

For propagation along the x axis the "source strength" of such converted energy is just $Re(\partial/\partial x)(\tau\dot{H}/T_0)$.

Thus, whereas liquid helium II provides the only example of complete thermal and mechanical interdependence, the more limited thermomechanical properties of ordinary substances are evident from general thermodynamic considerations. Further implications are being investigated.

The author expresses his sincere gratitude to Professor Philip M. Morse for helpful and stimulating discussions.

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* Since this study was undertaken, related work by several other investigators has appeared. Where overlap does now occur, the independently obtained results of the author are believed to present fresh viewpoints.

** A particularly clear derivation of the velocity of second sound has been given by Peshkov (see reference 3) based on the use of the second law.

*** In reference 5 the author gave the equivalent expression in mechanical terms. The thermodynamic properties of liquid helium II lead to a direct conversion between the two forms.

**** We are indebted to H. W. Woolley for an independent derivation of a form equivalent to (5).

† Peshkov mentions in reference 3 that the expression applies to materials other than helium II but without interpretation or consideration of the inherent decay.

†† The thermal impedance of a classical layer of thickness l backed by helium II is $(i\omega\rho c l k_e)^{-1} \coth((i\omega\rho c l/k_e)l) + \coth^{-1}(i\omega\rho c l/k_e)l / \rho c v_s$, where ρ , c , and k_e are, respectively, the density, heat capacity, and heat conductivity of the material. For a thin layer this approaches $(\rho c v_s)^{-1} + a/k_e$, independent of frequency and nearly equal to the thermal impedance of helium II.

¹ D. Osborne, *Nature* **162**, 213 (1948).

² R. Dingle, *Proc. Phys. Soc. London* **61**, 9 (1948).

³ V. Peshkov, *J. Exp. Theor. Phys., U.S.S.R.* **18**, 857 (1948).

⁴ J. Pellam, *Phys. Rev.* **75**, 1183 (1949).

⁵ J. Pellam, *Phys. Rev.* **73**, 608 (1948).

⁶ L. Tisza, *Phys. Rev.* **72**, 838 (1947).

⁷ J. Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1945).

Nuclear Spin 5/2 for ${}_{40}\text{Zr}^{91}$

O. H. ARROE AND J. E. MACK

University of Wisconsin, Madison, Wisconsin*

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IN interference spectrograms** of a Zr⁹¹-enriched sample of zirconium*** in a hollow cathode source,¹ the $4d^25s^2\ ^3P_0 - 4d^25s^2\ ^3P_1 - 5p\ ^3S_1$ line² at ν_{17777} presents the following structure, showing that the spin, I , of the interesting³ nucleus ${}_{40}\text{Zr}^{91}$ is undoubtedly 5/2 units:

$$\begin{array}{ll} (I+1) & -0.0701 \pm 0.0010 \text{ cm}^{-1,4} \\ \text{even isotopes} & (\text{arbitrary}) \quad 0.0000 \pm 0.0012 \text{ cm}^{-1,} \\ (I) & +0.0273 \pm 0.0010 \text{ cm}^{-1,} \\ (I-1) & +0.0983 \pm 0.0010 \text{ cm}^{-1,} \\ I, \text{ from the Landé interval rule } 2.69 & \begin{array}{l} +0.58 \\ -0.41. \end{array} \end{array}$$

Details, including the structure of other lines, will be submitted later.

* Work done on Navy contract N7 onr-285TO #1, NR 019 107.

** With auxiliary dispersion produced by a special Hilger spectrograph bought with funds granted by the University research committee.

*** Produced by the Y-12 plant, Carbide and Carbon Chemicals Corporation, and obtained by allocation from the AEC.

¹ O. H. Arroe and J. E. Mack, *Phys. Rev.* **76**, 173 (1949).

² W. F. Meggers and C. C. Kiess, *J. Research Nat. Bur. Stand.* **9**, 309 (1932).

³ E. Feenberg and K. C. Hammack, *Phys. Rev.* **75**, 1877 (1949), especially pp. 1882 and 1893.

⁴ The quantities preceded by the symbol \pm have been made larger than the spread of the data by amounts judged to be enough to make it highly improbable that the true values lie outside the ranges indicated.

The Binding Energy of the Triton

R. E. CLAPP*

*Lyman Laboratory of Physics, Harvard University,
Cambridge, Massachusetts*

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A VARIATIONAL calculation of the binding energy of the H³ nucleus has been performed using the Rarita-Schwinger¹ interaction (square well with tensor force), as modified to include a charge-exchange factor $(-\frac{1}{2}\tau_1 \cdot \tau_2)$. Except for the charge-

TABLE I. Computed triton binding energy.

Trial functions (see text)		Triton binding energy	
S-state	D-states	In Mev	In percent of 8.3 Mev
(Finite power series)			
I	—	1.80	21.7
I	II	3.74	45.1
I	II, III, IV	5.60	67.5
(Extrapolated)			
I	—	1.80	21.7
I	II	3.88	46.7
I	II, III, IV	5.80	69.9
(Gerjuoy and Schwinger; S-state and D-state)			
		39	
(Feshbach and Rarita)			
S-state alone		21	
S-state and D-state (estimated maximum)		40-50	

exchange factor, which has very small effect on the triton binding energy, this is the same problem as that studied by Gerjuoy and Schwinger² and by Feshbach and Rarita.³ The same variational method was used here, but a much more elaborate trial function was inserted. The calculations are still in progress, but the results so far obtained (and reported in Table I) show that other D-states not included in the previous calculations^{2,3} make a substantial contribution to the binding energy.

The basis for the present method is the construction of a denumerable series of orthogonal functions which form a complete set for the nuclear three-body problem. With the type of potential that was assumed, the triton ground state can include only a selected group of these functions, characterized by even parity, by $J = \frac{1}{2}$ for the total angular momentum, by $T = \frac{1}{2}$ for the total isotopic spin, and by $T_3 = -\frac{1}{2}$ for the third component of isotopic spin (to denote two neutrons and one proton). This sub-set of functions may then be arranged in order of increasing kinetic energy (which is roughly equivalent to the order of decreasing importance) and introduced systematically into the variational principle.

In setting up the system of orthogonal functions the usual relative coordinates, $\mathbf{r}: (r, \theta, \varphi)$ and $\mathbf{g}: (\rho, \eta, \psi)$, have been transformed to a six-dimensional spherical coordinate system, with a single radial variable,

$$R = (\frac{2}{3}r^2 + \frac{1}{2}\rho^2)^{\frac{1}{2}} \quad (1)$$

and five angular variables. The angular parts of the orthogonal functions are hyperspherical harmonics $Y_{K,L}$ which satisfy the defining equation:

$$\nabla^2(R^K \cdot Y_{K,L}) = 0 \quad (2)$$

with

$$\nabla^2 \equiv \frac{2}{3}\nabla_r^2 + 2\nabla_\rho^2. \quad (3)$$

These harmonics can be classified by their value of L into S, P, D , etc., states, and combined with suitable spin and isotopic spin functions to give the admissible set of ${}^2S_{\frac{1}{2}}, {}^2P_{\frac{1}{2}}, {}^4P_{\frac{1}{2}}$, and ${}^4D_{\frac{1}{2}}$ triton functions. With each harmonic $Y_{K,L}$ there will also be a radial factor $R^K \cdot f(R)$, where the function $f(R)$ need be defined only for $R > 0$ and may therefore be written as a function of R^2 . The form chosen was a Gaussian multiplied by a power series:

$$f(R) = e^{-\mu R^2}(a + bR^2 + cR^4 + \dots). \quad (4)$$

In Table I are listed the binding energies resulting from the successive inclusion of one 2S -state function (I) and three 4D -state functions (II, III, IV). The S-state is essentially the same as that used by Gerjuoy and Schwinger and by Feshbach and Rarita, and function II corresponds to their D-state, but functions III and IV contain D-states which they omitted. The binding energy is given in Mev and in percent of the experimental value which they used, 8.3 Mev.**

In the present calculations, the same Gaussian parameter μ was used for all of the four functions. This does no harm if each power series is sufficiently long. The power series for functions I,

II, III, and IV contained 7, 5, 3, and 3 terms, respectively.*** There were thus 18 adjustable coefficients and one Gaussian parameter, or a total of 19 independent variational parameters. In order that the rate and degree of convergence might be estimated, most of the 18 terms in the variational function were added one at a time. From the regularities which appeared it was possible to extrapolate with some assurance to the binding energy which would have been computed from the same four angular functions if it had been possible to use infinite rather than finite power series in the radial factors. The extrapolated results are also given in Table I. The effects of other 2S and 4D functions are being investigated, as well as the (very small) contributions of P -state functions. A detailed report will be submitted later. Although only the Rarita-Schwinger potential has been used so far, the method of analysis is designed to handle any proposed potential and to test its ability to account for the properties of the triton.

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* Coffin Fellow (General Electric Educational Fund), 1947-48; AEC Predoctoral Fellow, 1948-49. (The work reported on here extended over both years.) The writer is now at the Massachusetts Institute of Technology.

** The experimental triton binding energy given by L. Rosenfeld (*Nuclear Forces II* (Interscience Publishers, Inc., New York, 1949), p. 501) is 8.39 Mev. A still more recent value, 8.50 Mev, is based on the redetermination by Bell and Elliott of 2.235 Mev for the deuteron binding energy (Phys. Rev. **74**, 1552 (1948)). The old value, 8.3 Mev, was used in Table I for comparison with the previous calculations. It should be noted also that the Rarita-Schwinger constants are based on the value of 2.17 Mev for the deuteron binding energy.

*** To simplify the large secular equations, each power series was written as the sum of an equivalent series of orthogonal polynomials.

¹ W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

² E. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942).

³ H. Feshbach and W. Rarita, Phys. Rev. **75**, 1384 (1949).