TABLE II. Summary of β-spectrometer investigations.

| Isotope | β-max. kev | β-lines | hν |
|---|---|---|---|
| $Xe^{133}$ | 315 | 25.6 (Auger, medium) | |
| | | 30.9 (Auger, weak) | |
| | | 46.5 (*K* 1, very strong) | 82.4 |
| | | 59.3 (*K* 2? weak) | 95.2 (?) |
| | | 77.9 (*L* 1, medium) | 83.6 |
| | | 200 (*K* 3 weak) | 236 |
| $Xe^{135}$ | 930 | 214 (*K*) | 250 |
| | | 242 (*L*) | |
| $Xe^{138}$ | 2680 | | |

The activities of $Xe^{133}$, $Xe^{135}$, and $Cs^{138}$ on the collector plate, were sufficient for β-spectrometer investigations. The β-spectrometer data are summarized in Table II.

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[1] Bergström, Thulin, Svartholm, and Siegbahn, Ark. for Fysik 1, No. 11 (1949).

[2] I. Bergström and S. Thulin, Phys. Rev. 76, 313 (1949).

[3] Koch, Kofoed-Hansen, Kristensen, and Drost-Hansen, Phys. Rev. 76, 279 (1949).

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## Second Sound and Classical Heat Flow[a], *

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THE reversibility of the thermomechanical effect in liquid helium II and the inertial characteristics of the associated heat flow result in the true wave characteristics of second sound. In the portion of a second sound cycle where heat flows toward cooler regions, mechanical energy is stored as energy of internal convection. During the opposite half of the cycle heat flows toward warmer regions at the expense of this stored energy. The resulting average mechanical energy content of the wave provides a net mechanical energy flow or *intensity* in the direction of propagation.

Considerations of second sound are simplified by assuming square wave pulses, logical justification lying in the known absence of frequency dispersion. This obviates the distinction between instantaneous and average values. If heat flow density $\dot{H}$ (cal./sec. cm²) is sustained by liquid helium II during the generation of a one-dimensional square wave heat pulse, the temperature is raised by $\tau$ within a heated region which is progressing at the rate of second sound velocity $v_2$. This requires[1–4] that

$$\dot{H} = \rho c_v \tau v_2, \tag{1}$$

where $\rho$ is the density and $c_v$ the specific heat capacity per gram for helium II. The generation of this second sound actually requires slightly greater heat input than the heating rate $\dot{H}$, to provide the kinetic energy of internal convection stored in the pulse. At the abrupt front of the temperature pulse heat flows continuously out toward the ambient temperature region, cooler by amount $\tau$. According to the second law of thermodynamics** the rate of mechanical energy generation, or intensity $\gamma$, is related to temperature by

$$\gamma/\dot{H} = \tau/T_0, \tag{2}$$

where $T_0$ is the ambient absolute temperature (°K). The intensity becomes[3, 5]

$$\gamma = \tau\dot{H}/T_0 = \rho c_v v_2 \tau^2/T_0. \tag{3}$$

This expression*** is equally significant with respect to classical heat flow within ordinary materials.

Finally, combining (1) and (2) the expression for total energy flow (thermal plus mechanical) becomes

$$\text{energy flow} = \rho c_v v_2 \tau [1 + \tau/T_0] \tag{4}$$

indicating the flow of mechanical energy within a second sound packet to be but the fraction $\tau/T_0$ of the associated heat flow.

Second sound possesses a wave momentum and a radiation pressure. At a chosen position and instant, whichever fluid component happens to be moving in the direction of propagation possesses greater than ambient density, whereas the other component is necessarily retrogressing at less. Accordingly therefore, in the identical manner as for classical sound, the wave momentum flow $J$ for the pulses equals the mechanical energy density divided by wave velocity, equal in turn to radiation pressure $P_{\text{rad}}$.

$$P_{\text{rad}} = J = \rho c_v \tau^2/T_0 \tag{5}$$

or twice (5) for reflection from a thermally non-conducting barrier.**** Although of second-order magnitude, this is the sole existent pressure for second sound! This should not be confused with the small amount of first sound coupled[6] to second sound by the thermal coefficient of expansion of helium II. (It should be noted that the method of deriving the expression automatically includes both kinetic and potential energy density; for continuous waves a factor ½ would appear). Thus under appropriate experimental conditions there would be a "sound current" associated with second sound (as with ordinary sound) of velocity $c_v\tau^2/v_2T_0$.

Thermal boundary conditions early employed by the author (for setting up "thermal impedances") in design of second sound systems were the continuity of temperature and heat flow density between liquid helium II and adjacent classical solids. Thus for second sound normally incident upon a classical barrier

$$\tau_i + \tau_r = \tau_{tr}, \quad \dot{H}_i - \dot{H}_r = \dot{H}_{tr}, \tag{6}$$

where the subscripts (*i*) and (*r*) refer, respectively, to incident and reflected second sound, (*tr*) to transmitted classical thermal waves.

Employing (1) we obtain from (6)

$$\frac{\tau_i \dot{H}_i}{T_0} - \frac{\tau_r \dot{H}_r}{T_0} = \frac{\tau_{tr} \dot{H}_{tr}}{T_0}, \tag{7}$$

where complex conjugates are not used since instantaneous values are desired. Equations (1) and (6) can also be employed to show that for most cases not all of the incident second sound energy is reflected; thermal impedance[4] $\tau/\dot{H}$ equals $[\rho c_v v_2]^{-1}$ for helium II [see (1)] and is entirely real, and may for solid boundaries possess a real component as large as the imaginary component. The essence of (7) is therefore that the temperature wave† entering the solid possesses an *intensity* $\tau\dot{H}/T_0$ in the same manner as does second sound. (This is likewise required for the converse case of second sound waves in helium II being set up by thermal waves emerging from an adjacent solid.) But the well-known *expression for classical thermal waves involves rapid damping, so that the associated energy thereby lost reappears in other forms.*

Certain speculations may be made in this regard. The "kinetic energy" density of the thermal pulse may tentatively be regarded as associated with the mass flow inherent to thermal conduction. Correspondingly "potential energy" density should be related to pressure alterations accompanying thermal flow. The second law of thermodynamics as given by (2) thus holds for thermal conduction across a layer of classical material of thickness less than the thermal wave-length. Analysis based on the thermal impedance concept[4] reveals that second sound pulses should be transmitted without distortion†† or appreciable loss through layers of classical material of such thinness immersed in liquid helium II, thus conforming to conditions of complete reversibility. Arguments similar to those employed by Rayleigh[7] for a classical sound pulse indicate that the mechanical energy flow $\tau\dot{H}/T_0$ involved is shared equally between kinetic and potential forms, not only for second sound but also for classical thermal waves.

When penetration of thermal waves into classical materials exceeds several mean-free-path lengths, the reversibility between heat flow and mechanical energy stated by (2) is necessarily affected by the natural collisions occurring between particles or between phonons. Accordingly the well-known damping sets in and converts the "mechanical energy" content of the thermal wave to acoustical or thermal forms, depending upon the geometry and the substances (thermal coefficient of expansion) involved.

For propagation along the $x$ axis the "source strength" of such converted energy is just $Re(\partial/\partial x)(\tau \dot{H}/T_0)$.

Thus, whereas liquid helium II provides the only example of complete thermal and mechanical interdependence, the more limited thermomechanical properties of ordinary substances are evident from general thermodynamic considerations. Further implications are being investigated.

The author expresses his sincere gratitude to Professor Philip M. Morse for helpful and stimulating discussions.

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* Since this study was undertaken, related work by several other investigators has appeared. Where overlap does now occur, the independently obtained results of the author are believed to present fresh viewpoints.

** A particularly clear derivation of the velocity of second sound has been given by Peshkov (see reference 3) based on the use of the second law.

*** In reference 5 the author gave the equivalent expression in mechanical terms. The thermodynamic properties of liquid helium II lead to a direct conversion between the two forms.

**** We are indebted to H. W. Woolley for an independent derivation of a form equivalent to (5).

† Peshkov mentions in reference 3 that the expression applies to materials other than helium II but without interpretation or consideration of the inherent decay.

†† The thermal impedance of a classical layer of thickness $l$ backed by helium II is $(i\omega\rho_c c_c \kappa_c)^{-\frac{1}{2}}\coth[(i\omega\rho_c c_c/\kappa_c)^{\frac{1}{2}}l+\coth^{-1}(i\omega\rho_c c_c/\kappa_c)^{\frac{1}{2}}/\rho c_v v_2]$, where $\rho_c$, $c_c$, and $\kappa_c$ are, respectively, the density, heat capacity, and heat conductivity of the material. For a thin layer this approaches $(\rho c_v v_2)^{-1}+a/\kappa$, independent of frequency and nearly equal to the thermal impedance of helium II.

[1] D. Osborne, Nature **162**, 213 (1948).

[2] R. Dingle, Proc. Phys. Soc. London **61**, 9 (1948).

[3] V. Peshkov, J. Exp. Theor. Phys., U.S.S.R. **18**, 857 (1948).

[4] J. Pellam, Phys. Rev. **75**, 1183 (1949).

[5] J. Pellam, Phys. Rev. **73**, 608 (1948).

[6] L. Tisza, Phys. Rev. **72**, 838 (1947).

[7] J. Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1945).

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## Nuclear Spin 5/2 for $_{40}Zr^{91}$

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IN interference spectrograms** of a $Zr^{91}$-enriched sample of zirconium*** in a hollow cathode source,[1] the $4d^2 5s^2\,{}^3P_0 - 4d^2 5s({}^2P)5p\,{}^3S_1$ line[2] at $\nu_{17777}$ presents the following structure, showing that the spin, $I$, of the interesting[3] nucleus $_{40}Zr^{91}$ is undoubtedly 5/2 units:

| | | |
|---|---|---|
| $(I+1)$ | | $-0.0701\pm0.0010\ \mathrm{cm}^{-1}$,[4] |
| even isotopes | (arbitrary) | $0.0000\pm0.0012\ \mathrm{cm}^{-1}$, |
| $(I)$ | | $+0.0273\pm0.0010\ \mathrm{cm}^{-1}$, |
| $(I-1)$ | | $+0.0983\pm0.0010\ \mathrm{cm}^{-1}$, |

$I$, from the Landé interval rule $2.69\begin{matrix}+0.58\\-0.41.\end{matrix}$

Details, including the structure of other lines, will be submitted later.

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* Work done on Navy contract N7 onr-285TO #1, NR 019 107.

** With auxiliary dispersion produced by a special Hilger spectrograph bought with funds granted by the University research committee.

*** Produced by the Y-12 plant, Carbide and Carbon Chemicals Corporation, and obtained by allocation from the AEC.

[1] O. H. Arroe and J. E. Mack, Phys. Rev. **76**, 173 (1949).

[2] W. F. Meggers and C. C. Kiess, J. Research Nat. Bur. Stand. **9**, 309 (1932).

[3] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949), especially pp. 1882 and 1893.

[4] The quantities preceded by the symbol ± have been made larger than the spread of the data by amounts judged to be enough to make it highly improbable that the true values lie outside the ranges indicated.

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## The Binding Energy of the Triton

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August 8, 1949

A VARIATIONAL calculation of the binding energy of the $H^3$ nucleus has been performed using the Rarita-Schwinger[1] interaction (square well with tensor force), as modified to include a charge-exchange factor $(-\frac{1}{3}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$. Except for the charge-exchange factor, which has very small effect on the triton binding energy, this is the same problem as that studied by Gerjuoy and Schwinger[2] and by Feshbach and Rarita.[3] The same variational method was used here, but a much more elaborate trial function was inserted. The calculations are still in progress, but the results so far obtained (and reported in Table I) show that other $D$-states not included in the previous calculations[2,3] make a substantial contribution to the binding energy.

TABLE I. Computed triton binding energy.

| Trial functions (see text) <br> $S$-state | <br> $D$-states | Triton binding energy <br> In Mev | <br> In percent of 8.3 Mev |
|---|---|---|---|
| (Finite power series) | | | |
| I | — | 1.80 | 21.7 |
| I | II | 3.74 | 45.1 |
| I | II, III, IV | 5.60 | 67.5 |
| (Extrapolated) | | | |
| I | — | 1.80 | 21.7 |
| I | II | 3.88 | 46.7 |
| I | II, III, IV | 5.80 | 69.9 |
| (Gerjuoy and Schwinger; $S$-state and $D$-state | | | 39 |
| (Feshbach and Rarita) | | | |
| $S$-state alone | | | 21 |
| $S$-state and $D$-state (estimated maximum) | | | 40–50 |

The basis for the present method is the construction of a denumerable series of orthogonal functions which form a complete set for the nuclear three-body problem. With the type of potential that was assumed, the triton ground state can include only a selected group of these functions, characterized by even parity, by $J=\frac{1}{2}$ for the total angular momentum, by $T=\frac{1}{2}$ for the total isotopic spin, and by $T_3=-\frac{1}{2}$ for the third component of isotopic spin (to denote two neutrons and one proton). This sub-set of functions may then be arranged in order of increasing kinetic energy (which is roughly equivalent to the order of decreasing importance) and introduced systematically into the variational principle.

In setting up the system of orthogonal functions the usual relative coordinates, $\mathbf{r}:(r,\theta,\varphi)$ and $\boldsymbol{\rho}:(\rho,\eta,\psi)$, have been transformed to a six-dimensional spherical coordinate system, with a single radial variable,

$$R=(\tfrac{2}{3}r^2+\tfrac{1}{2}\rho^2)^{\frac{1}{2}} \tag{1}$$

and five angular variables. The angular parts of the orthogonal functions are hyperspherical harmonics $Y_{K,L}$ which satisfy the defining equation:

$$\nabla^2(R^K\cdot Y_{K,L})=0 \tag{2}$$

with

$$\nabla^2\equiv\tfrac{3}{2}\nabla_r^2+2\nabla_\rho^2. \tag{3}$$

These harmonics can be classified by their value of $L$ into $S$, $P$, $D$, etc., states, and combined with suitable spin and isotopic spin functions to give the admissible set of ${}^2S_{\frac{1}{2}}$, ${}^2P_{\frac{1}{2}}$, ${}^4P_{\frac{1}{2}}$, and ${}^4D_{\frac{1}{2}}$ triton functions. With each harmonic $Y_{K,L}$ there will also be a radial factor $R^K\cdot f(R)$, where the function $f(R)$ need be defined only for $R>0$ and may therefore be written as a function of $R^2$. The form chosen was a Gaussian multiplied by a power series:

$$f(R)=e^{-\mu R^2}(a+bR^2+cR^4+\cdots). \tag{4}$$

In Table I are listed the binding energies resulting from the successive inclusion of one ${}^2S$-state function (I) and three ${}^4D$-state functions (II, III, IV). The $S$-state is essentially the same as that used by Gerjuoy and Schwinger and by Feshbach and Rarita, and function II corresponds to their $D$-state, but functions III and IV contain $D$-states which they omitted. The binding energy is given in Mev and in percent of the experimental value which they used, 8.3 Mev.**

In the present calculations, the same Gaussian parameter $\mu$ was used for all of the four functions. This does no harm if each power series is sufficiently long. The power series for functions I,