Prog. Theor. Phys. **2**, 161 (1947), have considered the radiation produced by the decelera-

It might at first be thought that the general characteristics of this radiation could be inferred from a classical consideration of a point charge, initially at rest, that is instantaneously set in motion with constant velocity. The coordinate system is that in which the center of mass of the colliding nucleons is at rest. When the classically computed spectrum is cut off for angular frequencies greater than $E/\hbar$, the fraction of energy radiated is of order $\alpha \ln(E/mc^2)$ for $E \gg mc^2$, where $\alpha = e^2/\hbar c$, and m, e, and E are the rest mass, charge, and total energy of the particle. This result is in qualitative agreement with those obtained from quantum electrodynamics for a scalar particle of spin 0, and for a Dirac particle of spin $\frac{1}{2}\hbar$ and normal magnetic moment. For a vector particle of spin $\hbar$, however, it is known that the interaction with the electromagnetic field has much stronger high frequency components, which manifest themselves in much larger cross sections for bremsstrahlung and pair production at high energies.[5] This more singular interaction, which is associated with a change in direction of the spin magnetic moment, is shown below to give a fraction of energy radiated proportional to $\alpha(E/mc^2)^4$. Thus, for high energies of meson creation, such radiation will be of significance in connection with cosmic rays if the mesons produced have spin $\hbar$.

The present calculation is analagous to those performed several years ago by Bloch[6] and by Knipp and Uhlenbeck[7] in connection with the continuous gamma-radiation that accompanies radioactive beta-decay of nuclei. The method used by Bloch, which is the same as the second method used by Knipp and Uhlenbeck, consists in treating radiative beta-decay as a second-order process in which both electromagnetic and beta-couplings appear. The first method of Knipp and Uhlenbeck, on the other hand, assumes that the distribution of emitted electrons with regard to energy and angular momentum is given in advance, either by experiment or beta-decay theory. Then the initial electron state is taken to be an outgoing spherical wave, and the probability for a first-order radiative transition to a plane-wave final state is computed.

At the present time, not enough is known about the details of meson production in nuclear collisions to carry through the analog of the second-order calculation mentioned above. It is, however, perfectly feasible to perform the first-order calculation. This has in effect been done by Knipp and Uhlenbeck[7] for mesons of spin $\frac{1}{2}\hbar$ and normal magnetic moment, and the results are quoted below. The corresponding calculation for mesons of spin $\hbar$ is most readily accomplished with the help of a formulation of vector meson theory recently developed by Gunn[8] on the basis of earlier work by others.[9] Gunn's formulation is also applicable to spin 0 mesons; alternatively, the scalar relativistic wave equation can be used in this case. There seems to be little doubt that this type of calculation will give at least a good estimate of the radiation probability, even though it does not always agree precisely with the more exact second-order calculation in the case of radiative beta-decay.[10]

A separate and more serious question concerns the size to be assumed for the source of the outgoing spherical initial wave. While the low energy results for all three mesons, and the high energy results for mesons of spin 0 and $\frac{1}{2}\hbar$ do not depend significantly on the radius of the source, this is not the case for the high energy vector meson. Introduction of a cut-off at the Compton wave-length of the meson makes the fraction of energy radiated of order $\alpha(E/mc^2)^2$. However, the estimated multiplicity of meson production probably is also reduced by such a cut off,[2] so that the average energy per meson is increased. This would tend to counteract the reduction in the power of the energy that appears in the expression for the radiation probability. Thus, while it is not now clear whether or not such a cut-off should be introduced, a substantial amount of electromagnetic radiation is expected to accompany the creation of high energy vector mesons in either case.

II. CALCULATION FOR MESONS OF SPIN $\hbar$

In Gunn's notation[8] the wave equation for a free-vector meson is

$$i\hbar(\partial\psi/\partial t) = H\psi, \tag{1}$$

$$H = mc^2\beta + [\beta(1+\eta)/2m]\mathbf{p}^2 - (\beta\eta/m)(\boldsymbol{\sigma}\cdot\mathbf{p})^2. \tag{2}$$

The wave function ψ has six components, the other four redundant components of the original theory having been eliminated. The representation is fixed by the choices

$$\sigma_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \sigma_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & i \\ i & -i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

for the spin matrices, and the choices

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{4}$$

for the charge matrices.

tion of the primary proton, and obtained a logarithmic dependence on energy similar to our Eqs. (22) and (23). For high energies the center of mass coordinate system must be used, and this radiation is relatively unimportant.

[5] Oppenheimer, Snyder, and Serber, Phys. Rev. **57**, 75 (1940); R. F. Christy and S. Kusaka, Phys. Rev. **59**, 405 (1941).

[6] F. Bloch, Phys. Rev. **50**, 272 (1936).

[7] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936).

[8] J. C. Gunn, Proc. Roy. Soc. **A193**, 559 (1948).

[9] W. Heitler, Proc. Roy. Irish Acad. **A49**, 1 (1943). References to earlier work are given here and in Gunn's paper.

[10] P. Morrison and L. I. Schiff, Phys. Rev. **58**, 24 (1940). D. L. Falkoff and C. S. Chang [private communication and Phys. Rev. **73**, 1220 (1948)] have shown that the difference between the two theories is not very great for forbidden beta-decay.

The first-order wave Eq. (1) is achieved at the expense of making the Hamiltonian H non-Hermitian. An operator Ω represents an observable quantity if $\beta\Omega$ is Hermitian, and it is easily verified that (2) has this property. The charge density is $\psi^*\beta\psi$, where ψ^* is the Hermitian adjoint of ψ, and the expectation value of the energy is $\int\psi^*\beta H\psi d\tau$. Stationary solutions of (1) satisfy the equation $H\psi=E\psi$. It turns out that both the charge density and the frequency $E/\hbar$ can have positive and negative values, while the energy is always positive.

The inclusion of a radiation field represented by a divergenceless vector potential $\mathbf{A}$ is effected by replacing $\mathbf{p}$ by $\mathbf{p}-(e/c)\mathbf{A}$ in (2), and adding the term[11]

$$-(e\hbar/2mc)\beta(1+\eta)(\boldsymbol{\sigma}\cdot\mathbf{curl\ A}) \tag{5}$$

to H. The result is to replace H given in (2) by $H+H'$, where

$$\begin{aligned}H'=-(e/mc)\beta(1+\eta)(\mathbf{A}\cdot\mathbf{p})\\ +(e/mc)\beta\eta[(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{p})+(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{A})]\\ -(e\hbar/2mc)\beta(1+\eta)(\boldsymbol{\sigma}\cdot\mathbf{curl\ A}).\end{aligned} \tag{6}$$

The time-dependent perturbation theory will be used in the following to calculate the rate of transition from an outgoing spherical wave to a plane wave with emission of a quantum of radiation. This theory assumes that the various eigenfunctions of the unperturbed Hamiltonian are orthogonal; if they are not, a suitable weighting factor must be used. Since H is not Hermitian, the eigenfunctions are actually not orthogonal, and β must be used as a weighting factor. Thus, all transition matrix elements are for the operator $\beta H'$, not for H' itself.

The initial state wave function has the form

$$\psi^{(i)}=\begin{pmatrix}\psi_1^{(i)}\\ \psi_2^{(i)}\end{pmatrix}, \tag{7}$$

where $\psi_1^{(i)}$ and $\psi_2^{(i)}$ are each three-component functions. The spin matrices (3) operate on these components, while the charge matrices (4) operate in the ψ_1, ψ_2 space. Since $\psi^{(i)}$ is to represent an outgoing wave in the coordinate system in which the center of mass of the colliding nucleons is at rest, it has the form

$$\begin{aligned}\psi_1^{(i)}=a_1h_{j-1}^{(1)}(\kappa r)u_{j-1,j,m}+a_2h_{j+1}^{(1)}(\kappa r)u_{j+1,j,m}\\ +a_3h_j^{(1)}(\kappa r)u_{j,j,m},\\ \psi_2^{(i)}=b_1h_{j-1}^{(1)}(\kappa r)u_{j-1,j,m}+b_2h_{j+1}^{(1)}(\kappa r)u_{j+1,j,m}\\ +b_3h_j^{(1)}(\kappa r)u_{j,j,m}.\end{aligned} \tag{8}$$

Here $E=+(\hbar^2c^2\kappa^2+m^2c^4)^{\frac{1}{2}}$ is assumed to be positive for a positively charged meson, j is its total angular momentum quantum number, $h_l^{(1)}(z)\equiv(\pi/2z)^{\frac{1}{2}}H_{l+\frac{1}{2}}^{(1)}(z)$ is the spherical Hankel function of the first kind, and the u's are the normalized three-component angular eigenfunctions given in Gunn's Eqs. (8). Three of the six constant coefficients in (8) are arbitrary, except for normalization. If, for example, a_1, a_2, a_3 are specified, then the b's are given by the relations

$$b_1=\frac{E-mc^2}{E+mc^2}\frac{a_1-2[j(j+1)]^{\frac{1}{2}}a_2}{2j+1},$$

$$b_2=\frac{E-mc^2}{E+mc^2}\frac{2[j(j+1)]^{\frac{1}{2}}a_1+a_2}{2j+1},\quad b_3=\frac{E-mc^2}{E+mc^2}a_3.$$

The total outward current associated with (7) is

$$\frac{v}{\kappa^2}\frac{4Emc^2}{(E+mc^2)^2}(|a_1|^2+|a_2|^2+|a_3|^2), \tag{9}$$

expressed in mesons per unit time, where v is the speed.

The final state wave function also has the form (7), where now

$$\psi_1^{(f)}=\begin{pmatrix}A_1\\ A_2\\ A_3\end{pmatrix}e^{i\boldsymbol{\kappa}_1\cdot\mathbf{r}},\quad \psi_2^{(f)}=\begin{pmatrix}B_1\\ B_2\\ B_3\end{pmatrix}e^{i\boldsymbol{\kappa}_1\cdot\mathbf{r}}, \tag{10}$$

and $E_1=+(\hbar^2c^2\kappa_1^2+m^2c^4)^{\frac{1}{2}}$ is the total energy of the meson after it has radiated. As with (8), three of the coefficients are arbitrary; if the A's are specified, the B's are given by

$$B_1=\frac{E_1-mc^2}{E_1+mc^2}(A_1\cos^2\theta_1+A_2\sin^2\theta_1e^{-2i\phi_1}+\sqrt{2}A_3\cos\theta_1\sin\theta_1e^{-i\phi_1}),$$

$$B_2=\frac{E_1-mc^2}{E_1+mc^2}(A_1\sin^2\theta_1e^{2i\phi_1}+A_2\cos^2\theta_1-\sqrt{2}A_3\cos\theta_1\sin\theta_1e^{i\phi_1}),$$

$$B_3=\frac{E_1-mc^2}{E_1+mc^2}[\sqrt{2}A_1\cos\theta_1\sin\theta_1e^{i\phi_1}-\sqrt{2}A_2\cos\theta_1\sin\theta_1e^{-i\phi_1}-A_3(\cos^2\theta_1-\sin^2\theta_1)],$$

where θ_1, ϕ_1 are the polar angles of the propagation vector $\boldsymbol{\kappa}_1$, referred to the axis of quantization of the initial state as polar axis. The charge density associated with (10) is

$$[4E_1mc^2/(E_1+mc^2)^2](|A_1|^2+|A_2|^2+|A_3|^2), \tag{11}$$

expressed in mesons per unit volume.

The transition probability is proportional to the square of a matrix element that is of the form $\int\psi^{(f)*}\beta H'\psi^{(i)}d\tau$. The term in the vector potential $\mathbf{A}$ that corresponds to the emission of a quantum of propagation vector $\mathbf{k}$ and (unit) polarization vector $\boldsymbol{\varepsilon}$ is $\boldsymbol{\varepsilon}(2\pi\hbar c/kL^3)^{\frac{1}{2}}e^{-i\mathbf{k}\cdot\mathbf{r}}$, where it is convenient to quantize all fields in a large cubical box of edge length L. Now for an initial situation that is specified by some choice of the a's and of the total angular momentum quantum number j, there are $2j+1$ states of different m values, and the transition probabilities computed for all of these must be averaged. Since the result is independent of the choice of the axis of quantization of the initial states, there is no loss of generality in choosing $\mathbf{k}$ along

[11] The sign of (5) is given incorrectly in Gunn's paper.

the z axis and $\boldsymbol{\varepsilon}$ along the x axis. Thus in Eq. (6), $\mathbf{A}$ can be replaced by

$$A_x=(2\pi\hbar c/kL^3)^{\frac{1}{2}}e^{-ikz},\quad A_y=A_z=0. \tag{12}$$

The space integrals that appear in the calculation of the matrix elements can easily be reduced to the radial integral of a product of spherical Bessel and Hankel functions. If the initial wave source is assumed to be a point, the integral extends over r from 0 to ∞,[12]

$$\int_0^\infty j_l(qr)h_l^{(1)}(\kappa r)r^2dr=(q/\kappa)^l[i/\kappa(\kappa^2-q^2)], \tag{13}$$

where q is the magnitude of the vector sum of $\mathbf{k}$ and $\boldsymbol{\kappa}_1$. For a finite source, the integral must be cut off in some way at small values of r, and its value will then depend on the details of the cut-off.

In order to get a definite result, it is necessary to fix on a particular initial j value and coefficients a_1, a_2, a_3. These are chosen so that the initial states represent, in the non-relativistic limit, a positive meson of zero orbital angular momentum: $j=1$, $a_2=a_3=0$. The remaining constant a_1 is chosen so that the outgoing flux (9) is equal to unity. The final states are chosen successively to have only A_1, A_2, or A_3 different from zero and of magnitude such that (11) yields one meson in the box of volume L^3. It is necessary, therefore, to compute transition probabilities between each of three initial states ($m=1$, 0, -1) and three final states; the probabilities are then summed over the final states and averaged over the initial states. In practice, only six of the nine possible matrix elements need be computed, since the results for $m=-1$ are the same as those for $m=+1$.

The differential transition probability per unit time is given by the usual expression

$$dw=(2\pi/\hbar)\rho(f)\left|\int\psi^{(f)*}\beta H'\psi^{(i)}d\tau\right|^2,$$

where the energy density of final states in the volume L^3 is

$$\rho(f)=(L/2\pi)^6(E_1\kappa_1k^2dkd\Omega_kd\Omega_1/\hbar^2c^2).$$

The solid angle element $d\Omega_k$ for the emitted quantum can be replaced by 4π, since the choice of the z axis (direction of emission) is arbitrary. Also,

$$d\Omega_1=\sin\theta_1d\theta_1d\phi_1,\quad\text{and}\quad E=E_1+\hbar ck.$$

Since the initial state is so normalized that one meson is given off by the source per unit time, dw is the differential probability of finding a quantum and a meson in one of a group of neighboring final states when one meson has emerged from the source. It is therefore a dimensionless quantity.

The details of the calculation are quite lengthy, and are not reproduced here. The results are complicated in the general case, but become reasonably simple for initial meson energies close to mc^2 (non-relativistic case) and large compared to mc^2 (relativistic case). In the non-relativistic limit, for both directions of polarization of the quantum,

$$dw=\frac{\alpha}{2\pi}\frac{v_1^3}{vc^2}\frac{dk}{k}\sin^3\theta_1d\theta_1, \tag{14}$$

where v_1 is the speed of the meson after radiation, and θ_1 is the angle between quantum and meson. This gives a logarithmically divergent total radiation probability (infra-red catastrophe), so that it is natural to compute the fraction of the initial meson kinetic energy that appears in the form of electromagnetic radiation:

$$W=\int\frac{2\hbar ck}{mv^2}dw=\frac{4\alpha}{15\pi}\frac{v^2}{c^2}=\frac{8\alpha}{15\pi}\frac{E-mc^2}{mc^2}. \tag{15}$$

In the relativistic limit, the result that corresponds to (14) is

$$\begin{aligned}dw=(\alpha/54\pi)(\hbar/mc)^4(\kappa_1{}^3kdk/\kappa^3)[2k^2(2+3\cos^2\theta_1)\\+10k\kappa_1\cos\theta_1(1+\cos^2\theta_1)\\+5\kappa_1{}^2(1+\cos^2\theta_1)]\sin\theta_1d\theta_1.\end{aligned} \tag{16}$$

The fraction of energy radiated is, in analogy with (15),

$$W=\int\frac{\hbar ck}{E}dw=\frac{11\alpha}{4860\pi}\left(\frac{E}{mc^2}\right)^4. \tag{17}$$

In this case, the expression (16) for dw, which is the leading term for high energies, does not exhibit an infra-red catastrophe, although some of the neglected terms do. Thus, it is possible to compute the total transition probability, and hence, the average energy of an emitted quantum,

$$(\hbar ck)_{Av}=EW\Big/\int dw=(77/254)E. \tag{18}$$

These results refer to the coordinate system in which the center of mass of the colliding nucleons is at rest.

Since the quantity W given by (17) increases rapidly with increasing initial meson energy, it is natural to inquire as to the limit of validity of the perturbation theory treatment of the coupling between meson and electromagnetic field. In the present case, where the computed transition probability provides a measure of the distortion of the unperturbed initial state by admixed final states, it seems reasonable to require that the total transition probability $\int dw$ be small in comparison with unity. If then m is the mass of a π- or heavy-meson, the theory is limited to initial meson energies of the order of 10^9 ev or less in the center of mass coordinate system.

The existence of strong high frequency components for the interaction between a vector meson and the electromagnetic field suggests that the high energy expression for W would be somewhat smaller than (17) if the initial state source were taken to have a finite size. This is, in fact, the case. Introduction of a cut-off

[12] G. N. Watson, *Theory of Bessel Functions* (The Macmillan Company, New York, 1945), second edition, p. 401. The equivalent Eq. (44) of reference 7 agrees with Eq. (13) for $l=0$ and 1, but is incorrect for $l\geqq 2$.

in the radial integral (13) at the nucleon "radius," or roughly at $\hbar/mc$, shows that (15) is unchanged, but that the order of magnitude of W at high energies is

$$W \sim \alpha(E/mc^2)^2. \qquad (19)$$

The numerical coefficient is very difficult to compute and depends on the detailed way in which the integral is cut off.

III. CALCULATIONS FOR MESONS OF SPINS 0 AND $\frac{1}{2}\hbar$

The formalism utilized above is applicable to mesons of spin 0 if the spin matrices are properly chosen. Equations (3) must be replaced by $\boldsymbol{\sigma}=0$, and the charge matrices (4) now operate on a two-component wave function. The calculation can of course also be carried through with the help of the scalar relativistic wave equation, which is of second order in the time derivative. The results are easily written in closed form for all energies. The differential transition probability for an initial state of zero orbital angular momentum is

$$dw=\frac{\alpha}{2\pi}\frac{c}{v}\frac{(\hbar c\kappa_1)^3}{E}\frac{dk}{k}\frac{\sin^3\theta_1 d\theta_1}{(E_1-\hbar c\kappa_1\cos\theta_1)^2}. \qquad (20)$$

The corresponding result for mesons of spin $\frac{1}{2}\hbar$ and all energies is given by Knipp and Uhlenbeck[7] for an initial state that represents, in the non-relativistic limit, a meson of zero orbital angular momentum ($j=\frac{1}{2}$),

$$dw=\frac{\alpha}{2\pi}\frac{c}{v}\frac{\hbar c\kappa_1}{E}\frac{dk}{k}\left\{\frac{E^2+2E_1mc^2+E_1^2}{(E+mc^2)(E_1-\hbar c\kappa_1\cos\theta_1)} -\frac{m^2c^4}{(E_1-\hbar c\kappa_1\cos\theta_1)^2}-1\right\}\sin\theta_1 d\theta_1. \qquad (21)$$

Since (20) and (21) both show an infra-red catastrophe, only the fraction of energy radiated can be calculated. The results in the non-relativistic limit are identical with Eq. (15), as would be expected, since spin effects are of magnetic origin, and hence, of higher order in v/c than charge effects. In the relativistic limit, the expressions analogous to (17) are

$$W=(\alpha/\pi)[\ln(2E/mc^2)-3/2], \text{ for spin 0}, \qquad (22)$$

$$W=(4\alpha/3\pi)[\ln(2E/mc^2)-19/12], \text{ for spin } \tfrac{1}{2}\hbar. \qquad (23)$$

Equations (22) and (23) are unaffected by the assumption of a cut-off at the radius $\hbar/mc$.

It is interesting to note that at high energies, Eqs. (20) and (21) indicate a close angular correlation between quantum and meson in the center of mass coordinate system for spins 0 and $\frac{1}{2}\hbar$, the main contribution coming from angles $\theta_1 \lesssim mc^2/E_1$. For spin $\hbar$, on the other hand, Eq. (16) shows no such marked correlation.

IV. DISCUSSION OF RESULTS

The foregoing results can be used to estimate the amount of electromagnetic radiation produced in collisions of primary cosmic rays with nucleons. As a numerical example, a primary nucleon energy of 10^{15} ev will be assumed (Auger shower). From the calculation of Lewis, Oppenheimer, and Wouthuysen,[2] an average multiplicity of 280 mesons is estimated for such an encounter. While this is based on a pseudoscalar meson field, the high multiplicity is due to the strong singularity in the meson-nucleon interaction; since this also occurs for a vector meson field, it is reasonable to use the same result for mesons of spin $\hbar$. If now it is assumed that half of the primary energy (in the center of mass system) goes into meson production, the average initial meson energy in the center of mass coordinate system is about 2.5×10^9 ev, and Eq. (17) shows that $W\cong0.55$. This is about the limit of applicability of the perturbation theory; in particular, such a strong electromagnetic coupling would be expected to have a direct influence on the meson production calculation. If the π-mesons have spin 0, on the other hand, (22) shows that $W\cong0.005$.

For vector mesons, the multiplicity is proportional to $E_0^{\frac{1}{4}}$, so long as the primary energy E_0 is large in comparison with 10^9 ev; then W is proportional to $E_0^{\frac{3}{2}}$. Thus, it would seem at first that W becomes quite small for nucleon collisions in the 10^9-10^{11} ev range. However, wide fluctuations in the energies of individual mesons are to be expected, especially at the lower multiplicities; then, since the more energetic mesons radiate far more than the less energetic ones, the average W will be considerably greater than estimated from an equal division of energy among the mesons.

The assumption of an extended source, which leads to (19), is equivalent to the removal of the strong singularity from the meson-nucleon interaction, and hence, greatly reduces the multiplicity. This tends to increase the average meson energy, and hence, compensate for the decreased power of E in the expression for W. Considerations based on a cut-off must, however, be regarded as speculative at the present time.

The mesons discussed above are all assumed to have normal magnetic moments. In the case of spin 0 (scalar or pseudoscalar), no magnetic moment is possible, and the expression (22) for W is unique. For a meson of spin $\frac{1}{2}\hbar$, any additional magnetic moment would be expected to make the coupling with the radiation field more singular,[13] and hence make W substantially larger than that given by Eq. (23). There are good arguments from other directions, however, against π-mesons having spin $\frac{1}{2}\hbar$.[14] In the case of spin $\hbar$, the assumption of an anomalous magnetic moment would also be likely to increase rather than decrease W, so that a substantial amount of electromagnetic radiation is expected to accompany the creation of energetic vector mesons in any case. These results favor the assignment of spin $\hbar$ rather than spin 0 to the heavy (π) mesons of the cosmic radiation.

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[13] W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).
[14] R. E. Marshak, Phys. Rev. **75**, 700 (1949).